As to the vibration of circular disc or cylider, the same manner can be applied. When no coupling exists, the *r. a. f.* become

$$w_a^2=\frac{\pi^2E}{a^2\rho}, \qquad w_r^2=\frac{\zeta^2E}{r^2\rho}\cdot\frac{1}{1-\sigma^2}, \tag{3}$$

where ζ is the roots of the equation which contains Bessel functions, and takes a value of 2.03 when $\sigma=0.27$. When considered the coupling, the *r. a. f.* must be described as follows:

$$w_1^2=\frac{\pi^2E}{b^2\rho}\ u_1(p^2,\eta), \qquad w_2^2=\frac{\pi^2E}{b^2\rho}\ u_2'(p^2,\eta), \tag{4}$$

where $p^2=a^2/b^2=a^2\zeta^2/\pi^2r^2(1-\mu^2)$

and η is three dimentional coupling coefficient.

1) When $a/r\rightarrow o$, we get

$$w_1=w_r,\ \ w_2^2=\frac{\pi^2E}{a^2\rho}\cdot\frac{1}{1-\eta^2}. \tag{5}$$

w_2 represents the *r. a. f.* of thickness vibration of circular disc, and so it must be the same representation with the next formula

$$w_t^2=\frac{\pi^2}{t^2}\cdot\frac{\lambda+2\mu'}{\rho}=\frac{\pi^2E}{t^2\rho}\cdot\frac{1-\sigma}{(1+\sigma)(1-2\sigma)}, \tag{6}$$

where λ and μ' are Lamé's elastic constants. From the above two equations, we deduce the equation

$$\eta^2=\frac{2\sigma^2}{1-\sigma}=\frac{2\mu^2}{1-\mu}. \tag{7}$$

2) When $a/r\longrightarrow\infty$,

$$w_1=w_2,\ \ w_2^2=\frac{\pi^2E}{b^2\rho}\cdot\frac{1}{1-\eta^2}=\frac{\zeta^2E}{r^2\rho}\cdot\frac{1}{(1+\mu)^2(1-2\mu)}, \tag{8}$$

w_2 represents the *r. a. f.* of extentional radial vibration of infinitely long cylinder. From Airey's theory (J. R. Airey: *Arch. d. Mathem. u. Phys.* 3. 20 (1913), 294.), the *r. a. f.* of the same vibration is represented by

$$w^2=\frac{\xi^2E}{r^2\rho}\cdot\frac{1-\sigma}{(1+\sigma)(1-2\sigma)}, \tag{9}$$

where ξ is the roots of the equation which contains Bessel functions. From the above two equations, using the relation $\mu=\sigma$, we obtain the equation

$$\xi^2=\frac{\zeta^2}{1-\mu^2}. \tag{10}$$

Such relations could be verified by many experimental results employing $BaTiO_3$ ceramic vibrators.

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14. Study on High Dielectric Constant Ceramics. (XVI)

Langevin Type $BaTiO_3$ Ceramic Vibrator

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(Abe Laboratroy)

A Langevin type vibrator using $BaTiO_3$ ceramics as the electrostrictive material is expected to have an excellent electro-acoustic transducing property due to the nature of its piezoelectric properties. This expectation is realized by many experimental studies (K. Abe, T. Tanaka and co-workers: *Jour. of the Denki Hyoron*, **39**, No. 8 (1951), 12.; *Jour. of Acoust. Soc. of Jap.*, **7**, No. 1 (1951), 16, 20,), of which only the results are summarized here.

(1) Electro-acoustic properties.

A typical example of characteristics measured about a vibrator whose diameter is 6 cm. and resonant frequency is 50 k.c. is such as follows:

$fo = 49.88$ *k. c.* $\qquad M = 2m = 700$ *gr*

$\triangle f = 0.14$ *k. c.* $\qquad \triangle = \pi \cdot \triangle f = 440$ *rad/s*

$Y_{m0} = 52.8$ *mō* (*in air*) $\qquad r = 2m\triangle = 308{,}000$ *dyne/kein*

$C_a = 5800$ *pF.* $\qquad A = \sqrt{Ymo \cdot r} = 12.1 \times 10^7$ *dyne/e.s.u.V*

$Y_m = 5.3$ *mō* (one face immerced in oil).

The electro-mechanical efficiency η_{em} is usually very large, which is due to the small dielectric loss angle of ceramics

$$\eta_{em} \simeq 96\%.$$

The mechano-acoustic efficiency η_{ma} is

$$\eta_{ma} = \frac{r}{r_a + r} = 1 - \frac{Y_m}{Y_{m0}} = 89\%.$$

The electro-acoustic efficiency η_{ea} is therefore

$$\eta_{ea} = \eta_{em} \cdot \eta_{ma} \simeq 86\%.$$

The acoustic power output P_a, at resonant frequency is given by

$$P_a = \frac{r_a}{r_a + r} \cdot Y_m \cdot V^2,$$

and so, to get 2 Watts/cm^2. of acoustic power, for example, it is sufficient to apply only about 110 Volt of A. C. field in this case.

(2) Voltage Characteristics.

The characteristics of the vibrator are very much influenced by D. C. biasing field, which is due to the second order electro strictive phenomenon of $BaTiO_3$ ceramics. But after once treated under high D. C. field, the vibrator acts as a good transducer by the remanant polarization, and its characteristic scarcely varies by applying A. C. field correspond to 3～5 Watt/cm^2. Continuous application of D. C. negative high voltage decrease the remanant polarization and reduce the electro-acoustic characteristics.

(3) Temperature characteristics.

The measured value of the temperature coefficient of resonant frequency was about $+120 \times 10^{-6}$ and $+220 \times 10^{-6}$ at the temperature ranges from 0～30°C and 30～60°C respectively. Variation of the value the of force factor *A* could scarcely be observed at the temperature range from 0～60°C.

(4) Mechanical strength.

It has been said that the mechanical strength of a Langevin type vibrator is determined by the adhesive force of the cement between the metal and the piezoelectric material. The method of adhesion was experimentally studied and improved, and it was attained, that the adhesive force could overcome the mechanical fore of piezoelectric material. The experimental results showed the adhesive froce to be 420 kg./cm^2. and the tensile strength of piezoelectric material 110 kg./cm^2.

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15. Study on Surface Electricity. (XIV)

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The most important characters of U-effect II in applying it to electro-acoustic devices are the inner impedances of the elements, the frequency characters and the amplitude characters. Elements of the dia. (I) 0.76, (II) 0.49 and (III) 0.37 mm. were used in the experiments, each of which contained 40 mercury — 1 n. HCl aq. interfaces.

The inner impedances of the elements were measured by "Impedance matching method" (*e.g., This Bulletin*, **24**, 12 (1951) *etc.*) to be each 5, 20 and 30 kΩ (at 1,000 c. p. s.) and were inversely proportional to the cross sectional area of the elements.

The frequency characters were measured at constant amplitude of vibration ($1.2 \cdot 10^{-3}$ mm.) with loads obtained above. The results are:

Frequency characters.

Frequency \ Element	I	II	III
2,000 (c. p. s.)	7.9 (mV.)	16.8	21.1
1,000	7.1	20.0	23.7
500	5.6	15.9	23.7
100	2.7	1.8	0.5

Output voltages in mV.

In this experiment free type of vibration was used and elements of large, middle and small cross sections had natural peaks at low, middle and high frequencies, but when piston type is used, flat frequency characters can be obtained. (*This Bulletin*, **20**, 28 (1950)). From this we can deduce that it is better to use elements of small cross section with free type of vibration, in applying this effect to hydrophone of supersonic wave.

The amplitudes of mechanical vibration were measured by frequency modulation method. (*This Bulletin*, 28, 47 (1952)). The output-amplitude curves at constant frequency were quadratic and gave characters of the so-called