<table>
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<tr>
<td>Author(s)</td>
<td>Ishikawa, Motoo</td>
</tr>
<tr>
<td>Citation</td>
<td>Kyoto University (京都大学)</td>
</tr>
<tr>
<td>Issue Date</td>
<td>1978-07-24</td>
</tr>
<tr>
<td>URL</td>
<td><a href="https://doi.org/10.14989/doctor.k2086">https://doi.org/10.14989/doctor.k2086</a></td>
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<td>Type</td>
<td>Thesis or Dissertation</td>
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Kyoto University
STUDY ON PERFORMANCE CHARACTERISTICS

OF

DIAGONAL TYPE NONEQUILIBRIUM PLASMA MHD GENERATOR

BY

MOTO0 ISHIKAWA

FEBRUARY 1978

INSTITUTE OF ATOMIC ENERGY
KYOTO UNIVERSITY
JAPAN
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OF

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The electrical energy used not only in Japan but also in the whole world shows a remarkable yearly increase. However, the overall thermal efficiency of a present large scale steam power plant remains at about 40 percent and that of an atomic power plant is about 30 percent. Large thermal pollution with air contamination and efficient usage of fuels will be supposed to become serious problems unless the efficiency is improved. Thus, it is a pressing need to develop a method of energy conversion with high efficiency and low pollution.

An MHD (magnetohydrodynamic) -steam binary power plant is expected to have a potential to increase its overall thermal efficiency as large as to 50 or 60 percent. Moreover, the MHD generation will have the largest capacity among various methods giving high conversion efficiency.

In Japan, experimental researches of several open-cycle MHD power generators with an alkali-metal seeded combustion gas of fossil fuel have been made in the Electrotechnical Laboratory as a National Project since 1966 [1.1]. In foreign countries, experimental investigations of the open-cycle MHD generator have been made by Dicks et al. of UTSI, Petty et al. of AVCO [1.3] in USA, and by Sheindlin and Biturin et al. of High Temperature Institute in USSR [1.4]. In addition, numerous theoretical studies have been performed by Hurwitz [1.5], Dzung [1.6], Celinski [1.7], Oliver [1.8], Argyropoulos [1.9], et al. With these research and development, it is now expected that the open-cycle MHD-steam power plant will be operated commercially in the latter half of the 1980's.

On the other hand, for closed-cycle MHD power generation, in which nonequilibrium plasma of an alkali-metal seeded rare gas is
utilized as the working fluid, there have been made fundamental experiments and theoretical analyses, and such as a fossil fuel fired heat exchanger \[1.10\], a future nuclear fusion reactor, and a solar furnace are thought as heat sources. A low temperature working gas can be used in the closed-cycle MHD which will provide higher energy conversion efficiency and power density compared with those of the open-cycle MHD generator.

Theoretical studies on the closed-cycle MHD generator have been done by Velikhov [1.11], Solbes [1.12], Yoshikawa [1.13], Tanaka [1.14], and many other workers, and experimental studies by Bertolini [1.15], Brederlow [1.16], Zauderer [1.17], Nakamura [1.18], Rietjens [1.19], Shioda [1.20], et al. They have shown that the closed-cycle MHD power generation has potentiality of increasing the conversion efficiency, but the effective Hall parameter will be difficult to exceed 2 or 3 under the existence of ionization instabilities unless fully ionization of seed atoms is fulfilled.

As a working gas for the closed-cycle MHD power generation, alkali-metal seeded argon or helium is usually utilized. In this thesis, cesium-seeded helium is chosen as a working fluid.

So long as linear generator types are concerned, three types have been proposed so far. Although a Faraday type generator will reveal the best generator performance, it requires many DC-AC inverters corresponding to each output electrode pairs. Hence, the plant system containing a Faraday type generator will be complicated due to many number of inverters.

By contrast, a Hall type generator can be operated with only a single load in principle, but needs high Hall parameter to give good performance [1.21]. It will hence show fairly poor performance characteristics if the ionization instability is not suppressed.

As an intermediate of the above two types, a diagonal type generator is thought to be possessed of good characteristics even
for low Hall parameters [1.21], and it can be operated with a single load or a few, and hence requires a few DC-AC inverters. A plant system containing the diagonal type generator will be more simplified with respect to operational control than that containing a Faraday type generator. In addition, its performance characteristics are expected to be as close as to those of a Faraday type generator.

Consequently, diagonal type generators with nonequilibrium plasma will be promising for commercial scale generators, nevertheless they have not been investigated so far, although diagonal type combustion gas MHD generators have been well studies by Dicks [1.2], Petty [1.3], Dishchikov [1.22], et al.

The purpose of this thesis is to investigate theoretically the performance characteristics of a diagonal type MHD generator with a nonequilibrium plasma of cesium-seeded helium, and the thesis consists of two parts.

In part I, an optimization of a large scale diagonal type MHD generator is investigated. This part consists of the following three chapters.

In Chapter 1, studies of optimizations of MHD generators made until now are briefly introduced. Also, the purpose and significance of the investigation are stated.

In Chapter 2, quasi one-dimensional basic equations for the working gas flow are introduced in a diagonal and a Faraday type generators, in which the friction and heat losses, electrode voltage drops, and finite segmentation effects of electrodes are considered, and it is assumed that the end effects can be neglected. In addition, the relaxation of ionization is neglected, because it affects only electrical characteristics in the generator inlet region [1.15]. As an estimation function for the optimization of the generators, total thermal efficiency is adopted. Beside, the isentropic efficiency, output power density, etc. of the generator,
some constraints for the optimization, and expressions of the electrical conductivity and Hall parameter of the working gas are derived.

In Chapter 3, by the optimization theory derived in the preceding chapter, numerical calculation is made for the typical closed-cycle diagonal type MHD-steam power plant with about 2000 MW of thermal input. Influences of the Mach number, inclination parameter, duct length, inlet pressure and temperature of a diagonal type generator on the total thermal efficiency of the plant are investigated in detail. As the results the optimum duct with optimum Mach number, inclination parameter, and duct length giving the maximum efficiency is found out. In the similar way, the optimization of a Faraday type generator is further studied, and a comparative study of the generator performances of both types is made.

In part II, the electrical characteristics of a diagonal type nonequilibrium plasma MHD generator are investigated by two-dimensional analyses. Part II consists of the following six chapters.

In Chapter 1, two- or three-dimensional analyses of a Faraday, Hall, and diagonal type MHD generators carried out until now are introduced briefly.

In Chapter 2, basic equations, which are to be used in the analyses, are introduced. By use of the stream and potential functions, two-dimensional partial differential equations are derived from the basic equations together with boundary and subsidiary conditions required for a diagonal type generator. Expressions of the electrical efficiency and the specific Hall voltage are shown.

In Chapter 3, basic electrical characteristics of a generator duct are investigated by two-dimensional analyses for spatially constant and variable electrical conductivity and Hall parameters. Influences of the load factor, the Hall parameter, and the inclination parameter of sidebars on the current and potential distributions and the electrical efficiency are then investigated.
In Chapter 4, effects of a resistive electrode and of a distribution of the applied magnetic flux on the electrical characteristics of a generator are studied by two-dimensional analyses. Influences of the conductivity and configuration of wedge-shaped resistive electrodes on the current concentration at the electrode, electrical efficiency, etc. are studied. And also the electrical characteristics of a generator with the spatially attenuating magnetic flux in the region near the duct wall are compared with those of the generator with the spatially constant magnetic flux.

In Chapter 5, end effects are investigated by two-dimensional analyses. Influences of the attenuation of the magnetic flux, and position of the output electrodes of various types on the current distribution and the internal resistance are studied in detail for two cases, i.e., spatially constant and spatially variable electrical conductivity and electron mobility.

In Chapter 6, effects of internal or external connection of electrodes in a diagonal type generator are investigated by two-dimensional analyses in the plane perpendicular to the plasma flow. Current and potential distributions for two cases are then compared from view point of electrical efficiency for various parameters of the load factor, duct wall temperature, and inclination parameter.
ACKNOWLEDGMENTS

This doctoral thesis has been studied under the guidance of Professor Dr. Juro Umoto of Kyoto University. The author wishes to express his highest gratitude for his guidance and continuous encouragements enabling the author to complete the work. Professor J. Umoto also strictly read the manuscript of the thesis and gave accurate comments which were sincerely appreciated.

The author would like to thank Professor Yoshio Hattori of the Institute of Atomic Energy, Kyoto University for valuable discussions.

The author also wishes to express his thanks to Associate Professor Takehisa Hara and Assistant Masaharu Yoshida of Professor J. Umoto's Laboratory, and Associate Professor Kiyoshi Yoshikawa, Mr. Daijiro Tanaka, and Mr. Hisayuki Toku of the Institute of Atomic Energy for discussions. Mr. Jun Shimada, Mr. Masayuki Saito, and Mr. Ryutaro Himeno helped with the calculations.

Preparation of this thesis has been helped by Miss Etsuko Morisaki in typewriting. Lastly, the author wishes to thank to Mrs. Machiko Ishikawa for her encouragement.
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NOMENCLATURE

Latin symbols

\( A \)  
- cross-section of duct (Part I)

\( A \)  
- square matrix (APPENDIX III)

\( A_0, A_1 \)  
- cross-section of duct inlet and outlet, respectively

\( a \)  
- inclination parameter of electrodes

\( a_{\text{opt}} \)  
- optimum value of \( a \)

\( B \)  
- applied magnetic induction vector

\( B \)  
- \( z \)-component of \( B \)

\( B_w \)  
- magnetic flux density at duct wall

\( B_0 \)  
- magnetic flux density at center of duct

\( b \)  
- known column vector

\( C \)  
- specific heat at constant gas pressure

\( c \)  
- electrode width

\( c_f \)  
- friction factor

\( c_{\xi}, c_{\zeta} \)  
- coefficient defined by Eqs. (1.42) to (1.44)

\( D \)  
- hydraulic diameter of generator duct (Part I)

\( D \)  
- diagonal matrix (APPENDIX III)

\( E \)  
- electric field intensity vector

\( E' \)  
- electric field intensity vector in resistive electrode

\( E \)  
- lower triangular matrix

\( E_{x}, E_{y}, E_{z} \)  
- \( x, y, \) and \( z \) component of \( E \)

\( E'_{x}, E'_{y} \)  
- \( x \) and \( y \) component of \( E' \)

\( (E_{x})_{\text{open}} \)  
- \( x \) component of \( E \) for open circuit condition

\( e \)  
- electric charge of electron

\( F \)  
- upper triangular matrix

\( f \)  
- friction loss on duct wall

\( g \)  
- gradient of \( B \) in end region

\( h \)  
- duct height

\( h' \)  
- Planck's constant

\( I \)  
- load current of generator

\( I' \)  
- current defined by Eq. (2.94)
current defined by Eq.(2.69)
I_x x-component of I
I_y y-component of I
I_n current flowing into anode
I_y(n) current flowing into anode A_n
i mesh number in the x direction
i_z ratio of the current flowing into one conducting sidewall to I'
J current density vector
J' current density vector in resistive electrode
J_n current density perpendicular to cross-section C'A' in Fig.2.1
J_peak maximum current density on electrodes
(Part II, Chapter 4)
J_x, J_y, J_z x, y, and z component of J
J'_x, J'_y x and y component of J'
ave average current density on electrodes
(Part II, Chapter 4)
or on output electrodes (Part II, Chapter 5)
j mesh number in the y direction (Part II, Eq.2.44)
or electrode number where B begins decreasing in end region (Part II, Chapter 5)
k Boltzmann's constant
k_s equivalent sand roughness
l generator duct length
L_opt optimum value of L
M_opt optimum value of M_0
M_0 Mach number of the working fluid
m, m' constant in Eqs.(2.8) and (2.9)
electron mass
m_i mass of ion
m_s mass of seed atom
m_0 mass flow rate (Part I)
m_0 mass of parent gas atom (Part II)
\( n, n' \) constant in Eqs. (2.86) and (2.87)

\( n_e \) number density of electron

\( n'_e \) normalized electron density

\( n_{es} \) electron density obtained from Saha's equation

\( n_s \) number density of seed atom

\( n_{s0} \) original number density of seed atom before ionization

\( n_0 \) number density of parent gas atom

\( P_{com} \) driving power of compressor

\( P_{MHD} \) DC output of MHD generator

\( P_{st} \) AC output of steam generator

\( P_t \) total AC output of MHD-steam plant

\( P_w \) power density of generator

\( P \) static gas pressure

\( P_{com} \) pressure at compressor entrance

\( p_e \) partial pressure of electron

\( \rho \) Prandtl number

\( P_s \) stagnation pressure

\( P_{s0}, P_{sl} \) stagnation pressure at duct inlet and outlet

\( P_1 \) static gas pressure at duct outlet

\( Q_{ei} \) collision cross-section between electrons and ions

\( Q_H \) specific Hall voltage

\( Q_{in} \) thermal input to MHD-steam plant

\( Q_{is} \) collision cross-section between ions and seed atoms

\( Q_{i0} \) collision cross-section between ions and parent gas atoms

\( Q_{loss} \) heat loss of the generator

\( Q_s \) collision cross section between electrons and seed atoms

\( Q_{st-in} \) thermal input to steam cycle

\( Q_0 \) collision cross-section between electrons and parent gas atoms

\( q \) heat loss on duct wall

\( R \) gas constant of working fluid

\( R_b \) ballast resistance
$R_{b0}$  
$R_b$ for $g=0$

$R_{i}$  
internal resistance of end region

$R_{i0}$  
$R_i$ for $g=0$

$R_i'$  
resistance between $A_i$ and $C_i$ in Fig.2.34

$r$  
recovery factor

$s_i$  
arbitrary surface crossing the insulating wall
surfaces $A_i'$ and $C_i'$ in Fig.2.34

$s$  
electrode pitch

$t$  
static gas temperature

t$_{com}$  
temperature at compressor entrance

t$_e$  
electron temperature

t$_e'$  
normalized electron temperature

t$_{e0}$  
optimum value of $t_e$

t$_{e0}'$  
electron temperature at duct inlet and outlet

t$_s$  
stagnation temperature

$t_{s0}$  
stagnation temperature at duct inlet and outlet

t$_w$  
duct wall temperature

t$_0$  
gas temperature at the center of duct

t$_6$  
temperature given in Fig.1.2(a)

t$_8$  
temperature at exit of compressor

$u$  
unit matrix

$u$  
gas velocity vector

$u_0$  
x component of gas velocity

gas velocity at the center of duct

$v$  
duct volume (Part I)

$v$  
potential difference between output electrode pair
and $n$-th electrode in Fig.2.34 (Part II)

$v_{H}$  
potential difference per one electrode pitch

$v_i$  
potential difference between $A_i$ and $C_i$ in Fig.2.34

$v_i'$  
ionized voltage of seed atom

$v_{ideal}$  
ideal potential difference per one electrode pitch

$v_{sc}$  
quantity approximately proportional to the required
quantity of the super-conducting wire

$v_x'$  
potential difference per one electrode pitch
\( v_y \) potential difference between anode and cathode

\( V_0 \) no-load potential difference between the output electrode pair and n-th electrode in Fig.2.34

\( v \) volume of hexahedron

\( v_d \) ratio of electrode voltage drop to \( v_y \)

\( w \) width of the duct in the z direction

\( x \) \((y-1)M_0^2/2\)

\( x_i \) unknown column vector

\( x_1 \) defined by Eq.\((2.44)\)

\( x_1 \) x co-ordinate of the right edge of the insulating wall \( C_n \) in Fig.2.34

\( y_j \) defined by Eq.\((2.44)\)

\( z \) stage number of compressors

\( x, y, z \) space co-ordinates

Greek symbols

\( \alpha \) \( \gamma/(\gamma-1) \) (Part I)

\( \alpha \) angle between \( J \) and the direction normal to the boundary surface in Fig.2.21 (Part II)

\( \alpha' \) angle between \( J' \) and the boundary surface in Fig.2.21

\( \beta \) local Hall parameter for electrons

\( \beta_{\text{crit}} \) critical Hall parameter

\( \beta_{\text{eff}} \) effective Hall parameter

\( \beta_i \) local Hall parameter for ions

\( \gamma \) specific heat ratio

\( \Delta \) dimensionless voltage drop (Part I)

\( \Delta \) width of mesh in difference scheme (Part II)

\( \delta \) collision loss factor

\( \delta_B \) shown in Fig.2.28

\( \epsilon \) \( 1 + \beta \beta_i \)

\( \epsilon_s \) seed fraction

\( \epsilon_0 \) permittivity of free space

\( \zeta \) new variable derived from \( T_s \) (Part I)
or from y (Part II)

\( \eta \)  
total thermal efficiency of MHD-steam plant

\( \eta_b \)  
boiler efficiency

\( \eta_{com} \)  
compressor efficiency

\( \eta_{DA} \)  
conversion efficiency of a DC-AC inverter

\( \eta_d \)  
diffuser efficiency

\( \eta_e \)  
electrical efficiency

\( \eta_h \)  
heat exchanger efficiency

\( \eta_i \)  
isentropic efficiency of generator duct

\( \eta_{MHD} \)  
enthalpy extraction from generator duct

\( \eta_{re} \)  
recovery efficiency of heat loss of duct

\( \eta_{st} \)  
efficiency of steam cycle

\( \theta \)  
angle of inclination to the y axis of straight lines joining equipotential electrodes

\( \theta' \)  
angle between a total average current density \( J \) and x-axis

\( \kappa \)  
load factor of diagonal type generator

\( \kappa_f \)  
load factor of Faraday type generator

\( \Lambda \)  
given by Eq.(2.13)

\( \lambda \)  
parameter of finite segmentation effect of electrodes

\( \nu \)  
viscosity of working gas

\( \nu_e \)  
electron mobility

\( \nu_{es}, \nu_{e0}, \nu_{ei} \)  
average electron momentum-transfer collision frequency

\( \nu_{es}, \nu_{e0}, \nu_{ei} \)  
coefficient defined by Eq.(2.6)

\( \nu_r \)  
coefficient defined by Eq.(2.2)

\( \xi \)  
new variable derived from \( p_s \) (Part I)

or from x (Part II)

\( \rho \)  
mass density of working fluid

\( \sigma \)  
local electrical conductivity

\( \sigma_e \)  
electrical conductivity of resistive electrode

\( \sigma_{eff} \)  
effective electrical conductivity of gas

\( \sigma', \sigma'' \)  
given by Eq.(2.19)

\( <\sigma>_i \)  
average electrical conductivity of plasma between \( A_i \) and \( C_i \) in Fig.2.34
\( \phi \) potential function

\( \Psi \) stream function

\( \psi \) normalized stream function

\( \psi_{ij} \) value of \( \psi \) at the point \((x_i, y_j)\)

\( \Omega \) relaxation parameter matrix

\( \omega, \omega^*, \omega_0 \) relaxation parameter

\( \omega', \omega'' \) constants on relaxation parameter

\(< >\) shows average value per one electrode pitch
PART I

OPTIMIZATION OF DIAGONAL TYPE
NONEQUILIBRIUM PLASMA MHD GENERATOR
CHAPTER 1
INTRODUCTION

In designing an MHD generator duct, it is necessary that the generator duct is constructed in an optimum form. Hitherto, optimizations of the open-cycle diagonal type generator, in which an alkali-metal seeded combustion gas of fossil fuel is used, have been studied theoretically by Takano [1.23], Convey [1.24], Bobbio [1.25], Gorb [1.26], Yoshida [1.27], et al., and that of the closed-cycle Faraday type generator, in which alkali-metal seeded rare gas in nonequilibrium ionization is utilized, by Zauderer [1.28], Shioda [1.29], et al. On the other hand, optimization of the closed-cycle diagonal type generator has not been studied by anyone except the author [1.30]-[1.35].

In this part, an optimization of a large scale diagonal type MHD generator with nonequilibrium plasma of cesium-seeded rare gas is investigated. The generator is optimized so as to maximize the total thermal efficiency of an MHD-steam power plant by varying separately some parameters such as the Mach number, diagonal angle, duct length, inlet stagnation pressure or temperature, etc.

For grasping exactly performance characteristics of an MHD generator, a two- or three-dimensional analysis is required. However, by the usual two- or three-dimensional analysis, we can analyze only a local characteristics of the large scale generator. In this part, since the whole performance of the generator is studied the working fluid is assumed to be a quasi one-dimensional gas.

In Chapter 2, there are introduced quasi one-dimensional basic equations for the working gas flow in the diagonal and Faraday type generators, in which the friction and heat losses along the duct, the electrode voltage drop, the finite segmentation effect of electrodes, etc. are considered and it is assumed that the end effects of a generator and the relaxation of ionization can be neglected.

Next, as an estimation function for the optimization of the
generators, the total thermal efficiency is adopted. The isentropic efficiency, output power density, etc. of the generator, some constraints for the optimization, etc. are derived.

In Chapter 3, by the optimization theory derived in Chapter 2, numerical calculation is made for an example of the closed-cycle diagonal type MHD-steam power plant of the thermal input of about 2000 MW. Influences of the Mach number, inclination parameter, duct length, inlet pressure and temperature, etc. of the diagonal type generator on the total thermal efficiency of the plant are investigated in detail. As the results, an optimum duct of optimum Mach number, inclination parameter, duct length, etc., which maximize the efficiency, is found out. Next, in the similar way, the optimization of the Faraday type generator is studied, and a comparative study of the performances of both type generators is made.
2.1 Introduction

In this chapter, first, there are introduced quasi one-dimensional basic equations for the working gas flow in the diagonal and Faraday type generators, in which friction and heat losses along the generator duct, the electrode voltage drop, the finite segmentation effect of electrode, etc. are considered. Next, as an estimation function for the optimization of the generators, the total thermal efficiency is adopted. Besides, the isentropic efficiency, output power density, etc. of the generator, some constraints for the optimization are derived. The expressions of the electrical conductivity and the Hall parameter are derived, where ionization instabilities are considered.

Finally, for the constant Mach number type generators, the numerically solvable differential equations for the stagnation pressure and temperature are derived from the above basic equations.

2.2 Basic Equations in MHD Generator Duct

The MHD flow in a diagonal type generator duct [see Fig.1.1] is described by the following set of quasi one-dimensional basic equations, namely the continuity equation

\[ \rho u A = m_0, \quad (1.1) \]

the momentum equation

\[ \rho u \frac{du}{dx} + \frac{dp}{dx} = J_y B - f, \quad (1.2) \]

the energy equation

\[ \rho u \frac{d}{dx} \left( \frac{c_p T}{2} + \frac{u^2}{2} \right) = E_x J_x + E_y J_y - q, \quad (1.3) \]

the state equation

\[ p = \rho RT, \quad (1.4) \]
the current continuity equation

\[(J_x + aJ_y)A = I, \quad a = \tan \theta, \quad (1.5)**\]

and the generalized Ohm's law

\[
\begin{align*}
J_x &= \frac{\sigma_{\text{eff}}}{1 + \beta_{\text{eff}}^2} \left[ E_x + \beta_{\text{eff}} \{ uB(1-\Delta) - E_y \} \right], \\
J_y &= \frac{\sigma_{\text{eff}}}{1 + \beta_{\text{eff}}^2} \left[ \{ E_y - uB(1-\Delta) \} + \beta_{\text{eff}} E_x \right],
\end{align*}
\]

where

\[E_y = aE_x. \quad (1.7)**\]

In these equations and Fig.1.1, \(A\) is the duct cross section, \(a\) the electrode inclination parameter*, \(B\) the applied magnetic flux density*, \(C = \rho R\) the specific heat at constant pressure, \(E_x\) and \(E_y\) the x- and y-components of the electric field intensity, respectively, \(f\) the friction loss on the duct wall, \(I\) the load current*, \(J_x\) and \(J_y\) the x- and y-components of the current density, respectively, \(m_0\) the mass flow rate*, \(p\) the gas pressure, \(q\) the heat transfer loss on the duct wall, \(R\) the gas constant*, \(T\) the gas temperature, \(u\) the gas velocity, \(a = \gamma/(\gamma-1)*\), \(\beta_{\text{eff}}\) the effective Hall parameter, \(\gamma\) the specific heat ratio*, \(\sigma_{\text{eff}}\) the effective electrical conductivity, \(\Delta\) the dimensionless voltage drop*, and \(\theta\) the angle* of inclination to the y axis of the straight lines joining the equipotential electrodes. The duct cross section is treated as square.

2.3 Estimation Function and Constraints for Optimization

2.3.1 Estimation function

A total thermal efficiency \(\eta\) of an MHD-steam plant is adopted

* Quantities which are assumed constant when the basic equations are solved.

** See Appendix I.
as an estimation function of the generator performance. For comparison between the diagonal and Faraday type generators, the quantity \( V_{sc}' \) is evaluated, which is approximately proportional to a required volume of the super-conducting wire [1.36]. In addition, the isentropic efficiency \( n_i \) and the output power density \( P_w \) of the generator and the enthalpy extraction \( n_{MHD} \) from the generator are also calculated.

Figures 1.2(a) and (b) show the block diagram and rough T-S diagram, respectively, of the MHD-steam plant for the analysis in this thesis.

In the plant, \( n, n_i, n_{MHD}, P_w, \) and \( V_{sc}' \) are given by

\[
\begin{align*}
\eta &= \frac{P_t}{Q_{in}}, \\
P_t &= \eta_{DA} P_{MHD} + P_{st} - P_{com}, \\
Q_{in} &= m_0 C_p (T_s 0 - T_g), \\
n_i &= \frac{P_{MHD}}{m_0 C_p T_s 0 \{1 - (P_{sl}/P_{s0})^{1/\alpha}\}}, \\
n_{MHD} &= \frac{P_{MHD}}{m_0 C_p T_s 0}, \\
P_w &= \frac{P_{MHD}}{V},
\end{align*}
\]

(1.8) (1.9) (1.10) (1.11)

Fig. 1.1. Schema of a diagonal type MHD generator duct.
\[ V'_{sc} = \sqrt{A_1} (1.8\sqrt{A_1} + L), \quad (1.12) \]

respectively, where \( P_t \) is the total AC output, \( P_{\text{com}} \) the driving power of the compressor, \( P_{\text{MHD}} \) the DC output of the MHD generator, \( P_{st} \) the output of the steam generator, \( Q_{\text{in}} \) the thermal input, \( V \) the duct volume, \( p_{s0} \) and \( p_{s1} \) the stagnation pressure at the duct inlet.

---

**Fig. 1.2.** (a) Block diagram of MHD-steam plant. (b) T-S diagram.
and exit, respectively, \(T_{s0}\) the stagnation temperature at the duct inlet, \(T_8\) the temperature at the compressor exit, \(A_1\) the cross section of the duct exit, \(L\) the duct length and \(\eta_{DA}\) the inversion efficiency of DC-AC inverters.

Next, since it has been ascertained that Eq.(1.12) is available for \(B=8\) to 10 T, too. And \(P_{\text{MHD}}, P_{\text{st}}, P_{\text{com}}, T_8,\) and \(V\) are given by

\[
P_{\text{MHD}} = -\int_0^L (E_x J_x + E_y J_y) A dx
\]

\[
= \frac{IB(1-\Delta)}{1+a^2} \int_0^L \kappa u (\beta_{\text{eff}} - a) dx,
\]

(1.13)

\[
P_{\text{st}} = Q_{\text{st-in}} \eta_{\text{st}},
\]

(1.14)

\[
P_{\text{com}} = Z m_0 C_T \left\{ \left( \frac{p_{s0}}{p_{\text{com}}} \right)^{1/\alpha} - 1 \right\} / \eta_{\text{com}},
\]

(1.15)

\[
T_8 = T_{\text{com}} \left( \frac{p_{s0}}{p_{\text{com}}} \right)^{1/\alpha},
\]

(1.16)

\[
V = \int_0^L A dx,
\]

(1.17)

where \(Q_{\text{st-in}}\) is the thermal input to the steam cycle, \(T_{\text{com}}\) and \(p_{\text{com}}\) the temperature and the pressure at compressor inlet, \(Z\) the stage number of compressors, \(\kappa\) the load factor defined by Eq.(1.41) and \(\eta_{\text{com}}\) and \(\eta_{\text{st}}\) the efficiencies of the compressor and the steam cycle, respectively. Here, \(Q_{\text{st-in}}\) and \(p_{\text{com}}\) are given by

\[
Q_{\text{st-in}} = Q_{\text{loss}} \eta_{\text{re}} + m_0 C_p (T_d - T_b) \eta_b
\]

\[
+ m_0 C_p (T_b - T_s) \eta_h,
\]

(1.18)

\[
p_{\text{com}} = p_1 + (p_{s1} - p_1) \eta_d,
\]

(1.19)

where \(Q_{\text{loss}} = \int_0^L Q A dx\) is the heat loss of the generator, \(T_b, T_d\) and, \(T_s\) the temperatures at the boiler, the diffuser, and the heat-exchanger exits, respectively, \(p_1\) and \(p_{s1}\) the static and stagnation pressures at the duct exit, and \(\eta_b, \eta_d, \eta_h,\) and \(\eta_{\text{re}}\) the efficiencies of the boiler, the diffuser, the heat exchanger, and the recovery of \(Q_{\text{loss}}\) respectively. In addition, \(\eta_{\text{st}}\) varies usually
with the value of $Q_{\text{st-in}}$ [1.37].

2.3.2 Constraints

In this article, let us introduce some constraints to be considered when the estimation function is evaluated.

First, in order that the end effects and the boundary layers may not become too large, it is assumed that the ratio of the duct length $L$ to the width $\sqrt{A}$ is limited by

$$4 < \frac{L}{\sqrt{A}} < 18$$

on reference to [1.28], [1.38], and [1.39].

Next, since too much enthalpy extraction causes the gas dynamic instabilities such as choking of the gas flow and the separation of the boundary layer, the following constraint

$$\eta_{\text{MHD}} < 45\%$$

is adopted on reference to [1.28] and [1.40].

2.4 Electrical Conductivity, Hall Parameter, and Fluid Losses

2.4.1 Electrical conductivity and Hall parameter

In this part, the effects of the relaxation of ionization is not considered as already described. It is assumed that the ionization instability occurs in the gas plasma, the electrons obtain their energies only by the Joule heating and lose those only by collisions, and the fluctuation of the electron temperature can be neglected [1.15]. On these assumptions, the electron temperature $T_e$, the effective electrical conductivity $\sigma_{\text{eff}}$ and the effective Hall parameter $\beta_{\text{eff}}$ are obtained as

for $\beta < \beta_{\text{crit}}$:

$$T_e / T = 1 + 2\gamma M_0^2 \beta^2 (1-\Delta)^2 / \left\{ 3\delta (1+\alpha^2)(1+\beta_{\text{eff}}^2) \right\}$$

$$\times \left\{ (1+\alpha^2) + \kappa (\beta_{\text{eff}}^2 - \alpha^2) \ight.$$

$$\left. - (\beta_{\text{eff}}^2 - \alpha^2)(1-\kappa)\kappa \right\},$$

$$\sigma_{\text{eff}} = \lambda \sigma, \quad \beta_{\text{eff}} = \lambda \beta,$$

(1.22)

(1.23)
for \( \beta > \beta_{\text{crit}} \):

\[
T_e/T = 1 + 2\gamma M_0^2 \beta_{\text{crit}} (1-\Delta)^2 / \{3(1+a^2)(1+\beta_{\text{eff}}^2)\} \\
\times \{(1+a^2)^2 + \kappa(\beta_{\text{eff}}^2 - a^2) \}
\]

\[
\sigma_{\text{eff}} / \sigma = \beta_{\text{eff}} / \beta, \quad \beta_{\text{eff}} = \lambda \beta_{\text{crit}},
\]

where \( \sigma \) is the local scalar electrical conductivity, \( \beta \) and \( \beta_{\text{crit}} \) the local and critical Hall parameters for electrons, respectively, \( \lambda \) the parameter of the finite segmentation effect of the electrode, \( \Delta \) the collision loss factor, and \( M_0 \) the Mach number, in which \( \lambda \) is assumed constant [1.5], and moreover the grade of reduction of \( \beta_{\text{eff}} \) is assumed equal to that of \( \sigma_{\text{eff}} \) by reference to [1.21].

The local electrical conductivity \( \sigma \) and the Hall parameter \( \beta \) are given by

\[
\sigma = n_e e^2 / m_e v_e,
\]

\[
\beta = eB / m_e v_e,
\]

where \( e, m_e \) and \( n_e \) are electron charge, mass, and number density, respectively, and \( v_e \) the average momentum transfer collision frequency.

If the relaxation of ionization can be neglected, \( n_e \) is evaluated by Saha's equation, namely

\[
n_e^2 = n_s (2\pi m_e kT_e / h')^{3/2} \exp(-eV'_1 / kT_e),
\]

\[
n_s = n_{s0} - n_e,
\]

where \( h' \) is Planck's constant, \( k \) Boltzmann's constant, \( n_s \) the number density of seed atom, \( n_{s0} \) the original number density of seed atom before ionization, and \( V'_1 \) the ionization potential of seed atom.
Next, \( v_e \) is given by
\[
v_e = \left( \frac{8kT_e}{\pi m_e} \right)^{1/2} (n_s Q_s + n_0 Q_0 + n_e Q_{ei}),
\]
where \( n_0 \) is the number density of parent gas atom, and \( Q_0, Q_{ei}, \) or \( Q_s \) are the collision cross-sections between electron and parent gas atom, ion, or seed atom, respectively.

If \( T_e \) does not rise much, \( n_e \) becomes much smaller than \( n_s \). Then, we have
\[
n_s = \frac{\varepsilon_s p}{kT},
\]
where \( \varepsilon_s \) is the seed fraction, and \( n_e Q_{ei} \) becomes negligible in comparison with \( n_s Q_s \) and \( n_0 Q_0 \).

Consequently, substitution of Eqs. (1.28) to (1.30) into Eqs. (1.26) and (1.27) yields
\[
\sigma = \left( \frac{2\pi m_e k}{h^2} \right)^{3/4} \left( \frac{\pi \varepsilon_s / 8m_e}{8m_e} \right)^{1/2} \frac{1}{2} e^2 p^{-1/2} T_e^{1/2} T_e^{-1/4} / \{(1-\varepsilon_s)Q_0 + \varepsilon_s Q_s\}
\times \exp\left(-\frac{eV_i}{2kT_e}\right),
\]
\[
\beta = \left( \frac{k\pi}{8m_e} \right)^{1/2} eBp^{-1} T_T e^{-1/2} / \{(1-\varepsilon_s)Q_0 + \varepsilon_s Q_s\}.
\]

### 2.4.2 Friction and heat losses

It is assumed that the friction loss \( f \) and the heat loss \( q \) are given by
\[
f = 2pu^2 c_f / D,
\]
\[
q = 4\rho u c_p / D \cdot c_{f}/2 \cdot (T_s - T_w) / (1 + \sqrt{c_f/2\beta N}),
\]

respectively, where

\[
\frac{c_f}{\left\{ \frac{2.87 + 1.58 \log(x/k_s)}{1 + r(y-1)M_0^2/2} \right\}^{-2.5}}
\]

\[
r = \sqrt{\frac{p_r}{\gamma}}, \quad D = \sqrt{A},
\]

\[
\beta_N = 0.52\left(\frac{\rho u}{c_f/2k_s/\mu}\right)^{0.45} p^{0.8}
\]

where \(c_f\) is the friction factor, \(D\) the hydraulic diameter, \(k_s\) the equivalent sand roughness which represents the roughness level of a plate surface, \(p_r\) the Prandtl number, \(r\) the recovery factor, \(T_s\) the local stagnation temperature in the main stream, \(T_w\) the wall temperature, and \(\mu\) the viscosity of the gas [1.41]. Equations (1.33) to (1.35) are those which Schlichting has obtained for a compressible turbulent boundary layer on a rough plate [1.41].

2.5 Transformation of Basic Equations for Constant Mach Number Generator

2.5.1 Diagonal type generator

The basic flow equations (1.2) and (1.3) can be rewritten by the new variables defined by

\[
\begin{align*}
\xi &= \log(p_{s0}/p_s), \\
\zeta &= \log(T_{s0}/T_s),
\end{align*}
\]

(1.36)

and the adiabatic law

\[
\log(p_s/p) = a\log(T_s/T) = a\log(1 + x),
\]

(1.37)

where

\[
x = (\gamma-1)M_0^2/2,
\]

(1.38)

as follows:

\[
d\xi/dx = \frac{\sigma_{eff}uB^2}{\left(1-\Delta\right)}\{1 + \left(\frac{\beta_{eff}^2 - a^2}{a^2}\right)x/(1+a^2)\}
\]
\[\{(1 + \frac{\beta^2}{e_{\text{eff}}})p\} - \alpha x \xi / dx - \' / p, \quad (1.39)\]

\[d\xi / dx = I(\beta_{\text{eff}} - a) \sigma_k(1 - \kappa) / \{(\alpha + x)p(\alpha^2 + 1)\} + \gamma / \{(\alpha + x)u_p\}, \quad (1.40)\]

where

\[I = (1 - \kappa) \sigma_{\text{eff}}u_B(1 - \Delta)/(\beta_{\text{eff}} - a)/(1 + \beta_{\text{eff}}), \quad \kappa = E_x/(E_x)_{I=0} = (1 + \alpha^2)E_x / \{((\beta_{\text{eff}} - a)u_B(1 - \Delta))\}. \quad (1.41)\]

Next, using Eqs. (1.22) or (1.24), (1.23) or (1.25), (1.32), and (1.41), we can obtain the following equation

\[dT_e / dx = c_\xi d\xi / dx + c_\zeta d\zeta / dx, \quad (1.42)\]

where for \(\beta < \beta_{\text{crit}}:\]

\[c_\xi = T(f_1f_2p + f_2f_1p) / \{1 - T(f_1f_2e + f_1f_1e)\}, \quad \]

\[c_\zeta = T(-T_e / T + f_1f_2T + f_1f_1T) / \{1 - T(f_1f_2e + f_1f_1e)\}, \]

\[f_1 = 2yM_0^2\beta^2/(1 - \Delta)^2 \{3\delta(1 + \alpha^2)(1 + \beta_{\text{eff}}^2)\}, \quad \]

\[f_2 = 1 + \alpha^2 + \kappa(\beta_{\text{eff}}^2 - a^2) - (\beta_{\text{eff}} - a)^2(1 - \kappa), \quad \]

\[f_{1p} = -f_{1T} = 2f_1 / (1 + \beta_{\text{eff}}^2), \quad \]

\[f_{1e} = -f_1 / \{T_e(1 + \beta_{\text{eff}}^2)\}, \quad \]

\[c_{11} = 2a + \kappa(\beta_{\text{eff}} - a), \quad \]

\[c_{12} = \beta_{\text{eff}} \{2\kappa(\beta_{\text{eff}} - a)\beta_{\text{eff}} + (\beta_{\text{eff}}^2 - 2a\beta_{\text{eff}} - 1)\} / (1 + \beta_{\text{eff}}^2), \quad \]

13
\[ f_{2p} = c_{11}\left\{ \frac{1}{2} + (1-\kappa)(\beta_{\text{eff}} - a) + c_{12} \right\}, \]
\[ f_{2T} = c_{11}\left\{ -\frac{1}{2} + (1-\kappa)(\beta_{\text{eff}} - a) + c_{12} \right\}, \]
\[ f_{2e} = c_{11}\left\{ \frac{1}{4} + T_i/T_e - c_{12} \right\}/T_e, \]

and for \( \beta > \beta_{\text{crit}} \):

\[ c_{\xi} = (T_e - T + c_e)/c_0, \]
\[ c_{\zeta} = -(2T_e - T + c_e)/c_0, \]
\[ c_0 = 1 - \{ c_e(2T_i/T_e + 3/2) - (T_e - T)/2 \}/T_e, \]
\[ c_e = 2\gamma M_0^2 \beta_{\text{crit}}^2 (1-\Delta)^2 T(\beta_{\text{eff}} - a)(1-\kappa) \]
\[ x(a + (\beta_{\text{eff}} - a)\kappa)/\{3\delta(1+a^2)(1 + \beta_{\text{eff}}^2)\}. \]

Now, for obtaining the values of \( p, T, \) etc. in the diagonal type generator we have to assume beforehand one value of \( M_0, u, \)
\( \lambda, \) etc. or one relation among them. We assume that \( M_0 \) is constant by reference to the previous investigation of the Faraday type generator [1.42]. Then, Eqs. (1.39), (1.40), and (1.42) can be numerically solved with the assumed values of \( m_0, p_0, T_0, M_0, I, \) and \( L. \) Thus, the optimum generator duct can be obtained which makes maximum under the constraints (1.20) and (1.21).

2.5.2 Faraday type generator

The momentum and energy equations for the Faraday type generator, which correspond to Eqs. (1.2) and (1.3), respectively, in the case of \( J_x = 0, \) are transformed to the following equations

\[ d\xi/dx = \sigma_{\text{eff}} uB^2(1-\Delta)(1 - \kappa_{f})/p - \alpha \xi d\xi/dx + f/p, \]
\[ (1.45) \]
\[ d\zeta/dx = \sigma_{\text{eff}} uB^2(1-\Delta)^2 \kappa_{f}(1 - \kappa_{f})/\{\alpha(1+X)p\} \]
\[ + q/\{\alpha(1+X)up\}, \]
\[ (1.46) \]
similar to Eqs. (1.39) and (1.40), respectively, for the diagonal type, where $\kappa_f$ is the load factor of the Faraday type generator defined by

$$\kappa_f = \frac{E_y}{uB(1 - \Delta)}. \quad (1.47)$$

In addition, since two conditions are necessary to analyze the performances of the Faraday type generator, by reference to [1.42], it is assumed that the Mach number $M = M_0$ and the electron temperature $T_e = T_{e0}$ are constant. Then, $\kappa_f$ is written as

$$\kappa_f = 1 - \left\{ \frac{3\beta\delta(T_{e0} - T)}{(2\beta_{crit}\gamma T_0)} \right\}^{1/2}$$

$$/(M_0(1-\Delta)\beta), \quad (1.48)$$

2.6 Concluding Remarks

In this chapter, an optimization theory of the MHD generator has been described. The main conclusions are as follows.

(1) The numerically solvable differential equations for the stagnation pressure, temperature, and electron temperature were derived from the basic MHD equations for the constant Mach number generator. For these equations, the electrode voltage drop, effect of finite segmentation, friction and heat losses at the duct wall, and ionization instabilities were considered.

(2) As the estimation function of the generators the total thermal efficiency of an MHD-steam power plant for the optimization was derived, where the efficiencies of diffuser, heat exchanger, and compressor, etc. were considered, and some constraints required for the estimation function were introduced.
CHAPTER 3
NUMERICAL INVESTIGATION

3.1 Introduction

In this chapter, using the optimization theory derived in the preceding chapter, numerical calculation is made for a closed cycle diagonal type MHD-steam power plant with about 2000 MW of the thermal input. Influences of the Mach number, inclination parameter, duct length, inlet pressure and temperature, etc. of the diagonal type generator on the total thermal efficiency of the plant are investigated in detail. As the results, an optimum duct of optimum Mach number, inclination parameter, duct length, etc. is found out, which maximizes the efficiency. Then, the distributions of physical quantities in the optimum duct are discussed.

Next, in the similar way, the optimization of the Faraday type generator is discussed, and the optimum Mach number, duct length, etc. are obtained. Finally, a comparative study of the performances of both type generators is made.

3.2 Numerical Conditions

A numerical analysis is carried out for the cesium-seeded helium in nonequilibrium ionization. We assume that the thermal input \( Q_{in} = 2000 \text{ MW} \) as previously mentioned. Also the magnetic flux density \( B = 6 \text{ to } 10 \text{ T} \), the inlet stagnation pressure \( p_{s0} = 5 \text{ to } 15 \text{ atm} \), and the inlet stagnation temperature \( T_{s0} = 1600 \text{ to } 2200 \text{ K} \) are used. We adopt the seed fraction \( e_s = 0.3 \% \) and the compressor stage number \( Z = 2 \), which maximize the total thermal efficiency \( n \) under the conditions that \( L = 10 \text{ m} \), \( p_{s0} = 10 \text{ atm} \), and \( T_{s0} = 2000 \text{ K} \). Also \( n_{st} \) is given by

\[
\eta_{st} = \begin{cases}
38.25 & \text{for } Q_{st-in} < 380 \text{ MW}, \\
41.04 & \text{for } 380 \leq Q_{st-in} < 1000 \text{ MW}, \\
41.76 & \text{for } 1000 \leq Q_{st-in} < 1500 \text{ MW},
\end{cases}
\]

[1.37] [1.49]
Moreover, the case of $\eta_{st} = 42.0\%$ is considered, too, in which it is assumed that some MHD generators and a steam turbine generator are operated together.

In Table 1.1, the above mentioned and the other assumed conditions are listed.

Table 1.1 Values of quantities used in numerical calculations

<table>
<thead>
<tr>
<th>$B$ (T)</th>
<th>6 - 10</th>
</tr>
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<tbody>
<tr>
<td>$C_P$ (J/Kg deg)</td>
<td>$5.30 \times 10^3$</td>
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<tr>
<td>$k_s$ (m)</td>
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</tr>
<tr>
<td>$m_0$ (Kg/sec)</td>
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</tr>
<tr>
<td>$Pr$</td>
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<tr>
<td>$P_{s0}$ (atm)</td>
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<td>$Q_0$ ($m^2$)</td>
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<td>$Q_s$ ($m^2$)</td>
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<td>$\eta_{re}$ (%)</td>
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<tr>
<td>$\lambda$</td>
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</tr>
</tbody>
</table>

3.3 Optimization of Diagonal Type Generator

3.3.1 Influence of Mach number and inclination parameter

In Fig.1.3 the variations of $\eta$ by $M_0$ and $a$ are shown in the case of $P_{s0} = 10$ atm, $T_{s0} = 2000$ K, and $L = 10$ m. From the figure, it is seen that there exists an optimum Mach number $M_{opt}$ which maximizes $\eta$, $M_{opt}$ becomes larger with $|a|$, $M_{opt} = 1.38$ at $a = -2.2$, and the maximum value of $\eta$ varies little for $-1.8 < a < -2.6$. 
3.3.2 Influence of duct length

Figure 1.4 illustrates the influences of $L$ and $a$ on the maximum value of $\eta$ corresponding to $M_0 = M_{\text{opt}}$ in the case of $p_{s0} = 10$ atm and $T_{s0} = 2000$ K. The figure presents that there exists an optimum inclination parameter $a_{\text{opt}}$ which makes $\eta$ maximum and that the variation of $\eta$ by $L$ is fairly small except the case of $L = 8$ m where the constraint (1.20) prevents $\eta$ from increasing because of a short duct length. Then, $\eta$ has the maximum value 52.1% at $a = -2.2$ and $L = 10$ m. Next, Fig.1.5 shows the variations of $\eta$ by $M_0$ when $\eta_{st}$ has the constant value 42.0% and the variable values given by Eq.(1.45) under the same conditions as in Fig.4. It indicates that the value of $\eta$ is heightened about 0.2% by combining some MHD generators and one steam cycle.

3.3.3 Influence of inlet stagnation pressure
Fig. 1.4. Influence of \( L \) and \( a \) on \( \eta \).

Fig. 1.5. \( \eta \) vs. \( M_0 \) when \( \eta_{st} \) is assumed constant or variable.
Figure 1.6 denotes the variation of $\eta$ by $p_{s0}$ and $a$ in the case of $T_{s0} = 2000$ K and $L = 10$ m. It is shown that the smaller $p_{s0}$ is, the larger $\eta$ and $a_{\text{opt}}$ are, and for example, the maximum values of $\eta$ reach 53.4 and 53.1% for $p_{s0} = 5$ and 7.5 atm, respectively.

![Graph showing the influence of $p_{s0}$ and $a$ on $\eta$.]

3.3.4 Influence of inlet stagnation temperature

Figure 1.7 shows the influences of $T_{s0}$ and $a$ on $\eta$ in the case of $p_{s0} = 10$ atm and $L = 10$ m. The figure indicates that the value of $\eta$ increases by about 1.6% per increase of 100 K of $T_{s0}$ and that the maximum value of $\eta$ is little dependent on $a$ in the range of $|a| > 1.5$. These indicate that $T_{s0}$ should be elevated more than 1900 K for obtaining $\eta > 50\%$.

3.3.5 Influence of applied magnetic flux density

In Fig. 1.8, the influences of $a$ and $B$ on $\eta$ are shown when
From the figure, it is seen that $|a_{\text{opt}}|$ becomes large with increasing $B$, for example, $a_{\text{opt}} = -1.8$, $-1.95$, and $-2.2$ for $B = 6$, $8$, and $10$ T, respectively.

Moreover, the maximum value of $\eta$ exceeds $50\%$ for $B >$ about $6$ T, and it increases with increasing $B$ by the rate of about $1\%$ per $1$ T. The rate of increase of $\eta$ by $B$ does not become so much in the case where the ionization instability is considered as in this thesis, though the output power increases in proportion to $B^2$ if the ionization instability does not occur.

3.3.6 Distribution of various quantities in optimum duct

In Figs. 1.9 and 1.10, there are plotted the distributions of $T_{s0} = 2000$ K, $p_{s0} = 7.5$ atm, and $L = 10$ m.

![Graph](image)

Fig. 1.7. Influence of $T_{s0}$ and $a$ on $\eta$. 21
the quantities in the optimum generator duct [see Table 1.2], which gives the maximum efficiency $\eta$, where $B = 10$ T, $P_{s0} = 10$ atm, and $T_{s0} = 2000$ K. In this case, we have $\eta = 52.1\%$, $\eta_i = 73.6\%$, $\eta_{MHD} = 44.0\%$, and $P_w = 52.3$ MW/m$^3$ as listed in Table 1.2.

Figure 1.8 shows that $T_e$ decreases uniformly from $T_{e0} = 2000$ to $T_{e1} = 1900$ K, and so the difference $T_{e1} - T_{e0}$ is only about 5% of $T_{e0}$. Therefore $T_e$ may be regarded as almost constant. $\sigma_{\text{eff}}$ decreases from 1.16 to 0.31 $\Omega$/m and inversely $\beta$ increases from 4.8 to 32.2. Also $J_x$ changes its sign in the latter half of the generator duct, both $-E_x$ and $E_y$ have their peaks there, and $|E_x|$ does not exceed 9kV/m. Next, the ratio of the inlet cross section to the exit $A_1/A_0$ is 8.38.
3.4 Optimization of Faraday Type Generator

In the performance analysis of the Faraday type generator, $M_0$ and $T_{e0}$ are assumed constant along the flow, and the same numerical conditions as for the diagonal type listed in Table 1.1 are used.

The duct length of the optimum duct becomes about 10 m under the given conditions. Also, are obtained $T_{e_{\text{opt}}} = 1970$ K, $M_{\text{opt}} = 1.34$, $\eta = 52.3\%$, $\eta_i = 74.4\%$, $\eta_{\text{MHD}} = 44.4\%$, and $P_w = 56.1$ MW/m$^3$ in the case of $T_{s0} = 2000$ K and $p_{s0} = 10$ atm as shown in Table 1.2, where $T_{e_{\text{opt}}}$ is an electron temperature which maximizes $\eta$.

The distributions of the quantities in the optimum duct of the Faraday type generator are plotted in Figs.1.11 and 1.12, which

![Graph showing variations of quantities along the diagonal type generator duct (I).](image)
show that $\sigma_{\text{eff}}$ decreases from 0.81 to 0.36 $\mu$/m and $\beta$ increases from 6.3 to 32.4 along the duct. Also $|E_x|$ exceeds 10 kV/m in the first half of the duct and $A_1/A_0$ becomes 7.01.

3.5 Comparison between Diagonal and Faraday Type Generators

As shown in the previous sections, the values of the various quantities in the optimum generator ducts in the case of $T_{s0} = 2000$ K and $p_{s0} = 10$ atm are listed in Table 1.2. As seen in the table, $\eta$, $\eta_1$, and $P_w$ of the diagonal type generator become about 0.2 %, 0.8 %, and 3.8 MW/m$^3$, respectively, smaller than those of the Faraday type, and $V'_{sc}$ of the former becomes only a little

![Graph](image_url)

Fig. 1.10. Variation of quantities along the diagonal type generator duct (II).

24
larger than that of the latter. In both type generators, the optimum duct length $L_{\text{opt}}$, which makes $\eta$ maximum, is nearly equal, $M_{\text{opt}}$ becomes supersonic, and $M_{\text{opt}}$ of the diagonal type is a little lower than that of the Faraday type generator. Next, $|E_x|$ in the former is below 9 kV/m, but that in the latter exceeds 10 kV/m.

In the open cycle MHD generation, the performances of the diagonal type generator are a little inferior to those of the Faraday type generator [1.23]. On the other hand, the difference of the performance between both type generators in the closed cycle MHD generation is fairly smaller than that in the open cycle. Consequently, it can be said that the performances of the diagonal type generator sufficiently approach to those of the Faraday type.

Fig. 1.11. Variation of quantities along the Faraday type generator duct (I).
Fig. 1.12. Variation of quantities along the Faraday type generator duct (II).

Table 1.2 Values of various quantities in optimum duct

<table>
<thead>
<tr>
<th></th>
<th>Diagonal Faraday</th>
<th>Diagonal Faraday</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>-2.2</td>
<td></td>
</tr>
<tr>
<td>$I$ (kA)</td>
<td>16.2</td>
<td></td>
</tr>
<tr>
<td>$T_e$ (K)</td>
<td>1950</td>
<td>1970</td>
</tr>
<tr>
<td>$N_0$</td>
<td>1.38</td>
<td>1.34</td>
</tr>
<tr>
<td>$L$ (m)</td>
<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td>$n$ (%)</td>
<td>52.1</td>
<td>52.3</td>
</tr>
<tr>
<td>$n_i$ (%)</td>
<td>73.6</td>
<td>74.4</td>
</tr>
<tr>
<td>$n_{\text{MED}}$ (%)</td>
<td>44.0</td>
<td>44.5</td>
</tr>
<tr>
<td>$P_w$ (MW/m$^2$)</td>
<td>52.3</td>
<td>56.1</td>
</tr>
<tr>
<td>$P_{\text{MED}}$ (MW)</td>
<td>1310</td>
<td>1320</td>
</tr>
<tr>
<td>$P_{\text{st}}$ (MW)</td>
<td>453</td>
<td>478</td>
</tr>
<tr>
<td>$P_c$ (MW)</td>
<td>594</td>
<td>593</td>
</tr>
<tr>
<td>$P_t$ (MW)</td>
<td>1143</td>
<td>1179</td>
</tr>
<tr>
<td>$A_1/A_0$</td>
<td>8.38</td>
<td>7.01</td>
</tr>
<tr>
<td>$V_{\text{sc}}$ (m$^2$)</td>
<td>36.3</td>
<td>35.8</td>
</tr>
</tbody>
</table>
3.6 Concluding Remarks

The main conclusions, which were derived from the above described numerical analysis, are as follows.

(1) When the ionization instability of the nonequilibrium plasma is assumed to occur, the total thermal efficiency of the MHD-steam plant with the diagonal type generator exceeds 52 % in the case of $B = 10$ T, $p_{s0} = 10$ atm, and $T_{s0} = 2000$ K.

(2) The optimum values of the Mach number and the inclination parameter are considerably influenced by the value of $B$, $p_{s0}$, or $T_{s0}$, and we have $M_{opt} = 1.38$ and $a_{opt} = -2.2$ in the case of $B = 10$ T, $p_{s0} = 10$ atm, and $T_{s0} = 2000$ K.

(3) The optimum duct length of the diagonal type generator is nearly equal to that of the Faraday type generator, and about 10 m. In this connection, the variation of the thermal efficiency by $L$ is small.

(4) The smaller $p_{s0}$ is, the larger $\eta$ and $a_{opt}$ are. The small $p_{s0}$ gives the very large $|a_{opt}|$.

(5) The total thermal efficiency increases by about 1.6 % per increase of 100 K of $T_{s0}$ and it reaches about 50 % at $T_{s0} = 1900$ K.

(6) The efficiency decreases by about 1.0 % per decrease of 1 T of $B$, but it exceeds 50 % still in the case of $B = 6$ T.

(7) The performances of the diagonal type generator can sufficiently approach to those of the Faraday type, and the thermal efficiency of the former is only 0.2 % lower than that of the latter. In both types, a quantities proportional to the volume of the superconducting wire is almost equal each other.

Thus, it may be thought that the diagonal type nonequilibrium plasma MHD generator will be practically used in future in view of the total thermal efficiency of the MHD-steam power plant.
PART II

TWO-DIMENSIONAL ANALYSES OF DIAGONAL TYPE NONEQUILIBRIUM PLASMA MHD GENERATOR
CHAPTER 1
INTRODUCTION

Electrical characteristics of MHD generators can be totally investigated by a quasi one-dimensional theory. However, in the generator duct, there usually occurs two- or three-dimensional phenomena, which cannot be analyzed by the above theory, such as current concentration at the electrode end and end effects. These have large influence on the generator performance. Accordingly, at least, a two-dimensional analysis of the generator becomes necessary for grasping more accurately the performance.

The two- or three-dimensional current and potential distributions in the Faraday type generator have been investigated in fair detail by Hurwitz [1.1], Celinski [1.3], Oliver [2.1], Argyropoulos [2.2], Lengyel [2.3], and many others, and its electrical characteristics have been considerably clarified. Those distributions in the Hall type generator have been studied by Denzel [2.4], Tanaka [2.5], et al. Moreover, the generator characteristics of a diagonal type MHD generator have been a little discussed by a two-dimensional analysis by Kaidalov [2.6], et al., but the detailed analysis has not been made.

Therefore, the author et al. have investigated some basic electrical characteristics of the diagonal type nonequilibrium plasma MHD generator [2.7] to [2.12], two methods of improving the characteristics [2.13] to [2.15], the end effects in the generator [2.16] to [2.20], influences of difference between internal and external connections of the diagonal electrodes on the generator characteristics [2.21], [2.22], etc.

In this part, the electrical characteristics of the diagonal type nonequilibrium plasma generator will be investigated by two-dimensional analyses.

In Chapter 2, there are introduced the basic equations, which are used in the analyses, such as the Maxwell equations, the gen-
eralized Ohm's law, and the energy equation for electrons. Applying the stream and potential functions to the basic equations, the nonlinear two-dimensional partial differential equations to be numerically solved are obtained. Next, boundary and subsidiary conditions are shown, which are required for calculations of the current and potential distributions in the diagonal type generator. Further, for quantitative estimation of the electrical efficiency, the specific Hall voltage, an expression of grade of the current concentration at an electrode end, etc. are derived.

In Chapter 3, the basic electrical characteristics of the generator such as an electrical efficiency and a specific Hall voltage are investigated by a two-dimensional analysis. When the electrical conductivity and the Hall parameter are assumed constant, the current distribution is evaluated, and influences of the load factor, the Hall parameter, and the inclination parameter on the distribution are investigated. Then, influences of the inclination parameter, the Hall parameter, etc. on the electrical characteristics are studied. Next, when the electrical conductivity and the Hall parameter vary spatially, the distributions of current, potential, and electron temperature are shown, and influences of the load factor and the inclination parameter on the current distribution are discussed. Then, influences of the duct size and the temperatures of the gas plasma and the duct wall on the electrical characteristics are investigated.

In Chapter 4, effects of a resistive electrode and a distribution of the applied magnetic flux on the electrical characteristics of the generator are studied by a two-dimensional analysis. The configuration of the resistive electrode and the distribution of magnetic flux used in this chapter are shown. Next, influences of the conductivity and configuration of a wedge-shaped resistive electrode on the current concentration at the electrode, electrical efficiency, etc. are investigated. Next, the electrical characteristics of the generator with the attenuation of magnetic flux in
the region near the duct wall are compared with those of the generator with the constant magnetic flux.

In Chapter 5, end effects of the generator are investigated by a two-dimensional analysis. First, when electrical conductivity and electron mobility are assumed constant, influences of the attenuation of magnetic flux and type and position of the output electrodes on the current distribution, the internal resistance, etc. in the end regions are studied. Next, when nonuniformity of electrical conductivity and electron mobility are considered, effects of the attenuation of magnetic flux on the current distribution, the internal resistance, the ballast resistance, etc. are investigated.

In Chapter 6, effects of internal or external connection of electrodes in a diagonal type generator are investigated by a two-dimensional analysis in the duct cross section perpendicular to the plasma flow. First, the two-dimensional differential equations in the above cross section are derived from the basic equations in Chapter 2, and the boundary and subsidiary conditions are introduced. Next, numerical calculation results of the current and potential distributions in the generators with internally and externally connected electrodes are comparatively studied. Then, influences of the load factor, the duct wall temperature, and the inclination parameter on the current distribution, electrical efficiency, etc. are discussed.
CHAPTER 2
TWO-DIMENSIONAL THEORY

2.1 Introduction

As already noted, for grasping accurately the generator characteristics considering current concentration at the electrode end, etc., at least, a two-dimensional analysis of the generator is needed.

In this chapter, the basic equations are introduced such as the Maxwell equations, the generalized Ohm's law, and the energy equation for electrons. The nonlinear partial differential equations are derived by applying the stream and potential functions. Next, the boundary and subsidiary conditions are presented, which are required for the diagonal type generator. Further, the electrical efficiency, the specific Hall voltage, and expression of grade of the current concentration at an electrode end, etc. are derived.

2.2 Basic Equation

2.2.1 Basic equation

In a two-dimensional analysis of the electrical characteristics of the diagonal type nonequilibrium plasma MHD generator, it is assumed that the electric quantities such as the current density, electric field, etc. vary with x and y [see Fig.2.1] in Chapters 3 to 5, and in particular, do periodically in the period of the electrode pitch s in the x direction viz. along the gas flow in Chapters 3 and 4, and that the gas velocity and temperature in Chapters 3 to 5 depend on only y according to Eqs.(2.8) and (2.9) and the pressure is constant.

The Maxwell equations for the electromagnetic field, the continuity equation of electrons, the generalized Ohm's law, the energy equation of electrons, etc. are used as the basic equations
for the gas plasma in MHD duct.

First, the Maxwell equations are written as

\[ \nabla \times E = 0, \quad \nabla \cdot J = 0, \quad \nabla \cdot E = 0, \]  

(2.1)

where \( E \), \( J \), and \( B \) are the electric field intensity, current density, and magnetic flux density or magnetic induction vectors, respectively.

Next, the continuity equation for the electrons is

\[ \frac{\partial n_e}{\partial x} = n_e \left( n_{\text{es}}^2 - n_e^2 \right) \nu_r, \]

where

\[ \nu_r = 1.09 \times 10^{-20} T_e^{-9/2} \]

(2.2)

is the three body recombination coefficient, \( n_e \) the electron density, \( n_{\text{es}} \) the electron density obtained by the following Saha's equation

\[ n_{\text{es}}^2 = n_s (2\pi m_e k T_e / h^2)^{3/2} \exp(-e\nu_i / k T_e), \]

in which

\[ n_s = n_{s0} - n_{\text{es}}, \]

(2.3)

\( n_s \) and \( n_{s0} \) are the seed atom density and the original one before ionization, respectively, and \( u \) the x component of gas velocity.

The generalized Ohm's law is written as

\[ J = \sigma (E + u \times B) + \nabla p_e / n_e \] 

\[ + \beta_i (J \times B) x B^2, \]

(2.4)

which is nothing but the momentum equation for electrons, where \( u \) is the gas velocity vector, \( p_e = n_e k T_e \) the electron partial pressure, and \( \beta_i \) the Hall parameter for ion [see Eq.(2.12)].

The energy equation for electrons is given by
u\partial /\partial x \{ (3/2kT_e + eV_1)n_e \} /\partial x = J^{2}/\sigma - 3/2kn_em_e(T_e - T) - v_{es}/m_s + v_{e0}/m_0 + v_{ei}/m_i - v(kn_eT_e) \cdot J/en_e, (2.5)

where

\begin{align*}
  v_{es} &= (8kT_e/\pi m_e)^{1/2}n_sQ_s, \\
  v_{e0} &= (8kT_e/\pi m_e)^{1/2}n_0Q_0, \\
  v_{ei} &= (8kT_e/\pi m_e)^{1/2}n_eQ_{ei},
\end{align*}

(2.6)

$\rho$, $m_s$, $m_0$, and $m_i$ are the masses of seed atom, parent gas atom, and ion, respectively, $n_0$ the parent gas density, $Q_{ei}$ the collision cross section between electron and ion. In Eq. (2.5), the radiation term is neglected since the electron temperature is relatively low.

**Fig. 2.1.** Schema of diagonal type MHD generator duct for two-dimensional analysis.
under the numerical conditions in this thesis.

Next, we assume \( E, J, B \) and \( u \) as

\[
\begin{align*}
E &= (E_x, E_y, 0), \\
J &= (J_x, J_y, 0), \\
B &= (0, 0, B), \\
u &= (u, 0, 0),
\end{align*}
\]

in the chapters except Chapter 6.

The secondary magnetic flux caused by \( J \) is assumed to become negligible in comparison with the applied magnetic flux, since the magnetic Reynolds number of the gas plasma is much smaller than unity under the operating conditions.

Next, as described previously, it is assumed that the gas velocity \( u \) and temperature \( T \) in Chapters 3 to 5 depend on only \( y \) according to the following relations,

\[
\frac{u}{u_0} = \left(\frac{4y}{h} \cdot \left(1 - \frac{y}{h}\right)\right)^m
\]

and

\[
\frac{T - T_w}{T_0 - T_w} = \left(\frac{4y}{h} \cdot \left(1 - \frac{y}{h}\right)\right)^{m'}
\]

respectively, where \( h \) is the duct height, \( T_w \) the wall temperature, \( T_0 \) and \( u_0 \) the gas temperature and velocity at the center of flow, namely \( y=h/2 \), respectively, and \( m \) and \( m' \) constants.

Finally, the electrical conductivity \( \sigma \), Hall parameters \( \beta \) and \( \beta_i \) for electrons and ions are given by

\[
\sigma = n_e e^2 / m_e v_e,
\]

\[
\beta = \frac{1}{m_e} B, \\
\beta_i = e B (8kT_e / \pi m_e)^{1/2} \left\{ \frac{Q_{i0}}{(1 + m_s/m_0)} + \frac{e_s Q_{is} / \sqrt{2}}{(2)^{-1} / \beta} \right\}
\]

where the average electron momentum transfer collision frequency \( v_e \) is given by

\[
v_e = (8kT_e / \pi m_e)^{1/2} (n_s \Omega_s + n_0 \Omega_{00} + n_e \Omega_{ei}),
\]

\[2.12\]
in which
\[
Q_e = 3.90 \left( \frac{e^2}{\pi} \frac{kT_e}{e} \right)^2 \log \Lambda,
\]
\[
\Lambda = 12 \left( \frac{e^0 kT_e}{e} \right)^{3/2} \frac{n_e}{\nu_e},
\]
and \(Q_{10}\) and \(Q_{1s}\) the collision cross sections between ions and parent gas atoms and between ions and seed atoms, respectively, \(\nu_e\) the electron mobility, and \(\varepsilon_0\) the permittivity of free space.

2.2.2 Stream and potential functions

To evaluate the current distribution in the MHD generator duct, we use the conventional stream function \(\psi\) defined by
\[
J_x = \frac{\partial \psi}{\partial y}, \quad J_y = -\frac{\partial \psi}{\partial x},
\]
which satisfy \(\nabla J = 0\) in Eq. (2.1).

Also, to obtain the potential distribution, we adopt the potential function \(\phi\) defined by
\[
E = -\nabla \phi,
\]
which satisfies \(\nabla \times E = 0\) in Eq. (2.1).

Then, from Eqs. (2.1), (2.4), and (2.14) the following partial differential equation of the 2nd order for \(\psi\) is derived.
\[
\nabla^2 \psi + P \frac{\partial \psi}{\partial x} + Q \frac{\partial \psi}{\partial y} = R_1,
\]
where
\[
P = \sigma/\varepsilon \cdot \left\{ \frac{\partial (\varepsilon/\sigma)}{\partial x} - \frac{\partial (\beta/\sigma)}{\partial y} \right\},
\]
\[
Q = \sigma/\varepsilon \cdot \left\{ \frac{\partial (\varepsilon/\sigma)}{\partial y} + \frac{\partial (\beta/\sigma)}{\partial x} \right\},
\]
\[
R_1 = \sigma/\varepsilon \cdot \left\{ -\frac{\partial (\nu_p/\partial x e_n)}{\partial x} + \frac{\partial (\nu_p/\partial y e_n)}{\partial y} \right\},
\]
in which
\[
\varepsilon = 1 + \beta \beta_1.
\]
Also, from Eqs. (2.1), (2.4), and (2.15) the following equation for \(\phi\) is obtained,
\[
\phi^2 + P' \frac{\partial \phi}{\partial x} + Q' \frac{\partial \phi}{\partial y} = R',
\]
(2.18)
where
\[
P' = \frac{(\partial \sigma'/\partial x + \partial \sigma''/\partial y)}{\sigma'},
\]
\[
Q' = \frac{(-\partial \sigma''/\partial x + \partial \sigma'/\partial y)}{\sigma'},
\]
\[
R' = \sigma \left( \sigma (uB - \partial p_e/\partial y/e_n) + \sigma' \partial p_e/\partial x/e_n \right)
+ \frac{\partial \sigma}{\partial x}/\sigma' + \sigma' \left( \sigma'' \partial p_e/\partial x/e_n + \sigma'(-uB + \partial p_e/\partial y/e_n) \right)/\sigma',
\]
(2.19)
in which
\[
\sigma' = \sigma \varepsilon/(\varepsilon^2 + \beta^2), \quad \sigma'' = \sigma \beta/(\varepsilon^2 + \beta^2).
\]
In these equations \( P, P', \) etc. are the functions of \( \Psi \) or \( \Phi \) and so both Eqs.(2.16) and (2.18) become nonlinear equations.

2.3 Boundary and Subsidiary Conditions

For solving Eqs.(2.16) and (2.18), we need boundary conditions on an electrode surface and an insulating wall, periodicity conditions, and two subsidiary conditions peculiar to the diagonal type generator.

First, the boundary condition on the insulating wall is
\[
J_y = 0.
\]
(2.20)
Using Eqs.(2.4), (2.7), (2.14), and (2.15), this equation is changed into the following equation
\[
\Psi = \text{const.}
\]
(2.21)
or\[
\varepsilon (E_y + \partial p_e/\partial y/e_n) + \beta (E_x + \partial p_e/\partial x/e_n) = 0.
\]
(2.22)
Next, the boundary condition on a good conductor electrode surface is

\[ E_x = 0. \]  \hspace{1cm} (2.23)

Using Eqs. (2.4), (2.7), (2.14), and (2.15), the above equation is transformed into the following equation

\[ \varepsilon J_x / \sigma + \beta J_y / \sigma - \partial \rho_e / \partial x / e \sigma / e \]  

\[ = (\varepsilon \partial \Psi / \partial y - \beta \partial \Psi / \partial x - \sigma \partial \rho_e / \partial x / e) / \sigma = 0 \]  \hspace{1cm} (2.25)

or

\[ \phi = \text{const.} \]  \hspace{1cm} (2.25)

Next, the periodicity conditions for \( J \) and \( E \) are given

\[ J(x + s) = J(x), \quad E(x + s) = E(x). \]  \hspace{1cm} (2.26)

By Eqs. (2.14) and (2.15), these equations are rewritten as

\[ \Psi(x + s) = \Psi(x) + I'_y, \quad \phi(x + s) = \phi(x) + V'_x, \]  \hspace{1cm} (2.27)

respectively, where \( I'_y \) is the current flowing into an electrode and \( V'_x \) the potential difference per electrode pitch \( s \) in the \( x \) direction.

Finally, in the diagonal type generator, the potential difference must be zero between the electrodes C and A short-circuited each other in Fig. 2.1. Accordingly, the first subsidiary condition becomes

\[ \int_C^A E \cdot ds = 0, \]  \hspace{1cm} (2.28)

where \( ds \) is the line element vector on an optional integral path from C to A.

Next, the current which runs through an arbitrary surface \( S \) crossing the insulating wall surfaces C' and A' is equal to the load current \( I \). This gives the following second subsidiary condition
\[ \int_S \mathbf{J} \cdot d\mathbf{S} = I, \]  
\[ (2.29) \]

where \( d\mathbf{S} \) is the element vector of the surface \( S \).

The current and potential distributions in the diagonal type generator duct can be obtained by solving numerically Eqs. (2.16) and (2.18) under the conditions (2.20) to (2.29).

2.4 Formulation of Electrical Characteristics

As the quantities which are used for the estimation of the generator performance, we adopt the electrical efficiency \( \eta_e \) given by

\[ \eta_e = \frac{-\int_0^s \mathbf{E} \cdot J dx}{\int_0^s (\mathbf{J} \times \mathbf{B}) \cdot u dx} \]  
\[ (2.30) \]

and the specific Hall voltage \( Q_H \) defined by

\[ Q_H = \frac{V_H}{V_{\text{ideal}}} \]  
\[ (2.31) \]

In this equation, \( V_H \) and \( V_{\text{ideal}} \) are the Hall and ideal potential differences per one electrode pitch \( s \), which are given by

\[ V_H = -\int_0^s \mathbf{E} x dx \]  
\[ (2.32) \]

and

\[ V_{\text{ideal}} = (\langle \beta \rangle - a) u B k s / (1 + a^2), \]  
\[ (2.33) \]

respectively, where

\[ \kappa = 1 - I (1 + \beta_{\text{eff}}^2) / \{ h w \sigma_{\text{eff}} \langle u \rangle B (\beta_{\text{eff}} - a) \}. \]  
\[ (2.34) \]

In this equation, \( w \) is the duct width in the \( z \) direction, and \( \sigma_{\text{eff}} \) and \( \beta_{\text{eff}} \) the effective conductivity and Hall parameter, which are obtained by the following relations

\[ J_x = \sigma_{\text{eff}} / (1 + \beta_{\text{eff}}^2) \cdot \{ (1 - a \beta_{\text{eff}}) \langle E_x \rangle \\ + \langle u \rangle B \beta_{\text{eff}} \} \]  
\[ (2.35) \]

* See Appendix II.
\[ J_y = \frac{\sigma_{\text{eff}}}{(1 + \beta_{\text{eff}}^2) \cdot \left( (\beta_{\text{eff}} + a) \langle E_x \rangle - \langle u \rangle \right)}. \]

In Eqs. (2.33) to (2.35), \(< >\) shows the average value per one electrode pitch \(s\).

Moreover, a grade of the current concentration at an electrode end is estimated by the following relations

\[ \frac{J_{\text{peak}}}{\langle J \rangle_{\text{el}}} \] (2.36)

where \(J_{\text{peak}}\) and \(\langle J \rangle_{\text{el}}\) are the maximum and average current density on the electrode. In this connection, \(J_{\text{peak}}/\langle J \rangle_{\text{el}} = 1\) shows that the current distribution on the electrode surface is completely uniform.

### 2.5 Numerical Calculation of Basic Equations

#### 2.5.1 Normalized basic equations

Now putting

\[ \xi = \frac{x}{s}, \quad \zeta = \frac{y}{s}, \quad \Psi = \frac{\Psi}{(I/\omega)}, \] (2.37)

\[ T' = \frac{T}{T_0}, \quad n' = \frac{n_e}{n_0}, \]

where \(n_0\) is the electron density obtained by Eq. (2.3) in the case of \(T_e = T_0\).

Equations (2.16), (2.17), (2.21), (2.24), (2.27), and (2.5) are normalized as

\[ \nabla^2 \Psi + p'' \frac{\partial \Psi}{\partial \xi} + p''' \frac{\partial \Psi}{\partial \zeta} + R'' = 0, \] (2.38)

\[ p'' = \frac{\sigma}{\epsilon} \{ \frac{\partial (\epsilon/\sigma)}{\partial \xi} - \frac{\partial (\epsilon/\sigma)}{\partial \zeta} \}, \]

\[ q'' = \frac{\sigma}{\epsilon} \{ \frac{\partial (\beta/\sigma)}{\partial \xi} + \frac{\partial (\epsilon/\sigma)}{\partial \zeta} \}, \]

\[ R'' = \frac{w k e T_0/(I e n'_e)}{(I e n'_e)} \left\{ \frac{\partial T}{\partial \xi} \frac{\partial n'_e}{\partial \zeta} + \frac{w (I/e \cdot \omega)}{\partial \xi} \right\}, \] (2.39)

\[ \Psi = \text{const.}, \] (2.40)

\[ \frac{\partial \Psi}{\partial \xi} = \frac{\beta}{\epsilon} \frac{\partial \Psi}{\partial \xi} + \frac{k_0}{e \cdot \sigma / e \cdot T_0 / n_e} \frac{\partial (T')}{\partial \xi}, \] (2.41)
\[
\psi(\xi+1, \zeta) = \psi(\xi, \zeta) + I'_y/(I/w),
\]
\[
3n_0T_0m_e \delta n_{e}' \sigma(T'_{e} - T_0) \left( \nu_{es}/m_s + \nu_{e0}/m_0 \right) + \nu_{e1}/m_1)/2 = (I/sw)^2 \left( \frac{\partial \psi}{\partial \xi} \right)^2 + \left( \frac{\partial \psi}{\partial \zeta} \right)^2,
\]
respectively.

2.5.2 Difference approximation of normalized basic equations

Defining
\[
\psi_{ij} = \psi(x_i, y_j), \quad x_i = i \Delta, \quad y_j = j \Delta,
\]
i, j = 1, 2, ---
\[
(2.44)
\]
by the central difference approximation, Eqs.(2.38) and (2.41) are transformed to
\[
\begin{align*}
(1 + \Delta P_{ij}/2)\psi_{i+1,j} - 4\psi_{i,j} + (1 - \Delta P_{ij}/2) \\
\times \psi_{i-1,j} + (1 + \Delta Q_{ij}/2)\psi_{i,j+1} \\
+ (1 - \Delta Q_{ij}/2)\psi_{i,j-1} + \Delta^2 R_{ij} &= 0 \\
i, j &= 1, 2, ---
\end{align*}
\]
\[
(2.45)
\]
\[
\begin{align*}
\psi_{i,j+1} - \psi_{i,j-1} - \beta/\varepsilon \cdot (\psi_{i+1,j} - \psi_{i-1,j}) \\
+ k\varepsilon T_0^e/(eIn_{e}^e) \cdot (n_{e}^e i+1,j^e e i+1,j \\
- n_{e}^e i-1,j^e e i-1,j), \\
i, j &= 1, 2, ---
\end{align*}
\]
\[
(2.46)
\]
where \( P_{ij}, Q_{ij}, R_{ij}, n_{eij}', \) and \( T_{eij}' \) are the values of \( P''_e, Q''_e, R''_e, n_{eij}', \) and \( T_{eij}' \) at a point \( (x_i, y_j) \), respectively. Here \( \psi_{i,j-1} \big|_{j=1} \) and \( \psi_{i,j} \big|_{j=h/\Delta} \), which can not exist physically, can be easily eliminated from Eqs.(2.45) and (2.46) for the electrode surfaces.

The rectangular region \( s \times h \) is subdivided by 20 \( \times 40 \) rectangular meshes of an equal size. In this connection, it has been
ascertained that the calculation results by those meshes agree very well with that by 50 x 100 rectangular meshes.

In addition, with respect to the numerical example in this thesis, it has been seen that if the forward or backward difference approximation is applied to the electrode surface, it is very difficult to find out the converging solutions of the basic equations.

2.5.3 Relaxation parameter** in SOR method

The simultaneous difference equations (2.45) can be solved under Eqs. (2.46), (2.40), and (2.42) by means of the SOR method (successive over-relaxation method). However, for converging the calculation of the strong current concentration which rises often at an electrode end in an MHD generator duct, the basic difference equations require a considerably smaller relaxation parameter $\omega = \omega^* (= 0.1 \text{ to } 0.4)$ than the conventional value. Therefore the digital computation cannot rapidly converge and wastes a long time.

Accordingly, the above small value $\omega = \omega^*$ is used only for the numerical calculation near the electrode. On the other hand, within the duct, the other relaxation parameter $\omega = \omega_0 (=1 \text{ to } 2)$ of the conventional magnitude is used to accelerate the calculation. Moreover, if the residuals of the solutions of the difference equations decrease, $\omega_0$ is increased by a suitable value $\omega'$, and conversely if they increase, $\omega_0$ is reduced by another suitable value $\omega''$.

For example, when $\omega^* = 0.3$ and $\omega_0 = \omega^*$, the repetition calculation of 4451 times was required for the residuals to become smaller than $10^{-3}$, but when $\omega^* = 0.3$ and $\omega_0 = 1.0$ to 1.84 ($\omega' = 0.002$ and $\omega'' = 0.02$) only the repetition of 592 times was done. In these examples, the numerical solutions agreed very well with those by a conformal mapping [1.6] or the equivalent circuit in the case where $\sigma$, $\beta$, and $u$ were constant.

** See Appendix III.
2.6 Concluding Remarks

In this chapter, a two-dimensional theory for analysis of the diagonal type MHD generator has been introduced. The main conclusions are as follows.

(1) The nonlinear partial differential equations were derived from the basic equation, by which the current and potential distributions in the MHD generator duct can be numerically calculated.

(2) The electrical efficiency, the specific Hall voltage, etc. were derived, by which the electrical characteristics of the diagonal type generator can be estimated.

(3) The method of numerical calculation of the basic equations was introduced, which includes the normalized basic equations, the difference basic equations, and the modified SOR method.
CHAPTER 3
BASIC ELECTRICAL CHARACTERISTICS

3.1 Introduction

The two- or three-dimensional analyses of the current and potential distributions, etc. in the Faraday type generator duct have been carried out by many researchers [2.1] to [2.3] et al., and its basic electrical characteristics have been fairly clarified.

However, the usual Faraday type generator is a current-controlled generator, and on the other hand, the diagonal type generator is a potential-controlled one because of the diagonal connections of electrodes. Therefore, the electrical characteristics of the diagonal type generator duct differ fairly from those of the Faraday type so that they must be clarified independently for the practical use of the diagonal type generator.

In this chapter, first, when the electrical conductivity or the Hall parameter are assumed constant in the generator duct, the current distribution is evaluated and influences of the load factor, etc. on it are investigated. The current distribution in the diagonal type generator is compared with that in the Faraday or Hall type generators. Then, influences of the inclination parameter, the Hall parameter, etc. on the electrical characteristics such as electrical efficiency of the diagonal type generator are studied. Next, when the electrical conductivity or the Hall parameter vary spatially, the distributions of current, potential, and electron temperature are evaluated and influences of the load factor and the inclination parameter on the current distribution are discussed. Then, the influences of the duct size and the temperatures of plasma and duct wall on the electrical characteristics of generator duct are investigated.

These investigations are carried out by means of the theory described in Chapter 2. In this connection, the Saha equilibrium
is assumed in the numerical calculations, since the gas is assumed to be a subsonic flow.

3.2 Numerical Conditions

The numerical analysis in this chapter is carried out for the diagonal type MHD generator with a cesium-seeded helium plasma in nonequilibrium ionization in which

\[
\begin{align*}
    h &= 0.2, \quad w = 0.1, \quad s = 0.1, \quad c = 0.06 \text{ m}, \\
    B &= 4, 5 \text{ T}, \quad u_0 = 2000 \text{ m/s}, \quad T_0 = 1700 \text{ to } 2000 \text{ K}, \quad p = 5 \text{ atm}, \quad \varepsilon_s = 0.3 \%, \\
    \delta &= 5, \quad m = m' = 1/7
\end{align*}
\]

where \( \varepsilon_s \) is the seed fraction of the cesium, and it is assumed that the duct size is related to the somewhat large experimental duct, and the gas velocity is subsonic.

3.3 Numerical Calculation Processes

![Diagram](image)

Fig. 2.2. Current distribution for constant \( \sigma \) and \( \beta \) (1).
The value of the stream function $\psi$ is determined by solving the basic difference equations (2.44) and (2.45) under the boundary conditions (2.40) and (2.46) and the subsidiary conditions (2.28) and (2.29) by the modified SOR method. The calculation processes are as follows.

(1) The values of $\psi$ on the insulators are plausibly assumed to satisfy the subsidiary condition (2.29).

(2) The reasonable values of $I$, $\sigma$, $\beta$, $e$, $n_e$, and $T_e$ are given, and then the value of $\psi$ is obtained by solving the difference equation (2.45) under the conditions (2.40), (2.42), and (2.46) by the SOR method.

(3) Using the Newton-Raphson method, the value of $T_e$ is calculated by Eq. (2.43), in which the value of $\psi$ obtained in the above process (2) is used. Then, by Eqs. (2.10), (2.11), etc. $\sigma$, $\beta$, and

\[ \sigma = 2.20, \quad \beta = 2.00, \quad \kappa = 0.703, \quad \alpha = -1 \]

Fig. 2.3. Current distribution for constant $\sigma$ and $\beta$ (2).
\( n_e \) are calculated.

4. The digital computation in the processes (1) to (3) are repeated until \( \psi \) converges to a constant value for \( I \) given in the process (2).

5. To satisfy the subsidiary condition (2.28) the previous value of \( I \) is changed to a new value. After many repetitions of the processes (2) to (4), the desirable value of \( \psi \) is acquired.

In addition, the potential \( \phi \) can be obtained by the similar processes.

3.4 When Electrical Conductivity and Hall Parameter Are Assumed Constant

3.4.1 Current distribution and influence of load factor

In this section, the current distribution, influences of the load factor, the Hall parameter, etc. are discussed when \( \sigma \) and \( \beta \) are assumed constant in the generator duct.

When \( \sigma \) and \( \beta \) are spatially constant, \( P, Q, \) and \( R_1 \) in Eq. (2.16)
become zero, and the equation reduces to the Laplace equation

$$\nabla^2 \psi = 0.$$  \hfill (2.48)

If values of $\sigma$ and $\beta$ are assumed, the numerical solution of Eq. (2.48) is found through the calculation processes (1), (2), and (4) described in Section 3.3, and then the current distribution in the duct is obtained.

In Figs. 2.2 and 2.3, the current distributions are plotted in the case of $K = 0.375; J = 0.532 \text{ A/cm}^2$ and $K = 0.703; J = 0.684 \text{ A/cm}^2$, respectively, where $\sigma = 2.2 \text{ \mu/m}$, $\beta = 2$, $B = 4T$, and $a = -1 (\theta = 135^\circ)$. These figures show that the current distribution in the diagonal type generator remarkably varies with $K$. That is, the current flows from an upstream anode to a downstream cathode when $K < K^*$. On the other hand, it does from a downstream anode to an upstream cathode when $K > K^*$, where

$$K^* = \kappa |_{<J_x> = 0} = \frac{(1 - a \beta_{\text{eff}})(\beta_{\text{eff}} - a)}{\beta_{\text{eff}}(1 + a^2)}. \hfill (2.49)$$

---

*Fig. 2.5. Influence of $a$ on $\eta_e, Q_H, \sigma_{\text{eff}}/\sigma$, and $\beta_{\text{eff}}/\beta$.***
In this connection, the load current flows from the inlet to the exit of the generator duct in all cases, since an anode and cathode are diagonally connected.

This is explained by the following. Using Eqs. (2.35), the tangent of the angle $\theta'$ between the direction of a average current density $<J>$ and the x-axis [see Fig. 2.1] is expressed by the following equations

$$
tan \theta' = \frac{-<J_y>}{<J_x>}
$$

$$
= -\frac{\beta_{eff} <E_x> + <E_y> - <u>B}}{\{<E_x> - \beta_{eff} (<E_y> - <u>B)\}}
$$

$$
= \frac{1 + a^2 + (\beta_{eff} - a)(\beta_{eff} + a)K}{\beta_{eff}(1 + a^2) - (1 - a\beta_{eff})}
$$

$$
x (\beta_{eff} - a)K. \quad (2.50)
$$

![Diagram with labels B = 4, $<\sigma>$ = 222, $<\beta>$ = 2.00, $K$ = 0.449, $a$ = -1]

Fig. 2.6. Current distribution for nonuniform $\sigma$ and $\beta$ (1).
Accordingly,
\[
\frac{\theta (\tan \theta')}{\partial \kappa} = \frac{(1 + a^2)(1 + \beta_{\text{eff}}^2)(\beta_{\text{eff}} - a)}{\{\beta_{\text{eff}}(1 + a^2) - (1 - a\beta_{\text{eff}})\} x (\beta_{\text{eff}} - a)\kappa^2} > 0.
\tag{2.51}
\]

This equation shows that \(\theta'\) increases with \(\kappa\). Namely, when \(\kappa\) is small, \(\theta'\) is small and the current runs from an upstream anode to a downstream cathode. On the contrary, when \(\kappa\) becomes large, \(\theta'\) does large and the current from a downstream anode to an upstream cathode.

Moreover, since the duct must operate as a generator, the load factor \(\kappa > 0\). Therefore, we obtain the condition \(\theta' > \tan^{-1} \frac{1}{\beta_{\text{eff}}}\) from Eq.(2.50), and also the load current \(I\) has to flow from the inlet to the exit of the duct, consequently we obtain \(\theta' < \frac{3\pi}{2} - \theta\). Thus, we have the following constraint

![Diagram](image)

Fig. 2.7. Electron temperature distribution corresponding to current distribution in Fig. 2.6.
\[
\tan^{-1} \frac{1}{\beta_{\text{eff}}} < \theta' < 3\pi/2 - \theta
\]  
(2.52)

for \( \theta' \) in the generator duct.

From the above description, it is understood that the current distribution in a diagonal type generator varies very widely with \( \kappa \). On the other hand, since \( \langle j_x \rangle = 0 \) for the whole duct of the Faraday type generator and \( \langle j_x \rangle > 0 \) for the Hall type, the above-mentioned phenomena do not occur in both type ducts.

The state of the current concentration at the electrode end is fairly similar to that in the Faraday or Hall type generator shown in the references [2.1] of [2.5].

3.4.2 Influence of Hall parameter on electrical efficiency and specific Hall voltage

The variations of \( \eta_e', \beta_{\text{eff}}^I/\sigma \), and \( \beta_{\text{eff}}/\beta \) by \( \beta \) are shown in Fig.2.4 in the case of \( \kappa = 0.5, B = 4T, a = -1, \) and \( \sigma = 2.0 \Omega/m. \)

The figure shows that \( \beta_{\text{eff}}^I/\sigma \) and \( \beta_{\text{eff}}/\beta \) increase with \( \beta \), and

\[ B = 4, \langle \sigma \rangle = 2.22, \langle \beta \rangle = 2.00, \kappa = 0.449, a = -1 \]

Fig. 2.8. Potential distribution corresponding to current distribution in Fig.2.6.
then they saturate. These are because $V_H$ increases as $\beta$ becomes large, but when $\beta$ becomes larger the current concentration at electrode ends and the nonuniformity of the current distribution become intensive. $\eta_e$ has the maximum value near $\beta = 4$.

3.4.3 Influence of inclination parameter on electrical efficiency and specific Hall voltage

The relations of $\eta_e$, $Q_H$, $\sigma_{\text{eff}}/\sigma$, and $\beta_{\text{eff}}/\beta$ to $\alpha$ are plotted in Fig.2.5 in the case of $\kappa = 0.5$, $B = 4T$, $\sigma = 2.0 \mu\Omega/m$, and $\beta = 2$. It is seen that $\eta_e$ and $Q_H$ increase with $|\alpha|$, and that those of the Hall type generator ($\alpha = 0$ or $\theta = 90^\circ$) become fairly small, namely its electrical characteristics are inferior to those of the diagonal type generator.

Fig. 2.9. Variation of electric potential $\Phi$ between confronted anode and cathode corresponding to current distribution in Fig.2.6.
3.5 When Nonuniformity of Electrical Conductivity and Hall Parameter Are Considered

3.5.1 Current, electron temperature, and potential distributions

In this section, the current, electron temperature, and potential distributions are evaluated and influences of $\beta$, $a$, $h$, $T_0$, and $T_w$ on the generator characteristics is studied when $\sigma$, $\beta$, and $\beta_i$ vary according to Eqs. (2.10) to (2.12). In addition, it was made clear by the numerical calculation results that two terms of the ion-slip $\beta \beta_i (J \times B) \times B / B^2$ and the gradient of the electron pressure $V_p / e n_e$ in Eq. (2.4) are much smaller than the other two terms under the numerical conditions in this thesis.

In Figs. 2.6, 2.7, and 2.8 are plotted the distributions of current, electron temperature, and potential, respectively, and in Fig. 2.9 the variation of potential $\phi$ between a pair of confronted

Fig. 2.10. Current distribution for nonuniform $\sigma$ and $\beta$ (2).
anode and cathode in the case of $\kappa = 0.499$, $B = 4T$, $a = -1$, $T_0 = 1800$ K, $T_w = 1600$ K, $<\sigma> = 2.2 \, \Omega/m$, $<\beta> = 2.00$, and $<J> = 0.468$ A/cm$^2$. By comparison between Fig.2.6 and 2.2, it is found that the current concentration at the electrode end in the case of non-uniform $\sigma$ and $\beta$ becomes more intensive than that in the case of constant $\sigma$ and $\beta$, and that the eddy current is induced on the electrode surfaces. The phenomena similar to these arise in the Faraday type generator [2.2]. This is because the Joule heating due to current concentration makes $T_e$ and therefore $\sigma$ high in the case of the nonuniform $\sigma$ and $\beta$.

As seen in Fig.2.2, the current distribution is almost uniform in the main flow where the gas velocity becomes almost constant, and its nonuniformity in the x direction does not yet occur there. However, in Fig.2.6, the nonuniformity in the x direction appears clearly in the main flow.

\[ B = 5, \, <\sigma> = 3.44, \, <\beta> = 2.46, \, K = 0.630, \, a = -1 \]

Fig. 2.11. Current distribution for nonuniform $\sigma$ and $\beta$ (3).
Next, Fig. 2.7 shows that the distribution of the electron temperature becomes very nonuniform and $T_e$ varies from 1600 to 2200 K. In addition, $\sigma$ varies 0.019 to 7.72 $\text{m}/\text{m}$ and $\beta$ 1.63 to 2.03.

It is seen from the potential distribution between the confronted anode and cathode in Fig. 2.9 that the electrode voltage drop is about 13% of the potential difference between the confronted electrodes.

3.5.2 Influence of load factor on current distribution

In Fig. 2.10, the current distribution is plotted for $\kappa = 0.533$, $<\sigma> = 2.68 \text{ m}/\text{m}$, $<\beta> = 2.49$, and $\beta_{\text{crit}} = 1.77$, and in Fig. 2.11, that for $\kappa = 0.630$, $<\sigma> = 3.44 \text{ m}/\text{m}$, $<\beta> = 2.46$, and $\beta_{\text{crit}} = 1.61$, where $B = 5\text{T}$, $T_0 = 1800 \text{ K}$, and $T_w = 1600 \text{ K}$.

In Fig. 2.10 where $\kappa$ is small, the current flows from an upstream anode to a downstream cathode, but in Fig. 2.11 where $\kappa$ is large, it runs from a downstream anode to an upstream cathode in

![Image of current distribution diagram](image-url)

Fig. 2.12. Current distribution for nonuniform $\sigma$ and $\beta$ (4).
the similar way to the case of constant $\sigma$ and $\beta$. Such phenomena cannot occur in the Faraday and Hall type generators. Also, from Figs. 2.6, 2.10, and 2.11, it is seen that the state of the nonuniformity in the $x$ direction in the main flow is changed with $\kappa$. Besides, as seen in Figs. 2.10 and 2.11, when $<\beta>$ becomes high and exceeds $\beta_{\text{crit}}$, the streamer due to the ionization instability grows [1.11] to [1.13]. Moreover both figures indicate that the state of the streamers varies with $\kappa$, and hence the state of ionization instability does with $\kappa$.

3.5.3 Influence of inclination parameter on current distribution

In Figs. 2.12 and 2.13, the current distributions are shown for $\kappa = 0.652$ and $a = 0$, and $\kappa = 0.658$ and $a = -2$, respectively, where $B = 4T$, $T_0 = 1800$ K, and $T_w = 1600$ K.

These figures tell that the current distribution varies remarkably by the values of $a$ and that the current density increases with decreasing $|a|$. The reason for these is the following. From

\[ B = 4, <\sigma> = 157, <\beta> = 2.03, \kappa = 0.658, a = -2 \]

Fig. 2.13. Current distribution for nonuniform $\sigma$ and $\beta$ (5).
the subsidiary condition (2.29), the following equation is derived

\[ <J_n> = \frac{I}{wh\sqrt{1 + a^2}}, \]  \hspace{1cm} (2.53)

where \( <J_n> \) is the average value of the current density perpendicular to the cross-section C'A' in Fig.2.1. The above equation indicates that the current density in the Hall type duct (\( a = 0 \)) is \( (1 + a^2)^{1/2} \) times as large as that of the diagonal type (\( |a| > 0 \)), even when the load current \( I \) is equal. Therefore, the current concentration, the nonuniformity of its distribution, etc. in the Hall type generator become more intensive than those in the diagonal type generator.

3.5.4 Influence of duct height on current distribution and electrical characteristics

![Diagram](image)

\( B=5, \langle \sigma \rangle = 4.28, \langle \beta \rangle = 2.46, h = 0.3, a = -1 \)

Fig. 2.14. Current distribution for nonuniform \( \sigma \) and \( \beta \) (6).
In the case of $B = 5\, T$, $a = -1$, $T_0 = 1800\, K$, $T_w = 1600\, K$, and $I_x = 0\, A$, Figs. 2.14 and 2.15 show the current distributions for $h = 0.3$ and $0.4\, m$, respectively, where the influence of the boundary layer are excluded in this article. From the figures, it is seen that the current distribution is not largely influenced by the duct size when a nonequilibrium ionization grows. In Fig. 2.16, the variations of $n_e$, $Q_H$, $\sigma_{eff}/\langle\sigma\rangle$, and $\beta_{eff}/\langle\beta\rangle$ by $h/s$ are plotted. As seen from the figure, $n_e$ and $Q_H$ are little dependent on $h/s$, while $\sigma_{eff}/\langle\sigma\rangle$ increases with $h/s$. This is explained by the following. In Figs. 2.14 and 2.15, the current distribution has the repetition of the almost same pattern except the region near duct wall, and it is little influenced by the duct size, if the duct size is much larger than the size of the pattern. Accordingly, it is thought that our investigation can be sufficiently carried out by the

\[ B = 5, \langle\sigma\rangle = 5.04, \langle\beta\rangle = 2.44, h = 0.4, a = -1 \]

Fig. 2.15. Current distribution for nonuniform $\sigma$ and $\beta$ (7).
3.5.5 Influence of temperatures of central gas flow and duct wall on electrical characteristics

In Fig. 2.17 the variations of $n_e$, $Q_H$, etc., by $T_0$ are plotted in the case of $B = 4T$, $a = -1$, $T_0 - T_w = 200 K$, and $\kappa = 0.5$. The figure tells that the changes of $n_e$ and $\beta_{\text{eff}}/\langle\beta\rangle$ due to $T_0$ are large and they increase fairly with $T_0$, but on the other hand, the variations of $Q_H$ and $\sigma_{\text{eff}}/\langle\sigma\rangle$ by $T_0$ is very small.

The reason of the above phenomena is shown below. When $T_0$ is low, nonequilibrium ionization occurs more intensively than when $T_0$ high, and consequently the nonuniformity of the current distribution becomes severe in the former case. Accordingly, $n_e$ falls as $T_0$ decreases. On the other hand, the decrease of $T_0$ makes $\langle\sigma\rangle$ and $\langle J \rangle$ low, so that the effects of $\langle\sigma\rangle$ and $\langle J \rangle$ on $Q_H$ offset each other, since $V_H$ depends on $\langle J \rangle/\langle\sigma\rangle$. $V_H$ or $Q_H$ are scarcely changed.

Fig. 2.16. Influence of $h/s$ on $n_e$, $Q_H$, $\sigma_{\text{eff}}/\langle\sigma\rangle$, and $\beta_{\text{eff}}/\langle\beta\rangle$. 
Fig. 2.17. Influence of $T_0$ on $\eta_e$, $Q_H$, $\sigma_{\text{eff}}/\langle\sigma\rangle$, and $\beta_{\text{eff}}/\langle\beta\rangle$.

Fig. 2.18. Influence of $T_w$ on $\eta_e$, $Q_H$, $\sigma_{\text{eff}}/\langle\sigma\rangle$, and $\beta_{\text{eff}}/\langle\beta\rangle$ (1).
Next, Fig. 2.18 shows the variations of $\eta_e$, $Q_H$, $\sigma_{\text{eff}}/<\sigma>$, and $\beta_{\text{eff}}/<\beta>$ by $T_w$ in the case of $B = 4T$, $a = -1$, $T_0 = 1800$ K, and $\kappa = 0.5$. It is seen that $\eta_e$ and $\beta_{\text{eff}}/<\beta>$ decrease slightly, and on the other hand, $Q_H$ and $\sigma_{\text{eff}}/<\sigma>$ increase fairly with $T_w$. These can be explained as follows. The intensive current concentration at the electrode ends already grows and the current distribution in the duct varies only a little with $T_w$. Therefore, $T_w$ has only a little influence on $\eta_e$ which is much influenced by the current distribution as seen from Eq.(2.30). On the other hand, the drop due to thermal boundary layer increases with decreasing $T_w$, while the degree of decrease of $<\sigma>$ is small because $T_0$ is assumed constant. $V_H$, on which $<J>/<\sigma>$ has a large influence, namely $Q_H$ decreases with lowering $T_w$.  

Fig. 2.19. Influence of $T_w$ on $\eta_e$, $Q_H$, $\sigma_{\text{eff}}/<\sigma>$, and $\beta_{\text{eff}}/<\beta>$ (2).
Moreover, in Fig. 2.19 the influence of $T_w$ on $n_e$, $Q_H$, etc. is plotted in the case of $B = 5T$, $a = -1$, $T_0 = 1800$ K, and $l_x = 0$. The layer of the low electrical conductivity due to the decrease of $T_w$ is apt to make the current distribution uniform, for example the value of $J_{\text{peak}}/\langle J \rangle_{\text{el}}$ changes from 14.0 to 12.6 as $T_w$ decreases from 1800 to 1000 K, and also to prevent the current from flowing in the case of $B = 5T$, where the higher degree of the nonequilibrium ionization occurs. Accordingly, the electrical characteristics become best when $T_0 - T_w = 300$ to 400 K in the case of ionization instability.

3.6 Concluding Remarks

The basic electrical characteristics of the diagonal type generator have been investigated by a two-dimensional analysis. The main results are as follows.

(1) The current concentration at the electrode end is fairly similar to that in the Faraday and the Hall type generators.

(2) The current distribution in the diagonal type generator is remarkably affected by the values of the load factor and the inclination parameter, which differs from the states in the Faraday and the Hall type ducts.

(3) In the main flow, the large nonuniformity of the current distribution in the $x$ direction grows if nonequilibrium ionization is made strong. The state of the nonuniformity is varied remarkably by the values of the load factor and the inclination parameter.

(4) The effects of the duct height on the electrical characteristics become small as the nonequilibrium ionization instability grows.

(5) The electrical characteristics of the generator degrades as the temperatures of the central gas flow and the duct wall lower, where the former and the latter affect mainly the electrical efficiency and the specific Hall voltage, respectively.
CHAPTER 4

EFFECTS OF RESISTIVE ELECTRODE AND DISTRIBUTION
OF APPLIED MAGNETIC FLUX ON
ELECTRICAL CHARACTERISTICS

4.1 Introduction

As shown from the results which have been obtained in Chapter 3, the current concentration at the electrode ends makes the current distribution in the generator duct nonuniform, and the Joule heating by the concentration may damage the electrodes and break down the insulation between the adjacent electrodes.

The current concentration is caused by short-circuiting of the axial Hall electric field through a good conductor electrode. For the Faraday type generator, as one way removing the concentration, the use of a resistive electrode has been proposed and already its effects have been investigated in fair detail [2.25] to [2.28]. As another way, reduction of the applied magnetic flux in the direction perpendicular to both the flux density and the gas velocity in a region near the duct wall has been thought out and its effects have been discussed, too [2.29].

On the other hand, their influences on the current concentration at the electrode ends and the performance characteristics of the diagonal type generator have been yet little investigated. However, as found in Chapter 3, the current distribution in the diagonal type generator duct varies remarkably with the load factor, which noticeably differs from that in the Faraday type. Accordingly, it is necessary to study their effects on the performance characteristics of the diagonal type generator.

In this chapter, influences of resistive electrode and attenuation of the magnetic flux density in the neighbourhood of the duct wall on the performance characteristics of the diagonal type generator with nonequilibrium ionization plasma are investigated.
by a two-dimensional analysis. The configuration of resistive electrode and the distribution of magnetic flux adopted in this thesis are shown. Next, current distributions are calculated, and the influence of conductivity and configuration of wedge-shaped resistive electrode on the current concentration and electrical efficiency is investigated. Besides, the electrical characteristics of generator with the attenuation of the magnetic flux density in the region near duct wall are compared with those of the generator with the constant magnetic flux density.

4.2 Effects of Resistive Electrode

4.2.1 Basic equations in resistive electrode

The basic equations for the gas plasma in the generator duct has been already shown in Section 2.1. In this article, only the basic equations in a resistive electrode are introduced.

The basic equations in the resistive electrode are the Maxwell equations
\[ \nabla \times \mathbf{E}' = 0, \quad \nabla \cdot \mathbf{J}' = 0, \] (2.54)
and Ohm's law
\[ \mathbf{J}' = \sigma \mathbf{E}', \] (2.55)
where \( \mathbf{E}' \) and \( \mathbf{J}' \) are the electric field intensity and current density vectors in the resistive electrode, respectively, and \( \sigma \) the conductivity of the electrode [see Fig. 2.20]. They are assumed as
\[ \mathbf{E}' = (E'_x, E'_y, 0), \quad \mathbf{J}' = (J'_x, J'_y, 0) \]
\[ \sigma = \text{const.} \] (2.56)

Using a stream function \( \psi' \) of the same type as \( \psi \) in Eqs. (2.14), which is derived as
\[ J'_x = \partial \psi' / \partial y, \quad J'_y = -\partial \psi' / \partial x, \] (2.57)
to evaluate the current distribution in the resistive electrode,
the following Laplace equation

$$\nabla^2 \psi' = 0$$ \hspace{1cm} (2.58)

can be obtained.

![Diagram of diagonal type generator duct with resistive electrodes](image)

**Fig. 2.20.** Schema of diagonal type generator duct with resistive electrodes.

### 4.2.2 Boundary and subsidiary conditions

The boundary condition (2.23) on the electrode surface in Chapter 2 is replaced by the following two equations.

$$E_x' = E_x, \quad J_y' = J_y.$$ \hspace{1cm} (2.59)

Using the stream functions $\psi$ and $\psi'$, these equations are trans-
formed as
\[
\frac{\partial \psi'}{\partial y} = \sigma_e \left( \frac{\partial \psi}{\partial y} - \frac{\partial \psi}{\partial x} \right) - \frac{\partial p_e}{\partial x} \left( en_e \right),
\]
respectively. The other boundary and subsidiary conditions are the same as Eqs. (2.21) and (2.27) to (2.29) in Chapter 2; A and C in Eq. (2.28) must be replaced by collectors A and C in Fig. 2.20.

4.2.3 Configuration of resistive electrode

When let us express the angle between \( J' \) and the normal of the boundary surface between the gas plasma and the resistive electrode with \( \alpha' \), the angle of intersection between an equipotential surface in the resistive electrode and the boundary surface is \( \alpha' \) as seen from Fig. 2.21, since \( E' \) and \( J' \) are parallel each other.

This indicates that a collector should be arranged on an equipotential surface [2.25], which is made of a good conductor. The configuration of resistive electrode can be assumed to be the wedge shape as shown in Fig. 2.20.

Then, Eqs. (2.4), (2.55), and (2.59) yield
\[
tan \alpha' = \sigma_e (\beta - tan \alpha) / \sigma,
\]
where \( \alpha \) is an angle between \( J \) and the normal of the boundary surface [see Fig. 2.21].

4.2.4 Numerical conditions

In this chapter, the numerical calculation is made for the diagonal type generator with cesium-seeded helium plasma in nonequilibrium ionization.

Numerical conditions are given by
\[
\begin{align*}
\frac{h}{2.25} = 0.2, & \quad w = 0.1, & \quad s = 0.1, & \quad c = 0.06 \text{ m,} \\
B = 4 \text{ to } 6 \text{ T,} & \quad u_0 = 2000 \text{ m/s,} & \quad T_0 = 1800, \\
\end{align*}
\]

(2.62)
\[ T_v = 1600 \text{ K, } p = 5 \text{ atm, } \varepsilon_s = 0.3 \%, \]
\[ \delta = 5, \quad m = m' = 1/7, \quad \theta = 135^\circ. \]

The values of \( B, B_w, \alpha', \delta_B, \sigma_e, \) and \( \kappa \) are varied and influences of those variations on the current distribution, the electrical characteristics, etc. are investigated.

4.2.5 Effect of resistive electrode

As shown in Fig.2.11, the intensive current concentration occurs at the end of a good conductor electrode, and there \( J_{\text{peak}}/\langle J \rangle_{\text{el}} = 13.3. \)

Figure 2.22 shows the current distribution in the generator duct with the resistive electrodes, where \( B = 5T, \kappa = 0.744, \alpha = -1, \sigma_e = 0.4 \text{ V/m, } \alpha' = 30^\circ, \langle \sigma \rangle = 1.71 \text{ V/m, } \langle \beta \rangle = 2.53, \) and \( \beta_{\text{crit}} = 3.81. \) The figure indicates that the excessive current concentration at the electrode end as seen in Fig.2.11 can be eliminated by use of the resistive electrode. In this case \( J_{\text{peak}}/\langle J \rangle_{\text{el}} = 1.46. \)
Next, Fig. 2.23 represents the current distribution in the case of $B = 6T$, $\kappa = 0.273$, $\sigma_e = 0.4 \, \text{V/m}$, $\alpha' = 30^\circ$, $\langle \sigma \rangle = 1.50 \, \text{V/m}$, $\langle \beta \rangle = 3.06$, and $\beta_{\text{crit}} = 3.29$. The current distribution becomes a little more nonuniform than that in Fig. 2.22, but a streamer does not grow yet.

Figure 2.24 shows the current distribution for $B = 6T$, $\kappa = 0.716$, $\sigma_e = 0.4 \, \text{V/m}$, $\alpha' = 30^\circ$, $\langle \sigma \rangle = 1.94 \, \text{V/m}$, $\langle \beta \rangle = 3.02$, and $\beta_{\text{crit}} = 2.17$. From the figure we see that the current scarcely concentrates at the electrode end, though the streamer occurs. These denote that the wedge-shaped resistive electrode can suppress occurrence of the current concentration at the electrode end in the same way as the Faraday type generator, and also that it can suppress the growth of streamer in the gas plasma to some ex-
In Fig. 2.25 is presented the influence of $\kappa$ on $J_{\text{peak}}/<\langle J\rangle_{\text{el}}$ on the electrode surface when $B = 5T$, $\alpha' = 30^\circ$, and $\sigma_e = 0.4 \, \text{U/m}$. Under these numerical conditions, the variation of $J_{\text{peak}}/<\langle J\rangle_{\text{el}}$ by $\kappa$ is small and the values of $J_{\text{peak}}/<\langle J\rangle_{\text{el}}$ become smaller than 1.46, although $\kappa$ varies from 0.281 to 0.744. These show that the resistive electrode can hold its validity against a large variation of $\kappa$, though the current distribution remarkably varies by $\kappa$ in a diagonal type generator.

In Figs. 2.26 and 2.27, the variations of $\eta_e$, $Q_H$, $J_{\text{peak}}/<\langle J\rangle_{\text{el}}$, $I$, $P$, $\sigma_{\text{eff}}/<\sigma>$, and $\beta_{\text{eff}}/<\beta>$ by $\sigma_e$ are plotted for $\alpha' = 5^\circ$ and $30^\circ$, respectively, where $B = 4T$ and $\kappa = 0.5$. Figure 2.26, where $\alpha'$ is small, shows that the current concentration at the electrode end is largely weakened, and $\eta_e$ and $Q_H$ increase a little with de-

---

**Fig. 2.23.** Current distribution in generator duct with resistive electrodes (2).
creasing $\sigma_e$, but $I$ and $P$ decrease. Figure 2.27 tells that current concentration is most removed at $\sigma_e = 0.5 \ \mu$m. This value of $\sigma_e$ roughly coincides with the value obtained from Eq. (2.61) under the above numerical conditions.

Moreover, Figs. 2.26 and 2.27 inform that the variation of $\eta_e$, $Q_H$, etc. is small in the $\sigma_e$ range of about 0.1 to 1 \ \mu/m, and thus the use of the resistive electrode will not degrade the electrical characteristics of the generator. $\sigma_e = 0.1$ to 1 \ \mu/m will be suitable for the resistive electrode. In this connection, many investigations of electrode materials have been carried out, and the electrical conductivity of $ZrO_2-CeO_2$ mixture or spinels has been found to show above-mentioned values [2.30] and [2.31].

\[
\begin{align*}
B &= 6, \langle \sigma \rangle = 1.94, \langle \beta \rangle = 3.02, \\
\kappa &= 0.716, \beta_e = 0.4, \alpha' = 30^\circ
\end{align*}
\]

Fig. 2.24. Current distribution in generator duct with resistive electrodes (3).
4.3 Effects of Distribution of Applied Magnetic Flux

4.3.1 Distribution of applied magnetic flux

It has been proposed to eliminate the current concentration at the electrode ends by an attenuation of the magnetic induction at the region near duct wall in the $y$ direction, where the Hall effect decreases because of a low magnetic flux density [2.29].

Distribution of the applied magnetic flux $B$ is assumed to be given by the expression

$$B = \begin{cases} 
(B_0 - B_w) / \delta_B y/h + B_w, & 0 < y/h < \delta_B \\
B_0, & \delta_B < y/h < 1 - \delta_B \\
-(B_0 - B_w) / \delta_B y/h + (B_0 - (1 - \delta_B) B_w) / \delta_B, & 1 - \delta_B < y/h < 1,
\end{cases} \tag{2.63}$$

as shown in Fig. 2.28 where $B_w$ is the magnetic flux density at the duct wall, and $\delta_B$ is the length shown in the figure. The distr-
bution of magnetic flux can have approximately the above configuration when the generator duct protrudes from the magnet in the y direction.

4.3.2 Effects of distribution of applied magnetic flux

The current distributions in the duct are plotted in Figs. 2.29, 2.30, and 2.31 in the case of $k = 0.114, 0.362, \text{ and } 0.617$, respectively, where $B_0 = 5T$, $B_w = 2T$, and $\delta_B = 0.1$. These figures show that the streamers are formed in the x, y, and negative x directions for $k = 0.114, 0.362, \text{ and } 0.617$, respectively. Namely, the current distribution in the diagonal type duct is very dependent on the value of $k$.

![Fig. 2.26. Influence of $\sigma_e$ on $\eta_e$, $Q_H$, etc. (1).](image)
In Comparison with Fig.2.11 which shows the current distribution for $\kappa = 0.630$ and $B_w = B_0 = 5T$, Fig.2.29 tells that the circulating current near the electrodes is eliminated and the current distribution there becomes uniform to some extent when $B_w$ is small, but the current still concentrates at electrode end and the main streamer is influenced little by $B_w$. Furthermore, as seen in Figs. 2.30 and 2.31, the new streamer occurs in the neighborhood of the insulating walls in parallel with the insulating walls when $\kappa$ is large.

Figure 2.32 shows influences of $B_w$ on the generator performance when $\kappa = 0.6$ and $B_0 = 5T$. The figure indicates that $J_{\text{peak}} / \langle J \rangle_{\text{el}}$ decreases with $B_w$ and its value for $B_w = 0$ is about 75% of that for $B_w = B_0$, but $\eta_e$, $Q_H$, etc. are affected little by $B_w$. The current concentration at the electrode end can be eliminated to some extent by decreasing $B_w$, but the main streamer is hardly dependent on $B_w$. Consequently, the generator performance is affected

![Graph](image)

Fig. 2.27. Influence of $\sigma_e$ on $\eta_e$, $Q_H$, etc. (2).
Fig. 2.28. Distribution of applied magnetic flux.

Fig. 2.29. Current distribution in generator duct (1).
Fig. 2.30. Current distribution in generator duct (2).

Fig. 2.31. Current distribution in generator duct (3).
Fig. 2.32. Influence of $B_w$ on $\eta_e$, $Q_H$, etc.

Fig. 2.33. Influence of $B_w$ and $K$ on $J_{\text{peak}}/\langle J\rangle_{\text{el}}$. 

$B_0 = 5$, $K = 0.6$, $\sigma_e = \infty$
little by attenuation of $B$ in $y$ direction near the duct wall.

From Fig.2.33, in which the variation of $J_{\text{peak}}/\langle J \rangle_{el}$ by $\kappa$ is plotted, it is seen that $J_{\text{peak}}/\langle J \rangle_{el}$ becomes about 65.0, 72.5, and 75.0% of those for $B_w = B_0^*$ in the cases of $\kappa = 0.2$, 0.4, and 0.6, respectively. This indicates that current concentration is eliminated better as $\kappa$ is small.

Furthermore, it was ascertained that $J_{\text{peak}}/\langle J \rangle_{el}$ becomes 10.9, 11.2, and 10.6 for $\delta_B = 0.1$, 0.2, and 0.4, respectively, when $B_0 = 5T$, $B_w = 2T$, and $\kappa = 0.4$. This shows that variation of $J_{\text{peak}}/\langle J \rangle_{el}$ by $\delta_B$ is small. In addition, a little short-circuit current flows between the adjacent electrodes when $\delta_B = 0.4$.

4.4 Concluding Remarks

The effects of wedge-shaped electrode and attenuation of magnetic flux density in the region near duct wall on the performance of the diagonal type MHD generator with nonequilibrium plasma have been numerically investigated. The main results are as follows.

(1) The wedge-shaped resistive electrode can largely remove the current concentration at the electrode ends of the diagonal type MHD generator with nonequilibrium plasma.

(2) It is not necessary to vary the conductivity of the resistive electrode according to the load factor, although the current distribution is varied remarkably by the load factor in the diagonal type generator.

(3) The generator characteristics such as the electrical efficiency and the specific Hall voltage are not degraded by the use of wedge-shaped resistive electrode.

(4) Attenuation of magnetic flux density in the region near the duct wall can eliminate the current concentration to some extent.

(5) The elimination of the current concentration varies with the load factor and decreases with increasing the load factor.
CHAPTER 5
END EFFECTS

5.1 Introduction

An eddy current is apt to be generated in the end regions of a generator, since spatial differences of the induced electromotive forces are created there in the direction of attenuation of the magnetic flux density. It distorts a normal current distribution in the generator duct ends and degrades the total electrical characteristics of the generator. These may be intensive especially in the diagonal type generator, because the output electrodes are arranged in the inlet and exit regions of generator duct.

The end effects in the Faraday type generator have been analyzed in fair detail [2.32] to [2.34]. On the other hand, those in the diagonal type have been only little discussed [2.35] to [2.37]. Accordingly, the end effects in the diagonal type generator are studied by a two-dimensional analysis.

In this chapter, the arrangement of the output electrodes and the distribution of the applied magnetic flux are shown, which are used for the numerical investigation. Then, in the case of constant electrical conductivity and electron mobility, by the numerical analysis are investigated influences of the attenuation of the magnetic flux density and the types and position of output electrodes on the current distribution in the end regions, the potential difference between adjacent electrodes, the internal resistance, etc. in the end regions.

Next, in the case of variable conductivity and electron mobility, effects of the attenuation of magnetic flux density on the current distribution, the internal resistance, the ballast resistance, etc. are studied.

In addition, the current distribution in the inlet region becomes symmetric with the distribution in the exit region on the assumptions made in this thesis.
5.2 Arrangement of Output Electrodes and Distribution of Applied Magnetic Flux

5.2.1 Arrangement of Output Electrodes

(1) Arrangement of Output Electrodes

Among various types of arrangements of the output electrodes which have been proposed for the diagonal type generator, the effects of three types shown in Figs. 2.34, 2.35(a), and (b), which we call hereafter A, B, and C types for the convenience, will be investigated in this chapter. As seen in the figures, in A type the two electrodes $E_1$ and $E_2$ which are shorted each other, in B type two pairs of the diagonally connected end electrodes, and in C type two pairs of the Hall connected electrodes are equipped as the output electrodes, respectively. Of course, the anode $A_i$ and cathode $C_i$ ($i = 1, 2, \ldots, n$) except the output electrodes are connected diagonally.

(2) Subsidiary Conditions of End Region

The subsidiary and periodicity conditions (2.28) and (2.29) are replaced by

\[ V_i = - \int_{C_i} A_i E \cdot d\zeta = 0, \quad i = 1, 2, \ldots, n, \quad (2.64) \]

\[ \int_{S_i} J \cdot d\zeta = I, \quad i = 1, 2, \ldots, n, \quad (2.65) \]

respectively.

In this connection, behind the $n$-th electrode pair $A_n$ and $C_n$ [see Fig. 2.34], the electric quantities are assumed to vary periodically with one electrode pitch $s$ along the gas flow. Then, the periodicity condition (2.27) is rewritten as

\[ \varphi(x_1) = \varphi(x_1 - s) + \tau_y^{(n)}, \quad (2.66) \]

where $x_1$ is the x co-ordinate of the right edge of the insulating wall $C'_n$ in Fig. 2.34.
Fig. 2.34. Coordinate system and end region of diagonal type MHD generator duct (A type).

Fig. 2.35. Duct end geometries (B and C types).

Next, From Eqs.(2.14) and (2.65), we can derive the following equations

\[
\psi_i' - \psi'_i = 1, \quad i = 1, 2, \ldots, n
\]

\[
\psi = \psi/(I/\omega),
\]

(2.67)
where \( \psi_i^A \) and \( \psi_i^C \) are the values of \( \psi \) on the insulating wall surfaces \( A_i \) and \( C_i \), respectively, in Fig. 2.34.

First, if the values of \( I \) and \( \omega \) are assumed and \( \psi_i^A \) is given a plausible value, the value of \( \psi_i^C \) is decided by Eq. (2.67). When Eq. (2.16) is digitally solved by those values of \( \psi_i^A \) and \( \psi_i^C \) and the appropriately assumed values of \( u \), \( \sigma \), and \( \beta \), we can obtain the numerical solution of \( \psi \). By applying the solution to Eqs. (2.4) and (2.7), we can find the values of \( E_x \) and \( E_y \). Further, by substituting the values of \( E_x \) and \( E_y \) into the integral \( V_i = \int_{C_i}^{A_i} E \cdot ds \) in Eq. (2.64), we can decide the value of \( V_i \), but yet the value of \( V_i \) obtained is not necessarily equal to zero.

So let us consider the average resistance between the electrodes \( A_i \) and \( C_i \)

\[
R_{i}^1 = \frac{h}{\langle \sigma \rangle_{i} \cdot cw \cos(\pi - \theta)}, \quad i = 1, 2, \ldots, n, \quad (2.68)
\]

where \( \langle \sigma \rangle_{i} \) is the average electrical conductivity of gas plasma. We assume that an imaginary current defined by

\[
I_{i} = \frac{V_i}{R_{i}^1}, \quad i = 1, 2, \ldots, n, \quad (2.69)
\]

flows through the resistance \( R_{i}^1 \). To make \( V_i \) zero, it is necessary to flow the inverse current \( -I_i \) through \( R_{i}^1 \). Therefore, newly it is required to increase the current \( \omega(\psi_{i+1}^A - \psi_i^A) \) running into the anode \( A_i \) by \( -I_i \).

Again beginning with the new modified value of \( \psi_i^A \), we must repeat the above-mentioned calculation process. When \( V_i \) becomes adequately small after many repetitions of the above-mentioned process, then we can obtain a satisfactory numerical solution of \( \psi \).

5.2.2 Distribution of applied magnetic flux

For effective use of the magnet the MHD generator duct will be arranged in the attenuation domain of the magnetic induction \( B \). In order to investigate the influence of attenuation of \( B \) on the current distribution in the generator, we assume that \( B \) is constant in the central domain and decreases linearly from the left
edge of the \(j\)-th electrode in the end regions of the generator.

5.3 When Electrical Conductivity and Electron Mobility Are Assumed Constant

5.3.1 Numerical conditions

As a numerical example, we analyze the current and potential distributions in a diagonal type MHD generator duct, in which

\[
\begin{align*}
    h &= 0.3, \quad w = 0.3, \quad s = 0.075, \quad c = 0.056 \text{ m}, \\
    B_0 &= 4 \text{ T}, \quad I = 150 \text{ A}, \quad u_0 = 800 \text{ m/s}, \quad n = 9, \\
    \sigma &= 5.0 \text{ V/m}, \quad \mu_e = 0.5 \text{ and } 1.0, \quad \theta = 153^\circ,
\end{align*}
\]

(2.70)

where \(B_0\) is the magnetic flux density in the central region of the duct, and the five configurations of \(B\) are assumed as shown in Fig. 2.36, in which \(g\) is the gradient of \(B\) and \(j=8\).

In addition, the numerical results obtained in this section can be available for the open cycle generator.

![Diagram of applied magnetic flux](image)

Fig. 2.36. Distribution of applied magnetic flux for \(j=8\).
5.3.2 Effects of type of output electrodes and distribution of applied magnetic flux

(1) Current distribution

In Figs. 2.37 to 2.40, are plotted the current distributions in the duct of the A type arrangement in the cases of \( g = 0, 5.33, 8, \) and 10.6 T/m, respectively, and \( j = 8 \). In the figures and the later Figs. 2.41 to 2.44, the contour interval of current stream lines is 1/8 of the load current \( I \).

Fig. 2.37. Current distribution for \( g = 0 \) and \( j = 8 \) (A type).

Fig. 2.38. Current distribution for \( g = 5.33 \) and \( j = 8 \) (A type).
Fig. 2.39. Current distribution for \( g = 8 \) and \( j = 8 \) (A type).

Fig. 2.40. Current distribution for \( g = 10.67 \) and \( j = 8 \) (A type).

From Fig. 2.37, it is seen that the current concentrates only at the left end of the first output electrode \( E_1 \) and the second electrode \( E_2 \) acts little as the output electrode, and also that the large circulating current flows through the first electrode pair \( A_1-C_1 \) which are diagonally connected, when the magnetic flux density \( B \) does not attenuate.

Next, Fig. 2.38 shows that in the case of \( g = 5.33 \) the current distribution on the output electrodes becomes fairly uniform, but
an eddy current occurs in the duct end region, whose rate is about 13.3% of $I$.

Figure 2.39 indicates that the nearly equal currents flow into both output electrodes, and the current distribution on the electrodes becomes very uniform when $B$ decreases with $g = 8$. The figure also tells that the current concentration at the output electrode ends weakens, because $\beta$ becomes small in the area suffering a spatial attenuation of $B$. Further in this case, the eddy current vanishes.

Next, from Fig.2.40 we see that a new nonuniform current distribution occurs when $B$ is rapidly attenuated by $g = 10.67$. For example, the current stream is pushed toward the central part of duct and a shorting current flows between $E_2$ and $A_1$.

Figures 2.38 to 2.40 inform that arranging the output electrodes within the attenuating domain of the applied magnetic flux density $B$ has no appreciable influence on the current distribution in the central part of generator.

Next, Figs.2.41 and 2.42 show the current distributions in the ducts of the B and C type arrangements, respectively, in the case of $g = 5.33$ and $j = 8$. It will be seen that the current distributions in both figures are similar to the distribution in Fig.2.38. The percentages of the eddy current $I_{\text{eddy}}$ in Figs.2.41 and 2.42 are about 21.1 and 9.1% of $I$, respectively, and the B type arrangement induces a little larger eddy current than the other type arrangements. This is because the B type duct is not equipped with the output electrodes on the upper side in the inlet.

Figures 2.43 and 2.44 show the current distributions in the A type duct in the case of $j = 6$ and 4, respectively, and $g = 8$.

Figures 2.39, 2.43, and 2.44 also show that the current distribution varies with the positions of the output electrodes for the same gradient $g$, and getting the output electrodes nearer the
Fig. 2.41. Current distribution for $g = 5.33$
and $j = 8$ (B type).

Fig. 2.42. Current distribution for $g = 5.33$
and $j = 8$ (C type).

domain of constant $B$ makes eddy current large. In this connection, Table 2.1 shows the calculated values of $I_{\text{eddy}}/I$ for the $j = 8$, 6, and 4, where $I_{\text{eddy}}$ is the eddy current in the duct end.

In addition, when $\mu_e = 1.0$, the percentage of $I_{\text{eddy}}/I$ becomes about 12.5, 22.8, and 10.0 % in A, B, and C type, respectively, which are nearly equal to the percentages when $\mu_e = 0.5$. 
(2) Internal resistance and current concentration

For quantitative estimation of the end effects of the generator, the internal resistance $R_i$ of the end regions and the rate $\frac{J_{\text{peak}}}{<J>_{\text{el}}}$ of the current concentration on the output electrodes are calculated by

$$R_i = \frac{(V_0 - V)}{I} \quad (2.71)$$

and Eq. (2.36), respectively, where $V_0$ is no-load potential difference between the output electrodes and the $n$-th electrode and $V$

Fig. 2.43. Current distribution for $g = 8$ and $j = 6$ (A type).

Fig. 2.44. Current distribution for $g = 8$ and $j = 4$ (A type).
Table 2.1 \( R_i, J_{\text{peak/}}^{<J>}, I_{\text{eddy}}, \) and \( V_{\text{el}} \)
for \( j = 8, 6, \) and \( 4 \) (A type)

<table>
<thead>
<tr>
<th>( J )</th>
<th>( R_i (\Omega) )</th>
<th>( J_{\text{peak/}}^{&lt;J&gt;}_{\text{el}} )</th>
<th>( I_{\text{eddy}} / I )</th>
<th>( V_{\text{el}} (V) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>5.10</td>
<td>1.63</td>
<td>0.</td>
<td>34.5</td>
</tr>
<tr>
<td>6</td>
<td>11.52</td>
<td>2.84</td>
<td>0.200</td>
<td>85.5</td>
</tr>
<tr>
<td>4</td>
<td>16.93</td>
<td>6.00</td>
<td>0.433</td>
<td>234.</td>
</tr>
</tbody>
</table>

Fig. 2.45. Variation of internal resistance by \( g \) for \( j = 8 \).
the potential difference between these electrodes.

Table 2.1 shows the values of $R_i$ and $J_{\text{peak}}/<J>_{\text{el}}$, which are obtained with respect to Figs.2.39, 2.43, and 2.44, respectively. In Fig.2.45 is plotted the variation of $R_i$ against $g$ in the ducts of the A, B, and C type arrangements in the case of $j = 8$. The figure indicates that $R_i$ decreases with $g$ in all types, and for instance, the values of $R_i$ for $g = 8.0$ become 49.8, 69.7, and 58.3 % of those for $g = 0$ in the A, B, and C type ducts, respectively.

Figure 2.46 shows the relation between $J_{\text{peak}}/<J>_{\text{el}}$ and $g$. From the figure it is seen that in any type of duct the current

![Graph showing variation of current concentration on output electrodes by $g$ for $j = 8$.]
Fig. 2.47. Variation of potential difference for $j = 8$ (A type).

Fig. 2.48. Variation of potential difference for $j = 8$ (B type).
concentration considerably weakens in keeping with the attenuation of B. This indicates that arranging the output electrodes in the attenuation area of the magnetic flux density is useful to guard the output electrodes.

(3) Potential distribution

In Figs. 2.47 to 2.49 are plotted the calculated results of the potential differences

\[
\phi = - \int_{E_1}^{A_i} E \cdot ds, \quad i = 1, 2, \ldots, n
\]

between each electrode pair A_{i-1}-C_i to A_{8-8} and the electrode E_1 in the A, B, and C type arrangements, respectively. In the above equation, ds is a line element vector on an optional integral path from E_1 to A_i.

Figure 2.47 shows that the relatively large potential differ-
ence arises between the adjacent electrodes when \( B \) does not attenuate in the duct end, and accordingly large ballast resistors are necessary to use effectively the electrodes \( E_1 \) and \( E_2 \) as the output electrodes. On the other hand, the potential difference becomes smaller as \( g \) becomes larger, and in the range of \( g = 5.33 \) to \( 8 \), the potential difference between the adjacent electrodes \( E_2 \) and \( A_1 \) is almost negligible. Thus, by a proper attenuation of \( B \), more output electrodes than two can be used without large ballast resistors.

From Figs. 2.48 and 2.49, it is seen that the potential distributions in the B and C type ducts are similar to that in the A type duct for \( g = 5.33 \) and \( j = 8 \).

Also Table 2.1 shows the potential difference \( \nu_{e1} \) between \( E_2 \) and \( A_1 \) in the A type duct.

5.4 When Nonuniformity of Electrical Conductivity and Electron Mobility Are Considered

5.4.1 Numerical conditions and ballast resistance

Numerical analysis is carried out for the diagonal type MHD generator with the nonequilibrium plasma, in which

\[
\begin{align*}
h &= 0.2, \quad w = 0.1, \quad s = 0.1, \quad c = 0.06 \text{ m}, \\
B_0 &= 4 \text{ and } 5 \text{ T}, \quad u_0 = 2000 \text{ m/s}, \\
T_0 &= 1800 \text{ K}, \quad T_w = 1600 \text{ K}, \quad p = 5 \text{ atm}, \\
\varepsilon_s &= 0.3 \%, \quad \delta = 5, \quad m = m' = 1/7, \\
g &= 0 \text{ to } 10 \text{ T/m}
\end{align*}
\]

Also in this section, it is assumed that the applied magnetic flux has the six configurations and \( j = 5 \). The numerical calculation is carried out for the A type arrangement in this section with reference to the results obtained in the previous section 5.3. Then, the load current \( I \) is assumed to flow equally into both output
electrodes $E_1$ and $E_2$ through a ballast resistance $R_b$ defined by

$$R_b = -\int_{E_1}^{E_2} \frac{E \cdot ds}{I}. \quad (2.74)$$

5.4.2 Effects of distribution of magnetic flux

(1) Current distribution

In Figs. 2.50, 2.51, and 2.52, the current distributions are plotted in the case of $g = 0$, 6, and 10 T/m, respectively, $B_0 = 4$ T and $I = 70$ A, where the contour interval of the current streamlines is $1/20$ of the load current $I$. In the figures, $<J>_{el} = 0.583 \text{ A/cm}^2$, $<\sigma> = 1.84 \text{ u/m}$, and $\beta_{crit} = 2.48$, where the values of $<\sigma>$ and $\beta_{crit}$ are calculated in the n-th electrode region.

Figure 2.50 shows that the current concentration at the output electrode ends is very intensive when $g = 0$, viz. $B$ is constant. On the other hand, Figs. 2.51 and 2.52 indicate that the concentration weakens as $g$ increases, since $\beta$ becomes small in the area suffering a spatial reduction of $B$. Also it is seen that the current flowing into a diagonally connected electrode pair reduces with increasing $g$ in the entrance region of duct, for instance, the currents of about 60, 25, and 15% of $I$ flow into $C_1$ for $g =$

![Fig. 2.50. Current distribution for $g = 0$, $B_0 = 4$, and $j = 5$.](image-url)
Fig. 2.51. Current distribution for $g = 6$, $B_0 = 4$, and $j = 5$.

Fig. 2.52. Current distribution for $g = 10$, $B_0 = 4$, and $j = 5$.

0, 6, and 10 T/m, respectively. Also the figures show that the eddy current is not induced when the output electrodes are disposed in the attenuation domain of $B$ [2.33], and that arranging the output electrodes within the attenuation domain has little appreciable influence on the current distribution in the central part of generator duct. These results are similar to the ones when electrical conductivity and electron mobility are assumed constant.

(2) Internal resistance and current concentration

Figure 2.53 shows the variations of $R_1/R_{10}$, $R_b/R_{b0}$, and
Fig. 2.53. Influence of $g$ on $R_i/R_{i0}$, $R_b/R_{b0}$, and $J_{\text{peak}}/\langle J\rangle_{\text{el}}$ for $B_0 = 4$.

$J_{\text{peak}}/\langle J\rangle_{\text{el}}$ by $g$, where $R_{i0}$ and $R_{b0}$ are $R_i$ and $R_b$ for $g = 0$, respectively, and $B = 4T$. From the figure, it is seen that $R_i$ decreases with $g$, for instance, $R_i$ for $g = 6$ becomes about 80% of $R_{i0}$, and that $J_{\text{peak}}/\langle J\rangle_{\text{el}}$ first decreases with $g$, reaches the minimum value 1.90 at $g = 8$ T/m, and next increases with $g$. This shows that the current concentration at the output electrode ends is almost diminished when $g = 8$ T/m. Accordingly, arranging the output electrodes in the attenuation area of the magnetic flux density is useful to guard the output electrodes.

In Fig. 2.54, the current distribution is plotted for $g = 6$. 

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T/m, $B_0 = 5T$, $I = 150$ A, $<J>_{el} = 1.25$ A/cm$^2$, $<v> = 2.85$ m/s, $<\beta> = 2.48$, and $\beta_{crit} = 1.90$. The figure indicates that the streamer is induced in the central part of generator, while the current distribution becomes successively uniform as $B$ attenuates along the generator duct and the current concentration is almost swept away near the output electrodes. Therefore it is seen that arranging the output electrodes within the attenuating domain of $B$ is effective also for the case where the streamer is generated in the central region of generator duct.

Fig. 2.54. Current distribution for $g = 6$, $B_0 = 5$, and $j = 5$.

(3) Potential distribution and ballast resistance

Figure 2.55 shows the variation of the potential difference between each electrode pair $A_1-C_1$ to $A_8-C_8$ and the electrode $E_1$ in the case of $B = 4T$. From the figures, it is seen that the relatively large potential difference arises between the two output electrodes $E_1$ and $E_2$ when $B$ does not attenuate, namely $g = 0$. On the other hand, the potential difference becomes smaller as $g$ becomes larger, it almost vanishes for $g = 6$, and the inverse difference appears for $g > 7$. Also Fig.2.55 denotes that the potential differences in the central part of generator duct is little influenced by the decrease of the magnetic induction.

Figure 2.53 tells that $R_b/R_{b0}$ decreases with $g$, becomes almost zero at $g = 6.5$, and then increases with $g$. Therefore, it is seen
that many output electrodes will require large ballast resistors when \( B \) does not attenuate or \( g \) exceeds 8 T/m, but they can be used without large ballast resistors in the range of \( g = 6 \) to 7 T/m.

In addition, the gradient of the magnetic flux density in the entrance of the apparatus described in Reference [1.3] is almost 8 T/m, and \( g = 6 \) to 7 T/m will be realized in the future super-conducting magnet.

Fig. 2.55. Variation of potential difference for \( B_0 = 4 \).
5.5 Concluding Remarks

The main results obtained in this chapter are as follows.

(1) The variation of the type of output electrode arrangement has little effect on the current distribution in the generator. The selection of the type can be decided from a viewpoint of advantage in manufacturing.

(2) A suitable distribution of the magnetic flux can make the current distribution very uniform near the end region of generator duct, both when the streamer is not induced and when it is induced in the central region.

(3) Disposing the output electrodes within the attenuation area of magnetic flux density has little influence on the current distribution in the central part of generator duct.

(4) The internal resistance in the end region of the generator duct decreases as the magnetic flux attenuates.

(5) The current concentration at the output electrode ends can be fairly eliminated by attenuating magnetic flux.

(6) When the output electrodes are disposed in the region with a suitably decreased magnetic flux, the potential difference and the ballast resistance between two output electrodes become very small. Accordingly it is thought that many output electrodes can be used without large ballast resistors.

It was made clear that the output electrodes of the diagonal type MHD generator should be arranged in the attenuating region of magnetic flux, since arranging them in the region becomes useful not only for the improvement of the electrical characteristics in generator duct ends but also for the effective use of magnet.
6.1 Introduction

In the diagonal type generator, a pair of anode and cathode to be shorted is connected internally or externally. What influence the difference between these connections gives the electrical characteristics of the generator must be studied to grasp sufficiently the generator performance. However, as it cannot be examined by the usual two-dimensional analysis for the vertical duct cross-section to magnetic flux, it has been little discussed [2.38].

In this chapter, by a two-dimensional analysis in the cross-section perpendicular to the plasma flow [2.39], [2.40], are investigated comparatively the current distribution, the electrical efficiency, etc. in the nonequilibrium plasma generators with the internally connected electrodes viz., the diagonal conducting wall (the so-called windframe generator) and those with the externally connected electrodes viz., the insulating sidewall, which let us call the DCW and the GIS, respectively, for simplicity. As in the previous chapters, numerical investigation is made for an example of the cesium-seeded helium in nonequilibrium ionization, where it is assumed that the ionization instability does not occur.

First, a two-dimensional differential equation in the duct cross-section perpendicular to the plasma flow is derived from the basic equations such as the Maxwell equations and the generalized Ohm's law, and then boundary and subsidiary conditions are shown for both the internal and external connections.

Next, the numerical calculation results of the current and potential distributions in the DCW and the GIS are comparatively studied. Then, influences of the load factor, the temperature of the duct wall, and the inclination parameter on the distributions,
the efficiency, the effective conductivity, etc. of the generator are investigated in detail.

6.2 Two-Dimensional Theory

6.2.1 Basic equations

In this chapter, it is assumed that the electrical quantities, such as the current and electric field, are independent of $x$ and they vary with $y$ and $z$ [see Fig. 2.56(a)], that the gas velocity and temperature depend on $y$ and $z$ according to Eqs. (2.86) and (2.87) presented later, and that the pressure is kept constant.

Next, $E$, $J$, $E$, and $u$ are assumed as

$$E = (E_x, E_y, E_z), \quad J = (J_x, J_y, J_z), \quad E = (0, 0, B), \quad u = (u, 0, 0).$$

Then, $\nabla \times E = 0$ in Eq. (2.1) and the assumption that the electrical quantities do not vary with $x$ yield

$$E_x = \text{const.},$$

thus $E_x$ is regarded as a fundamental parameter in the calculation in this chapter.

When a stream function $\psi$ is defined newly by the following relations

$$J_y = -\psi/\partial z, \quad J_z = \psi/\partial y.$$  \hfill (2.77)

Then, from Eqs. (2.1), (2.4), (2.15), and (2.77), the following differential equations can be derived for $\psi$ and $\phi$.

$$\frac{\partial^2 \psi}{\partial y^2} + \left(\frac{\epsilon^2}{\varepsilon} + \frac{\beta^2}{\varepsilon}\right) \frac{\partial^2 \psi}{\partial z^2}
+ \sigma \frac{\partial (1/\sigma)}{\partial y} \psi + \frac{\partial (1/\sigma)}{\partial z} \psi
+ \frac{\partial}{\partial z} \left(\frac{E_x + \psi}{\partial z} \frac{\partial \psi}{\partial z}\right)
+ \frac{\partial}{\partial z} \left(\frac{E_x + \psi}{\partial z} \frac{\partial \psi}{\partial z}\right)
+ \sigma \frac{\partial^2}{\partial y^2} \frac{\partial \psi}{\partial y} - \frac{\partial (1/\sigma) \psi}{\partial y} \frac{\partial \psi}{\partial y}
- \sigma \frac{\partial}{\partial z} \frac{\partial u_b}{\partial z} = 0,$$ \hfill (2.78)
In this connection, the solution of Eq. (2.78) gives the orthogonality condition

\[ \frac{\partial w}{\partial x} + \frac{\partial v}{\partial y} = 0 \]

where

\[ \begin{align*}
\frac{\partial w}{\partial x} &= \frac{\partial w}{\partial x} + \frac{\partial v}{\partial y} \\
\frac{\partial v}{\partial y} &= \frac{\partial w}{\partial x} + \frac{\partial v}{\partial y}
\end{align*} \]
6.2.2 Boundary and subsidiary conditions

Equations (2.78) and (2.79) require the boundary conditions on the electrode and insulating wall surfaces, and the subsidiary condition peculiar to the diagonal type generator.

First, the boundary condition on the electrode surfaces becomes

\[ E_z = 0 \text{ at } y = 0 \text{ and } h. \]  

(2.82)

Using Eqs. (2.4), (2.77), and (2.15), the above condition is changed with \( \psi \) or \( \phi \) as

\[
\begin{align*}
\frac{\partial \psi}{\partial y} &= \frac{\sigma}{\varepsilon} e_n e_p \frac{\partial}{\partial z}, \\
\phi &= \text{const., }
\end{align*}
\]

at \( y = 0 \) and \( h \).  

(2.82)'

respectively.

Next, from the assumption that the physical quantities are independent of \( x \), the duct walls at \( z = 0 \) and \( w \), which is the duct width in the \( z \) direction, become the infinite segmented electrodes in the DCW. Therefore, the boundary condition on the duct sidewalls is given by

\[ E_y = a E_x, \quad a = \tan \theta. \]  

(2.83)

By Eqs. (2.4), (2.15), and (2.77), Eq. (2.83) reduces to

\[
\begin{align*}
\frac{\partial \psi}{\partial z} &= -\sigma' (\alpha + \beta) E_x / \varepsilon - \sigma' \frac{\partial e_p}{\partial y} e_n e_p, \\
\frac{\partial \phi}{\partial y} &= -a E_x, \text{ at } z = 0 \text{ and } w.
\end{align*}
\]

(2.83)'

In the generator of the GIS, the boundary condition is

\[ J_z = 0, \text{ at } z = 0 \text{ and } w. \]  

(2.84)

Substitution of Eqs. (2.4), (2.15), and (2.77) into Eq. (2.84) yields

\[ \psi = \text{const.}, \]
or
\[ \frac{\partial \phi}{\partial z} = \frac{\partial p_e}{\partial z} / e_n. \]

Finally, in the diagonal type generator, the potential difference must be zero between a pair of the anode and cathode which are shorted each other. Hence, the subsidiary condition is given by
\[ - \int^C_A E \cdot ds = 0, \tag{2.85} \]
where \( ds \) is the line element vector on an optional integral path \( C \) to \( A \) [see Fig.2.56 (b)].

Using Eqs. (2.75) and (2.76), Eq. (2.85) reduces to
\[ \int^h_0 E_y dy = ahE_x. \tag{2.85}' \]

6.2.3 Gas velocity and temperature distributions

As mentioned before, the gas velocity \( u \) has only the \( x \) component \( u_x \), and \( u_x \) and \( T \) are assumed to vary in the \( y \) and \( z \) direction according to the relations
\[ \frac{u}{u_0} = \{4y/h \cdot (1 - y/h)\}^m \{4z/w \cdot (1 - z/w)\}^n \tag{2.86} \]
and
\[ \frac{(T - T_w)}{(T_0 - T_w)} = \{4y/h \cdot (1 - y/h)\}^{m'} \times \{4z/w \cdot (1 - z/w)\}^{n'}, \tag{2.87} \]
where \( m, m', n, \) and \( n' \) are the constants, \( T_0 \) and \( u_0 \) the gas temperature and velocity, respectively, at the center of flow; namely at \( y = h/2 \) and \( z = w/2 \), and \( T_w \) the duct wall temperature.

6.2.4 Electrical efficiency

To estimate the performance characteristics of generator, the electrical efficiency
\[ \eta_e = - \int_0^h \int_0^w E \cdot J dydz / \int_0^h \int_0^w (J \times E) \cdot ud ydz \]  

(2.88)

is used. Also an effective electrical conductivity \( \sigma_{\text{eff}} \) and Hall parameter \( \beta_{\text{eff}} \) are evaluated, which are obtained by the definitions

\[
\sigma_{\text{eff}} = \frac{\langle J \rangle^2}{\langle J \rangle \cdot \langle E^* \rangle}, \\
\beta_{\text{eff}} = \frac{|\langle J \rangle \times \langle E^* \rangle|}{\langle J \rangle \cdot \langle E^* \rangle},
\]

(2.89)

where \( E^* = E + \mathbf{u} \times E \) and \( \langle J \rangle \) and \( \langle E^* \rangle \) denote the average value of \( J \) and \( E^* \).

6.3 Numerical Investigations

6.3.1 Numerical conditions

Numerical conditions are given by

\[
\begin{align*}
  h &= 0.2, \quad w = 0.2 \text{ m}, \quad B = 5 \text{ T}, \\
  u_0 &= 2000 \text{ m/s}, \quad T_0 = 1800 \text{ K}, \\
  p &= 5 \text{ atm}, \quad \varepsilon_s = 0.3 \%, \quad \delta = 5, \\
  m &= m' = n = n' = 1/7.
\end{align*}
\]

(2.90)

In addition, since the investigation in Chapter 3 has clarified that the terms of the ion-slip and the gradient of electron pressure are negligible under the present conditions, it can be assumed that \( \varepsilon = 1 \) and \( \partial p_e / \partial y = \partial p_e / \partial z = 0 \).

6.3.2 Influence of load factor

Figures 2.57 and 2.58 show the current and potential distributions on the y-z plane in the DCW, respectively, and Fig. 2.59 presents the current distribution in the GIS, where \( E_x = -800 \text{ V/m}, \quad T_w = 1600 \text{ K}, \quad \kappa = 0.053, \) and \( a = -1 (\theta = 135^\circ) \). Figure 2.57 indicates that some current flows into the conducting sidewall, namely \( J \) has not only the y component \( J_y \) but also the z component \( J_z \) near the...
sidewall, and Fig. 2.58 shows that the large electrode voltage drop occurs, when $E_x$ and accordingly $\kappa$ are small. Also Fig. 2.59 shows that the current density becomes large in the central part of the duct cross-section and it is very small near the insulating sidewall.

In Figs. 2.60 and 2.61, the current and potential distributions in the DCW are plotted, respectively, and in Fig. 2.62, the current distribution in the GIS in the case of $E_x = -8000$ V/m, $T_w = 1600$ K, $\kappa = 0.513$, and $a = -1$. From Figs. 2.60 and 2.61, it is seen that the current flowing into the conducting sidewall and the electrode voltage drop diminish against Figs. 2.57 and 2.58. Also Fig. 2.62 indicates that the current near the insulating sidewall tends to bend toward inside.

Next, Fig. 2.63 gives the variation of $n_e$ and $\sigma_{\text{eff}}/\langle \sigma \rangle$ by $E_x$ in the case of $T_w = 1600$ K and $a = -1$, where $\langle \sigma \rangle$ is the average value of $\sigma$ in

![Fig. 2.57. Current distribution in the DCW when $E_x = -800$ V/m, $T_w = 1600$ K, and $a = -1$.](image)

![Fig. 2.58. Potential distribution in the DCW corresponding to current distribution in Fig. 2.57.](image)
the y-z plane. It shows that the magnitudes of $n_e$ and $\sigma_{eff}/\langle \sigma \rangle$ of the DCW become a little larger than those of the GIS, and these differences increase with $E_x$. This is because the current in the DCW tends to flow not only in the middle area of the duct cross-section but also near the sidewall due to presence of the electrodes on the sidewall, and on the other hand, in the GIS it is difficult for the current to run near the sidewall because of the low temperature and low conductivity, and the current tends to gather in the middle area of the duct.

In addition, the value of $\beta_{eff}/\langle \beta \rangle$ is nearly equal to unity in both the DCW and GIS, where $\langle \beta \rangle$ is the average value of $\beta$ in the y-z plane, and it is not plotted in the figure.

Figure 2.64 shows the relations between $E_x$ or $\kappa$ and the x component $I_x$ of the load current $I$, the
Fig. 2.61. Potential distribution in the DCW corresponding to current distribution in Fig. 2.60.

Fig. 2.62. Current distribution in the GIS under the same condition as Fig. 2.60.
ratio $i_z$ of the current flowing into one conducting sidewall to $I'$, or the ratio $v_D$ of electrode voltage drop to $V_y$ in the DCW, where

$$\kappa = \frac{E_x}{(E_x)_{\text{open}}}$$

$$= -(1 + a^2)E_x/(\beta_{\text{eff}} - a)<u>B$$, \hspace{1cm} (2.91)

$$I = I_x + I_y,$$

$$I_x = \int_0^h J_x dy\,dz, \hspace{0.5cm} I_y = ah\int_0^h J_y \,dz,$$ \hspace{1cm} (2.92)

$$i_z = \int_0^{h/2} J_z\,dy/I',$$ \hspace{1cm} (2.93)

$$I' = |\int_0^w J_y\,dz| \hspace{0.5cm} \text{at} \hspace{0.5cm} y = h/2,$$ \hspace{1cm} (2.94)

$$V_y = -\int_0^h E_y\,dy.$$ \hspace{1cm} (2.95)

![Graph](image)

**Fig. 2.63.** Influence of $E_x$ on $\eta_e$ and $\sigma_{\text{eff}}/\Theta$ when $T_w = 1600$ K and $a = -1$. 109
From the figure, it is found that the sign of $I_x$ changes from positive to negative when $E_x$ or $\kappa$ becomes large, namely the direction of the current $I_x$ is reversed. The values of $\kappa$ and $I_x$ in the DCW become nearly equal to those in the GIS. Then, $i_z$ and $v_D$ decrease and converge to a constant value when $\kappa$ becomes large, for example, $i_z = 14.3$, 5.8, and 4.0 % for $\kappa = 0.05$, 0.5, and 0.7, respectively [see Figs. 2.57 and 2.58].

![Graph showing influence of $E_x$ on $I_x$, $i_z$, $v_D$, and $\kappa$](image)

**Fig. 2.64.** Influence of $E_x$ on $I_x$, $i_z$, $v_D$, and $\kappa$ when $T_w = 1600$ K and $a = -1$.

### 6.3.3 Influence of duct wall temperature

Figure 2.65 shows the influence of duct wall temperature $T_w$ on the electrical characteristics both of the DCW and of the GIS in the case of $E_x = -5000$ V/m and $a = -1$. From the figure, it is seen that the values of $\eta_e$ and $i_z$ are nearly independent of $T_w$, but $\sigma_{eff}/<\sigma>$ and $I_x$ increase with $T_w$, and also that $\eta_e$ and $\sigma_{eff}/<\sigma>$ of the DCW are a little larger than those of the GIS.

Figure 2.66 informs the influence of $T_w$ on the performance
characteristics of the DCW when $E_x = -8000$ V/m and $a = -1$. The figure indicates that $\eta_e$ and $\kappa$ are nearly independent of $T_w$, and $\sigma_{\text{eff}}/\langle \sigma \rangle$, $I_x$, and $i_z$ increase with $T_w$, for example the values of $I_x$ and $i_z$ for $T_w = 1800$ K are 1.50 and 1.58 times as large as those for $T_w = 1300$ K, respectively. This is because the current is prevented from flowing near the duct wall since a low value of $T_w$ hinders the elevation of $T_e$ there, and so it is apt to flow in the central region of generator duct. The values of $\eta_e$ and $\sigma_{\text{eff}}/\langle \sigma \rangle$ are nearly equal to those in Fig.2.65 but the variations of $I_x$ and $i_z$ are larger than those in Fig.2.65.

6.3.4 Influence of inclination parameter

In Fig.2.67, we plot the relations of $\eta_e$, $\sigma_{\text{eff}}/\langle \sigma \rangle$, $I_x$, and $i_z$ when $\kappa = 0.5$ and $T_w = 1600$ K. It is shown that $\eta_e$ increases
largely and \( \sigma_{\text{eff}} / \langle \sigma \rangle \) does only a little when \( |a| \) becomes large. Also \( I_x \) becomes negative and its variation due to \( a \) is small for \( |a| > 0.4 \), but \( I_x \) is positive for \( |a| < 0.4 \), and especially \( I_x \) becomes very large at \( a = 0 \), namely in the Hall type generator.

Next, \( i_z \) is nearly constant in the range of \( 0 < |a| < 1 \), but it increases fairly for \( |a| > 1 \).

Fig. 2.66. Influence of \( T_w \) on \( I_x, i_z, \eta_e, \kappa, \)
and \( \sigma_{\text{eff}} / \langle \sigma \rangle \) when \( E_x = -8000 \) V/m
and \( a = -1 \).
Fig. 2.67. Influence of \( a \) on \( I_x \), \( i_z \), \( \eta_e \), and \( \sigma_{\text{eff}}/\langle \sigma \rangle \) when \( \kappa = 0.5 \) and \( T_w = 1600 \text{ K} \).
6.4 Concluding Remarks

The main conclusions, which have been derived from the above described two-dimensional analysis, are as follows.

(1) In the DCW, the current flowing into the electrodes on the sidewall increases with the load factor. In the GIS, it is difficult for the current to run near the sidewall and the most part of the current flows in the mid area of the generator duct. Also the electrode voltage drop becomes large with decreasing load factor.

(2) The generator characteristics such as the electrical efficiency and effective conductivity of the DCW are better than those of the GIS and this difference increases with the load factor and the wall temperature within the numerical conditions used in this chapter.

(3) The decreases of the duct wall temperature causes a reduction of the ratio $i_x$ and a little degradation of generator performance of the DCW.

(4) The electrical characteristics of the generator improves as the magnitude of the inclination parameter increases.

Thus, it may be thought that the diagonal type MHD generator with nonequilibrium plasma should be equipped with the conducting sidewall within the assumptions in this chapter.
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APPENDIX I

DERIVATION OF EQUATIONS (1.5) AND (1.7)

Equations (1.5) and (1.7) of the quasi one-dimensional basic equations are peculiar to a diagonal type MHD generator.

When the current continuity equation is applied to a hexahedron \( v \) which has the width of unit length in the \( z \) direction and an area \((A'C'D'B')\) in the \( x-y \) plane in Fig.1.1, we obtain the following equation

\[
\int_{v} \text{div} \mathbf{J} \, dv = 0, \quad (A.1)
\]

where \( dv \) is an element of \( v \). By Gauss' divergence theorem, the above equation is transformed to

\[
\int_{S} \mathbf{J} \cdot dS = S_{1}J_{n1} - S_{2}J_{n2} = 0, \quad (A.2)
\]

where \( dS \) is element vector of the surface \( S' \) of the hexahedron \( v \), \( S_{1} \) and \( S_{2} \) the cross-sections \( A'C' \times 1 \) and \( B'D' \times 1 \), respectively, and \( J_{n1} \) and \( J_{n2} \) the current components perpendicular to the cross-sections \( S_{1} \) and \( S_{2} \), respectively.

\[
S_{1}J_{n1} = S_{2}J_{n2} = SJ_{n} \quad \text{is equal to the load current} \ I, \quad (A.3)
\]

where \( S \) is a cross-section which intersects a normal cross-section \( A \) by the angle \( \theta \), \( J_{n} \) the current component perpendicular to \( S \).

Using the \( x \) and \( y \) components \( J_{nx} \) and \( J_{ny} \) of \( J_{n} \) and \( A \), \( J_{n} \) and \( S \) are written as

\[
J_{n} = J_{nx} - J_{ny} = -(J_{x} \cos \theta + J_{y} \sin \theta), \quad (A.4)
\]

\[
S = -A/\cos \theta. \quad (A.5)
\]

From Eqs. (A.4) and (A.5), the following equations

\[
(J_{x} + aJ_{y})A = I, \quad a = \tan \theta \quad (A.6)
\]

are derived. These equations are nothing but Eqs.(1.5).
Next, since an electrode pair $C$ and $A$ is short-circuited, the potential difference between $C$ and $A$ becomes zero. Using this relation, as the component of $E_x$ in the CA direction is $E_x \sin \theta$ and that of $E_y$ is $-E_y \cos \theta$, the total electric field in the CA direction becomes

$$E_x \sin \theta - E_y \cos \theta = 0.$$  \hspace{1cm} (A.7)

Accordingly,

$$E_y = aE_x.$$  \hspace{1cm} (A.8)

This gives the relation in Eq.(1.7).
APPENDIX II

DERIVATION OF EQUATIONS (2.33) TO (2.35)

Neglection of the ion-slip and electron partial pressure terms in Ohm's law i.e. Eq.(2.4) for the diagonal type MHD generator yields

\[ J_x = \sigma \left( E_x + \beta (uB - E_y) \right) / (1 + \beta^2), \quad (A.9) \]
\[ J_y = \sigma \left( \beta E_x - (uB - E_y) \right) / (1 + \beta^2), \quad (A.10) \]

with respect to the x and y components \( J_x \) and \( J_y \) of \( J \).

If the quasi one-dimensional approximation can be assumed, the following equations

\[ (J_x + aJ_y)A = I, \quad a = \tan \theta, \quad (A.11) \]
\[ E_y = aE_x \]

are formed as shown in the previous appendix.

From Eqs.(A.9) to (A.12), \( E_x \) is derived as

\[ (E_x)_{I=0} = -(\beta - a)uB/(1 + a^2) \quad (A.13) \]

for no-load condition. Here, when the results of two-dimensional analysis are used, \( \beta \) must be replaced with \( \langle \beta \rangle \).

Let us define an ideal potential difference per one electrode pitch \( V_{\text{ideal}} \) by

\[ V_{\text{ideal}} = -(E_x)_{I=0} \kappaS = (\langle \beta \rangle - a)u_0B_0S_\kappa/(1 + a^2), \quad (2.33) \]

where \( \kappa \) is the actual load factor [see Eqs.(A.14) and (A.15)]. This \( V_{\text{ideal}} \) becomes the potential difference per one electrode pitch when the ideal generator is operated with \( \kappa \).

Next, when \( J_x, J_y, E_x, \) and \( E_y \) are replaced with \( \langle J_x \rangle, \langle J_y \rangle, \langle E_x \rangle, \) and \( \langle E_y \rangle \), respectively, \( \sigma \) and \( \beta \) are replaced by the effective electrical conductivity \( \sigma_{\text{eff}} \) and the effective Hall parameter \( \beta_{\text{eff}} \), respectively, defined by
\[ <J_x> = \sigma_{\text{eff}} \frac{\langle E_x \rangle + \beta_{\text{eff}} (\langle u \rangle_B - \langle E_y \rangle)}{1 + \beta^2_{\text{eff}}}, \quad (A.9)' \]
\[ <J_y> = \sigma_{\text{eff}} \frac{\beta_{\text{eff}} \langle E_x \rangle - (\langle u \rangle_B - \langle E_y \rangle)}{1 + \beta^2_{\text{eff}}}, \quad (A.10)' \]
\[ <E_y> = a <E_x>, \quad (A.11)' \]
\[ (<J_x> + a <J_y>) A = I. \quad (A.12)' \]

with respect to the two-dimensional analysis.

When the load factor is defined by
\[ \kappa = \frac{\langle E_x \rangle}{\langle E_x \rangle} \bigg|_{I=0}, \quad (A.14) \]
from Eqs. (A.9)' to (A.12)' and (A.14) the following equation is derived
\[ \kappa = 1 - I(1 + \beta_{\text{eff}}^2) / \{ h w \sigma \text{eff} \langle u \rangle_B (\beta_{\text{eff}} - a) \}. \quad (A.15) \]
This is nothing but Eq. (2.32).

Equation (2.33) can be derived from Eqs. (A.8)' to (A.10)'.

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APPENDIX III

RELAXATION PARAMETER IN SOR METHOD

The convergence of the author's modified SOR method is explained as follows.

In general, simultaneous linear equations are written in the following matrix form

\[ Ax = b, \quad A = D - E - F, \]  \hspace{1cm} (A.16)

where \( A, D, E, \) and \( F \) are square, diagonal, lower triangular, and upper triangular matrices, respectively, and \( b \) and \( x \) known and unknown column vectors.

Now let us consider the following relaxation parameter matrix

\[ \Omega = \begin{pmatrix} \omega_1 & 0 & \cdots & 0 \\ 0 & \omega_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \omega_m \end{pmatrix}, \]  \hspace{1cm} (A.17)

where \( \omega_i (i = 1, 2, \ldots, m; m: \text{total node number}) \) is the relaxation parameter for the \( i \)-th node. Then, the \( (k+1) \)-th approximation \( x^{(k+1)} \) for \( x \) is given by

\[ x^{(k+1)} = x^{(k)} + \Omega^{-1} (b + E x^{(k+1)}) \]

\[ + F x^{(k)} - x^{(k)}. \]  \hspace{1cm} (A.18)

Further, this equation is rewritten as

\[ x^{(k+1)} = M x^{(k)} + g 
\]

\[ = M x^{(1)} + (M^{(k-1)} + \cdots + M + U) g, \]  \hspace{1cm} (A.19)

where \( U \) is the unit matrix, and

\[ M = (U - \Omega D^{-1} E)^{-1} (U + \Omega D^{-1} F - U), \]

\[ g = (U - \Omega D^{-1} E)^{-1} \Omega D^{-1} b. \]  \hspace{1cm} (A.20)

Next, if \( \lim_{k \to \infty} \| M \| < 1 \), where \( \| M \| \) is the norm of \( M \),
\[ \lim_{k \to \infty} x^{(k+1)} = (U - M)^{-1} g = A^{-1} b, \quad (A.21) \]

Namely, when \( k \) tends to \( \infty \), the value of \( x^{(k+1)} \) converges to the solution of Eq. (A.16).

Moreover, if \( A \) is a symmetric positive matrix, it can be proved that \( x \) converges for \( 0 < \omega_i \ (i = 1, 2, \ldots, m) < 2 \).