

B, gray	0.150	0.154
C, brown	0.158	
D, white	0.094	
clay powder (higher grade)	0.138	
" (lower grade)	0.208	

* These are the average values obtained from two samples.

** These are the values obtained after precipitating $\text{Fe}(\text{OH})_3$ and $\text{Al}(\text{OH})_3$.

The brown or gray-black color of the rock are the deeper, the more the iron content. The whitest and high grade paper clay contains very small quantities of iron. By the proper selection of the rock the color of paper clay will be improved.

13. Study on High Dielectric Constant Ceramics. (XV)

Coupled Vibration in Electrostrictive Vibrators

Kiyoshi ABE, Tetsuro TANAKA and Koji UO

(Abe Laboratory)

Theoretical analysis of coupled vibration, which can be seen in a mechanical vibrator having more than two modes of vibrations, was considered in previous report (K. Abe, T. Tanaka and K. Uo: *Jour. of the Denki Hyoron*, 39, No. 12 (1951), 2.), and it was concluded that such theory agrees quite well with the experimental result in the case of BaTiO_3 ceramic vibrator having the shape of rectangular plate. The same manner can be applied in the treatment of hollow cylinder or circular disc or cylinder.

Consider a thin hollow cylinder having axial length a and radius r , then the resonant angular frequencies (*r. a. f.*) are given by the next formula when no coupling effect exists between the two vibrations:

$$w_a^2 = \frac{\pi^2 E}{a^2 \rho}, \quad w_r^2 = \frac{E}{r^2 \rho}. \quad (1)$$

If coupling effect is considered, *r. a. f.* become as follows, substituting $p = a/\pi r$:

$$\begin{aligned} w_1^2 &= \frac{E}{r^2 \rho} \cdot \frac{(p^2 + 1) - \sqrt{(p^2 + 1)^2 - 4p^2(1 - \mu^2)}}{2p^2(1 - \mu^2)} = \frac{E}{r^2 \rho} u_1(p^2, \mu) \\ w_2^2 &= \frac{E}{r^2 \rho} \cdot \frac{(p^2 + 1) + \sqrt{(p^2 + 1)^2 - 4p^2(1 - \mu^2)}}{2p^2(1 - \mu^2)} = \frac{E}{r^2 \rho} u_2(p^2, \mu) \end{aligned} \quad (2)$$

where μ is the coupling coefficient. Compared with Love's same result about thin hollow cylinder (A. E. H. Love: "Mathematical Theory of Elasticity". Chap. XXIV, p. 546, 4th. ed. 1927), it is concluded that μ is equal to Poisson's ratio σ .

As to the vibration of circular disc or cylinder, the same manner can be applied. When no coupling exists, the *r. a. f.* become

$$w_a^2 = \frac{\pi^2 E}{a^2 \rho}, \quad w_r^2 = \frac{\zeta^2 E}{r^2 \rho} \cdot \frac{1}{1 - \sigma^2}, \quad (3)$$

where ζ is the roots of the equation which contains Bessel functions, and takes a value of 2.03 when $\sigma = 0.27$. When considered the coupling, the *r. a. f.* must be described as follows:

$$w_1^2 = \frac{\pi^2 E}{b^2 \rho} u_1(p^2, \eta), \quad w_2^2 = \frac{\pi^2 E}{b^2 \rho} u_2(p^2, \eta), \quad (4)$$

where $p^2 = a^2/b^2 = a^2 \zeta^2 / \pi^2 r^2 (1 - \mu^2)$

and η is three dimensional coupling coefficient.

1) When $a/r \rightarrow 0$, we get

$$w_1 = w_r, \quad w_2^2 = \frac{\pi^2 E}{a^2 \rho} \cdot \frac{1}{1 - \eta^2}. \quad (5)$$

w_2 represents the *r. a. f.* of thickness vibration of circular disc, and so it must be the same representation with the next formula

$$w_2^2 = \frac{\pi^2}{t^2} \cdot \frac{\lambda + 2\mu'}{\rho} = \frac{\pi^2 E}{t^2 \rho} \cdot \frac{1 - \sigma}{(1 + \sigma)(1 - 2\sigma)}, \quad (6)$$

where λ and μ' are Lamé's elastic constants. From the above two equations, we deduce the equation

$$\eta^2 = \frac{2\sigma^2}{1 - \sigma} = \frac{2\mu^2}{1 - \mu}. \quad (7)$$

2) When $a/r \rightarrow \infty$,

$$w_1 = w_2, \quad w_2^2 = \frac{\pi^2 E}{b^2 \rho} \cdot \frac{1}{1 - \eta^2} = \frac{\xi^2 E}{r^2 \rho} \cdot \frac{1}{(1 + \mu)^2 (1 - 2\mu)}, \quad (8)$$

w_2 represents the *r. a. f.* of extentional radial vibration of infinitely long cylinder. From Airey's theory (J. R. Airey: *Arch. d. Mathem. u. Phys.* 3, 20 (1913), 294.), the *r. a. f.* of the same vibration is represented by

$$w^2 = \frac{\xi^2 E}{r^2 \rho} \cdot \frac{1 - \sigma}{(1 + \sigma)(1 - 2\sigma)}, \quad (9)$$

where ξ is the roots of the equation which contains Bessel functions. From the above two equations, using the relation $\mu = \sigma$, we obtain the equation

$$\xi^2 = \frac{\zeta^2}{1 - \mu^2}. \quad (10)$$

Such relations could be verified by many experimental results employing BaTiO₃ ceramic vibrators.

14. Study on High Dielectric Constant Ceramics. (XVI)

Langevin Type BaTiO₃ Ceramic Vibrator

Kiyoshi ABE, Tetsuro TANAKA, Shigeru MIURA and Kiyoshi OKAZAKI

(Abe Laboratory)

A Langevin type vibrator using BaTiO₃ ceramics as the electrostrictive