4. On the Dielectric Measurement in the Centimeter Wave Region

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Received September 26, 1952

Introduction

Many methods of dielectric measurement in the meter wave region such as the lumped constant circuit method, the distributed constant circuit method, the dynatron method, the bridge method, *etc.*[1] have been proposed and the respective theories formed in simple and sufficiently exact ways.

The theories of dielectric measurement in the centimeter wave region are more complicated because the wave length in this region is of the order of the sample dimension and the exact explicit expression of the complex dielectric constant $\frac{\epsilon^*}{\epsilon_0} = \frac{\epsilon'}{\epsilon_0} - j\frac{\epsilon''}{\epsilon_0}$ treated in the theory of wave guide has not been given, though for special cases the exact expressions have been proposed, which were all treated circuit-theoretically so far as the authors know.

In 1946, S. Roberts and A. von Hippel[2] proposed for the first time the method using standing wave measurement but in their theory $\frac{\epsilon^*}{\epsilon_0}$ was expressed implicitly by the equation which contains a transcendental function, so the explicit expressions of ϵ'/ϵ_0 and ϵ''/ϵ_0 could not be obtained. Accordingly for practical measurement they utilized the graphical solutions or the approximate expressions.

Afterward, improvements of Hippel's method have been made by T. W. Dakin and others[3], but the explicit expressions of ϵ'/ϵ_0 and ϵ''/ϵ_0 are only the simplified approximate forms of Hippel's expressions. W. H. Surber and G. E. Crouch[4] have given the explicit expressions of ϵ^*/ϵ_0 circuit-theoretically by the shortcircuit–opencircuit method which needs, however, in order to avoid experimental errors, the extreme mechanical accuracy of apparatus, as is described later in this paper. Moreover, another method they proposed in the same paper[4] is based on the approximate equation which is connected with the electrical length of the sample and practically the variation of the sample length is attended with difficulties for some samples.

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In this paper the authors introduce the general explicit expressions of ϵ'/ϵ_0 and ϵ''/ϵ_0 for any electrical length of the sample and show that our general equations include Surber's theory as a special case. From these equations many new methods can be derived, some of which are more convenient and more practical than Surber's method and we can also show one of the causes of the errors which attend the shortcircuit-opencircuit method.

On the comparison of our methods with the experiments we will discuss in the next paper.

1. The Fundamental Equation of Measurement

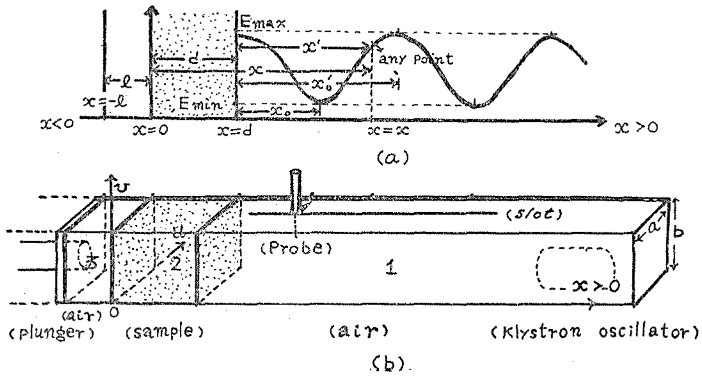

Fig. 1

As shown in Fig. 1 (b), we use rectangular coordinates (u, v, x), x being the direction of the wave guide axis. Within the rectangular wave guide in which the length l of the air column behind the sample of the length d is varied by moving the plunger, the field of H_{01} mode which is most practical for our purpose is given by the following expressions

$$\left.\begin{aligned} E_i(x)_u &= A_i\left(e^{\gamma_i x} + r_i e^{-\gamma_i x}\right) \\ H_i(x)_v &= \frac{A_i}{Z_i}\left(e^{\gamma_i x} - r_i e^{-\gamma_i x}\right) \end{aligned}\right\} \quad i = 1, 2, 3 \tag{1}$$

with

$$\left.\begin{aligned} A_i &= \frac{\pi}{b}\sin\frac{\pi v}{b}, \quad \gamma_1 \equiv \gamma_3 \\ Z_i &= j\omega\mu_i/\gamma_i, \quad \mu_1 = \mu_3\,, \; Z_1 \equiv Z_2 \end{aligned}\right\} \quad i = 1, 2, 3, \tag{2}$$

[5]

where $i = 1, 2, 3$ show respectively the field in the air near the klystron, that in the sample and that in the air near the plunger, and Z_i is the field impedance in the x-direction, For simplification, the suffixes u, v of E_{iv} and H_{iv} are omitted in the following.

At $x=d$ and $x=O$, we obtain from (1)

$$\left.\begin{aligned}\frac{E_1(d)}{H_1(d)}&=Z_1\frac{e^{\gamma_1 d}+r_1 e^{-\gamma_1 d}}{e^{\gamma_1 d}-r_1 e^{-\gamma_1 d}}\\ \frac{E_2(d)}{H_2(d)}&=Z_2\frac{e^{\gamma_2 d}+r_2 e^{-\gamma_2 d}}{e^{\gamma_2 d}-r_2 e^{-\gamma_2 d}}\end{aligned}\right\} \qquad (3)$$

and

$$\left.\begin{aligned}\frac{E_3(0)}{H^3(0)}&=Z_1\frac{1+r_3}{1-r_3}\\ \frac{E_2(o)}{H_2(o)}&=Z_2\frac{1+r_2}{1-r_2}\end{aligned}\right\} \qquad (4)$$

At the boundary surface of two media neither of which is not perfectly conducting, the tangential component of the electric vector and that of the magnetic vector are in general continuous. Therefore the boundary conditions are written as

$$\frac{E_1(d)}{H_1(d)}=\frac{E_2(d)}{H_2(d)} \quad \text{and} \quad \frac{E_2(o)}{H_2(o)}=\frac{E_3(o)}{H_3(o)} . \qquad (5)$$

As the medium extending over $x\leqslant -l$ is a perfect conductor, we obtain

$$E_3(-l)=A_3(e^{-\gamma_1 l}-r_3 e^{\gamma_1 l})=0 ,$$

that is

$$r_3=-e^{-2\gamma_1 l} . \qquad (6)$$

When we substitute (6) into the first equation of (4) and use (5), the equations of (3) and (4) become

$$\left.\begin{aligned}Z_1\frac{e^{\gamma_1 d}+r_1 e^{-\gamma_1 d}}{e^{\gamma_1 d}-r_1 e^{-\gamma_1 d}}&=Z_2\frac{e^{\gamma_2 d}+r_2 e^{-\gamma_2 d}}{e^{\gamma_2 d}-r_2 e^{-\gamma_2 d}}\\ Z_2\frac{1+r_2}{1-r_2}&=Z_1\tanh\gamma_1 l\end{aligned}\right\} . \qquad (7)$$

In most cases $\mu_1=\mu_2\ (\equiv\mu)$ holds, so from the second equation of (2),

$$\gamma_1 Z_1=\gamma_2 Z_2=jw\mu \qquad (8)$$

is derived. From (8) and (7), we obtain

$$T\equiv\frac{e^{2\gamma_1 d}+r_1}{e^{2\gamma_1 d}-r_1}=\frac{\gamma_1}{\gamma_2}\left(\frac{e^{2\gamma_2 d}+r_2}{e^{2\gamma_2 d}-r_2}\right) \qquad (9)$$

and

$$\frac{1+r_2}{1-r_2}=\frac{\gamma_2}{\gamma_1}\tanh\gamma_1 l . \qquad (10)$$

Eliminating r_2 from (9) and (10), we have

$$e^{2\gamma_2 d}\left(\frac{\frac{\gamma_2}{\gamma_1}T-1}{\frac{\gamma_2}{\gamma_1}T+1}\right)=\frac{\frac{\gamma_2}{\gamma_1}\tanh\gamma_1 l-1}{\frac{\gamma_2}{\gamma_1}\tanh\gamma_1 l+1} . \qquad (11)$$

Now, if we let T_1 and T_2 be the values of T corresponding to the positions of the plunger at $l=l_1$, and $l=l_2$ respectively, from (11) the following relation can be derived:

$$\left(\frac{\gamma_2}{\gamma_1}\right)^2 = \frac{(K-L)+(T_2-T_1)}{(K-L)\,T_1T_2+(T_2-T_1)KL}, \tag{12}$$

where

$$\left.\begin{aligned} K &\equiv \tanh\,\gamma_1 l_1 = j\,\tan\frac{2\pi}{\lambda_1}\,l_1 \\ L &\equiv \tanh\,\gamma_1 l_2 = j\,\tan\frac{2\pi}{\lambda_1}\,l_2 \\ T_i &\equiv \frac{e^{2\gamma_1 d}+\gamma_{1i}}{e^{2\gamma_1 d}-\gamma_{1i}} \end{aligned}\right\} \tag{13}$$

and λ_1 is the wave length in the air-filled guide, and the attenuation of the wave in the guide is assumed to be zero.

2. Transformation of T

If we put

$$r_1 \equiv e^{-2\phi} \text{ and } \phi \equiv \rho + j\psi, \tag{14}$$

from (1) the VSWR is expressed in the form

$$\Gamma \equiv \frac{E_1\,max}{E_1\,max} = \frac{1+|r_1|}{1-|r_1|} = \frac{1+e^{-2\rho}}{1-e^{-2\rho}}\ (\equiv \coth\,\rho) \tag{15}$$

or

$$e^{-2\rho} = \frac{\Gamma-1}{\Gamma+1}. \tag{16}$$

Now, when x_0 is the distance from the face $(x=d)$ of the dielectric sample to the point at which E_1 is minimum, and x_0' is that to the point at which E_1 is maximum, the following relations are obtained from the phase parts of (1):

$$\frac{2\pi}{\lambda_1}(x_0+d) = -2\psi - \frac{2\pi}{\lambda_1}(x_0+d) + (2n+1)\pi \tag{17}$$

and

$$\frac{2\pi}{\lambda_1}(x_0'+d) = -2\psi - \frac{2\pi}{\lambda_1}(x_0'+d) + 2n\pi, \tag{18}$$

where n is any positive or negative integer. From these equations

$$\exp(-j2\psi) = -\exp\left(j4\pi\frac{x_0+d}{\lambda_1}\right) \tag{19}$$

and

$$\exp(=j2\psi) = +\exp\left(j4\pi\frac{x_0'+d}{\lambda_1}\right) \tag{20}$$

follow. Substituting respectively (16), (19) and (16), (20) into (14), the following equations are derived:

$$r_1 = -\frac{\Gamma-1}{\Gamma+1}\,e^{j2\beta_1(x_0+d)} \tag{21}$$

and

$$r_1 = +\frac{\Gamma-1}{\Gamma+1}\,e^{j2\beta_1(x_0'+d)} \tag{22}$$

Furthermore, if we substitute these equations into (13), the expressions of T_i are transformed into the following forms:

$$T_m = \frac{(\Gamma_m+1)-(\Gamma_m-1)\ e^{j2\beta_1 x_{0m}}}{(\Gamma_m+1)+(\Gamma_m-1)\ e^{j2\beta_1 x_{0m}}} = \frac{\Gamma_m+j\cot\beta_1 x_{0m}}{1+j\Gamma_m\cot\beta_1 x_{0m}}\ (m=1,2) \quad . \tag{23}$$

and

$$T_m = \frac{(\Gamma_m+1)+(\Gamma_m-1)\ e^{j2\beta_1 x'_{0m}}}{(\Gamma_m+1)-(\Gamma_m-1)\ e^{j2\beta_1 x'_{0m}}} = \frac{\Gamma_m-j\tan\beta_1 x_{0m}}{1-j\Gamma_m\tan\beta_1 x_{0m}}\ (m=1,2) \quad . \tag{24}$$

Now, if x' is the distance measured from the face ($x=d$) of the dielectric sample to any point in the medium 1 (Fig. 1 (b)), $E_1(x)$ at $x=x'+d$ is expressed by

$$E_1(x) = (A_1 e^{\gamma_1 d})\, e^{\gamma_1 x'} + (A_1 r_1 e^{-\gamma_1 d})\, e^{-\gamma_1 x'} \quad . \tag{25}$$

Then, the reflection coefficient using (25) is given by the relation, $r' = A_1 r_1 e^{-\gamma_1 d}/A_1 e^{\gamma_1 d}$, i. e., $r = r_1' e^{2\gamma_1 d}$, so by substituting this into (21) and (22), we obtain

$$r_1' = -\frac{\Gamma-1}{\Gamma+1}\ e^{j2\beta_1 x_0} \tag{21$'$}$$

and

$$r_1' = +\frac{\Gamma-1}{\Gamma+1}\ e^{j2\beta_1 x_0'} \quad . \tag{22$'$}$$

3. Determination of ϵ'/ϵ_0 and ϵ''/ϵ_0

We will derive the equations which determine $\frac{\epsilon^*}{\epsilon_0}=\frac{\epsilon'}{\epsilon_0}-j\frac{\epsilon''}{\epsilon_0}$ from (12). The following is a well known characteristic relation of the wave guide:

$$\gamma_2^2=\left(\frac{2\pi}{\lambda_c}\right)^2-\omega^2\mu\,\epsilon^*=\left(\frac{2\pi}{\lambda_c}\right)^2-\omega^2\mu\,\epsilon_0\left(\frac{\epsilon'}{\epsilon_0}-j\frac{\epsilon''}{\epsilon_0}\right)$$
$$=-\left(\frac{2\pi}{\lambda}\right)^2\left[\frac{\epsilon'}{\epsilon_0}-\left(\frac{\lambda}{\lambda_c}\right)^2-j\frac{\epsilon''}{\epsilon_0}\right] , \tag{26}$$

where λ_c is the cutoff wave length and λ the wave length in free space. For γ_1^2 also

$$\gamma_1^2=-\left(\frac{2\pi}{\lambda_1}\right)^2=\left(\frac{2\pi}{\lambda_c}\right)^2-\omega^2\epsilon_0\,\mu=-\left(\frac{2\pi}{\lambda}\right)\left\{1-\left(\frac{\lambda}{\lambda_c}\right)^2\right\} \tag{27}$$

is written and so the following relation is derived:

$$\left(\frac{\gamma_2}{\gamma_1}\right)^2=\frac{\frac{\epsilon'}{\epsilon_0}-\left(\frac{\lambda}{\lambda_c}\right)^2-j\frac{\epsilon''}{\epsilon_0}}{1-\left(\frac{\lambda}{\lambda_c}\right)^2} \tag{28}$$

or

$$\frac{\epsilon'}{\epsilon_0}-j\frac{\epsilon''}{\epsilon_0}=\left\{1-\left(\frac{\lambda}{\lambda_c}\right)^2\right\}\left(\frac{\gamma_2}{\gamma_1}\right)^2+\left(\frac{\lambda}{\lambda_c}\right)^2 \quad . \tag{29}$$

Substituting (13) and (23) or (13) and (24) into (29), we obtain finally the following results:

$$\left.\begin{aligned} &\frac{\epsilon'}{\epsilon_0} = 1 + \left(\frac{\lambda}{\lambda_c}\right)^2 \left(1 - \frac{AA'+BB'}{A'^2+B'^2}\right) \\ \text{and} \quad &\frac{\epsilon''}{\epsilon_0} = \left(1 - \left(\frac{\lambda}{\lambda_c}\right)^2\right) \frac{AB'-BA'}{A'^2+B'^2} \end{aligned}\right\}, \tag{30}$$

where, A, B, A' and B' are given as follows.

1) Case using x_0 (the point of E_{min}),

$$\left.\begin{aligned} A \equiv\ & (\Gamma_2-\Gamma_1)\left(1+\cot\frac{2\pi}{\lambda_1}x_{01}\cdot\cot\frac{2\pi}{\lambda_1}x_{02}\right) - \left(\Gamma_1\cot\frac{2\pi}{\lambda_1}x_{01} + \Gamma_2\cot\frac{2\pi}{\lambda_1}x_{02}\right)\left(\tan\frac{2\pi}{\lambda_1}l_1-\tan\frac{2\pi}{\lambda_1}l_2\right) \\ A' \equiv\ & \Gamma_2\left(\tan\frac{2\pi}{\lambda_1}l_1\cdot\tan\frac{2\pi}{\lambda_1}l_2+\cot\frac{2\pi}{\lambda_1}x_{01}\cdot\cot\frac{2\pi}{\lambda_1}x_{02}\right) - \Gamma_1\left(\cot\frac{2\pi}{\lambda_1}x_{01}\cdot\cot\frac{2\pi}{\lambda_1}x_{02}\cdot\right. \\ & \left.\tan\frac{2\pi}{\lambda_1}l_1\cdot\tan\frac{2\pi}{\lambda_1}l_2+1\right) - \left(\tan\frac{2\pi}{\lambda_1}l_1-\tan\frac{2\pi}{\lambda_1}l_2\right)\left(\Gamma_1\cdot\cot\frac{2\pi}{\lambda_1}x_{01}+\Gamma_2\cdot\cot\frac{2\pi}{\lambda_1}x_{02}\right) \\ B \equiv\ & (\Gamma_2-1)\cot\frac{2\pi}{\lambda_1}x_{02} - (\Gamma_1-1)\cot\frac{2\pi}{\lambda_1}x_{01} + \left(\tan\frac{2\pi}{\lambda_1}l_1-\tan\frac{2\pi}{\lambda_1}l_2\right)\cdot \\ & \left(1-\Gamma_1\Gamma_2\cot\frac{2\pi}{\lambda_1}x_{01}\cdot\cot\frac{2\pi}{\lambda_1}x_{02}\right) \\ B' \equiv\ & \left(\tan\frac{2\pi}{\lambda_1}l_1\cdot\tan\frac{2\pi}{\lambda_1}l_2-1\right)\cot\frac{2\pi}{\lambda_1}x_{01} - \left(\Gamma_2-\tan\frac{2\pi}{\lambda_1}l_1\cdot\tan\frac{2\pi}{\lambda_1}l_2\right)\cot\frac{2\pi}{\lambda_1}x_{02} \\ & +\left(\tan\frac{2\pi}{\lambda_1}l_1-\tan\frac{2\pi}{\lambda_1}l_2\right)\left(\Gamma_1\Gamma_2-\cot\frac{2\pi}{\lambda_1}x_{01}\cdot\cot\frac{2\pi}{\lambda_1}x_{02}\right) \end{aligned}\right\} \tag{31}$$

2) Case using x_0' (the point of E_{max}),

$$\left.\begin{aligned} A \equiv\ & (\Gamma_1-\Gamma_2)\left(1+\tan\frac{2\pi}{\lambda_1}x_{01}\cdot\tan\frac{2\pi}{\lambda_1}x_{02}\right) + \left(\Gamma_1\tan\frac{2\pi}{\lambda_1}x_{01} + \Gamma_2\tan\frac{2\pi}{\lambda_1}x_{02}\right)\left(\tan\frac{2\pi}{\lambda_1}l_1-\tan\frac{2\pi}{\lambda_1}l_2\right) \\ A' \equiv\ & \Gamma_2\left(\tan\frac{2\pi}{\lambda_1}l_1\cdot\tan\frac{2\pi}{\lambda_1}l_2+\tan\frac{2\pi}{\lambda_1}x_{01}\cdot\tan\frac{2\pi}{\lambda_1}x_{02}\right) - \Gamma_1\left(\tan\frac{2\pi}{\lambda_1}x_{01}\cdot\tan\frac{2\pi}{\lambda_1}x_{02}\right. \\ & \left.\tan\frac{2\pi}{\lambda_1}l_1\cdot\tan\frac{2\pi}{\lambda_1}l_2+1\right) + \left(\tan\frac{2\pi}{\lambda_1}l_1-\tan\frac{2\pi}{\lambda_1}l_2\right)\left(\Gamma_1\tan\frac{2\pi}{\lambda_1}x_{01}+\Gamma_2\tan\frac{2\pi}{\lambda_1}x_{02}\right) \\ B \equiv\ & (\Gamma_1-1)\tan\frac{2\pi}{\lambda_1}x_{01} - (\Gamma_2-1)\tan\frac{2\pi}{\lambda_1}x_{02} + \left(\tan\frac{2\pi}{\lambda_1}l_1-\tan\frac{2\pi}{\lambda_1}l_2\right)\cdot \\ & \left(1-\Gamma_1\Gamma_2\tan\frac{2\pi}{\lambda_1}x_{01}\cdot\tan\frac{2\pi}{\lambda_1}x_{02}\right) \\ B' \equiv\ & \left(\Gamma_2-\tan\frac{2\pi}{\lambda_1}l_1\cdot\tan\frac{2\pi}{\lambda_1}l_2\right)\tan\frac{2\pi}{\lambda_1}x_{02} - \left(\tan\frac{2\pi}{\lambda_1}l_1\cdot\tan\frac{2\pi}{\lambda_1}l_2-1\right)\cdot \\ & \tan\frac{2\pi}{\lambda_1}x_{01} + \left(\tan\frac{2\pi}{\lambda_1}l_1-\tan\frac{2\pi}{\lambda_1}l_2\right)\left(\Gamma_1\Gamma_2-\tan\frac{2\pi}{\lambda_1}x_{01}\cdot\tan\frac{2\pi}{\lambda_1}x_{02}\right) \end{aligned}\right\} \tag{32}$$

4. New Methods of Dielectric Measurement

The equations (30) are the exact explicit expressions of ϵ'/ϵ_0 and ϵ''/ϵ_0. If the value of ϵ^* given by (12) can be determined by measuring x_0 or x_0',

Γ, l_1 and l_2, then ϵ'/ϵ_0 and ϵ''/ϵ_0 are determined from (30), but these expressions are practically too complicated to be applied to the measurement, so in this paragraph we discuss on several special cases where these expresions are simplified.

Here we present only the methods corresponding to the measurement of x_0 (the position of E_{min}), Γ, l_1 and l_2.

Case 1) Starting from $l_1=\lambda_1/4,\ 3\cdot\lambda_1/4,\cdots\cdots(K=\infty)$, we obtain

$$\left(\frac{\gamma_2}{\gamma_1}\right)^2=\frac{1}{T_1T_2+L(T_2-T_1)}\quad, \tag{33}$$

where x_{01} is the position of E_{min} corresponding to $l=l_1$.

a) By adjusting l_2 (the position of the plunger) so that the position of E_{min} may be $x_{02}=\lambda_1/4,\ 3\cdot\lambda_1/4,\ \cdots\cdots$ (in this case (23) is reduced to $T_2=\Gamma_2$), (33) becomes

$$\left(\frac{\gamma_2}{\gamma_1}\right)^2=\frac{1}{T_1\Gamma_2+L(\Gamma_2-T_1)}\ . \tag{34}$$

b) By adjusting l_2 (the position of the plunger) so that the position of E_{min} may be $x_{02}=0,\ \lambda_1/2,\lambda_1,\cdots\cdots$ (in this case (23) is reduced to $T_2=1/\Gamma_2$), (33) becomes

$$\left(\frac{\gamma_2}{\gamma_1}\right)^2=\frac{\Gamma_2}{T_1+L(1-T_1\Gamma_2)}\ . \tag{35}$$

c) By letting x_{02} be the position of E_{min} such that the position of the plunger may be $l_2=0,\ \lambda_1/2,\ \lambda_1,\cdots\cdots$ $(L=0)$, (33) is reduced to

$$\left(\frac{\gamma_2}{\gamma_1}\right)^2=\frac{1}{T_1T_2}\ . \tag{36}$$

Case 2) Starting from $l_2=0,\ \lambda_1/2,\ \lambda_1,\cdots\cdots$ $(L=0)$, we obtain

$$\left(\frac{\gamma_2}{\gamma_1}\right)^2=\frac{K+(T_2-T_1)}{KT_1T_2}\quad, \tag{37}$$

where x_{02} is the position of E_{min} corresponding to $l=l_2$.

a) By adjusting l_1 (the position of the plunger) so that the position of E_{min} may be $x_{01}=\lambda_1/4,\ 3\cdot\lambda_1/4,\ \cdots\cdots$ $(T=\Gamma_1)$, (37) is reduced to

$$\left(\frac{\gamma_2}{\gamma_1}\right)^2=\frac{K+(T_2-\Gamma_1)}{KT_2\Gamma_1}\ . \tag{38}$$

b) By adjusting l_1 (the position of the plunger) so that the position of E_{min} may be $x_{01}=0,\ \lambda_1/2,\ \lambda_1,\cdots\cdots$ $(T_1=1/\Gamma_1)$, (37) is reduced to

$$\left(\frac{\gamma_2}{\gamma_1}\right)=\frac{K\Gamma_1+(T_2\Gamma_1-1)}{KT_2}\ . \tag{39}$$

Case 3) Adjusting l_i $(i=1,\ 2)$ (the positions of the plunger) so that the positions of E_{min} may be respectively $x_{01}=\lambda_1/4,\ \frac{3}{4}\lambda_1,\cdots\cdots$ $(T_1=\Gamma_1)$ and $x_{02}=0,\ \lambda_1/2,\ \lambda_1,\cdots\cdots$ $(T_2=1/\Gamma_2)$, we obtain

$$\left(\frac{\gamma_2}{\gamma_1}\right)^2=\frac{(K-L)+(\frac{1}{\Gamma_2}-\Gamma_1)}{(K-L)\frac{\Gamma_1}{\Gamma_2}+(\frac{1}{\Gamma_2}-\Gamma_1)\,KL}. \tag{40}$$

Considering that Γ_1 and Γ_2 are real, and K and L purely imaginary, we know (40) is a very practical expression. We can also treat the method corresponding to the measurement of x_0' (the position of E_{max}), Γ, l_1 and l_2 in the same manner as above.

5. The Agreement of Case 1) c) with the Expression by Surber and Crouch[4)]

In the paper by Surber and Crouch, the position of E_{min} is measured from the point which is $n\ \lambda_1/2$ (n is an integer) apart from the front face of the dielectric sample and expressed with $\bar{x}_0$ as shown in Fig. 2. Then $x=n\lambda_1/2-\bar{x}_0$ and $\cot\ (2\pi x_0/\lambda_1)=-\cot\ (2\pi\bar{x}_0/\lambda)$. Therefore, from (23)

$$\frac{1}{\mathrm{T}_i}=\frac{\Gamma_i+j\tan\theta_i}{1+j\Gamma_i\tan\theta_i}\ ,\qquad \theta_i\equiv\frac{2\pi}{\lambda_1}\bar{x}_{0i} \tag{41}$$

is derived.

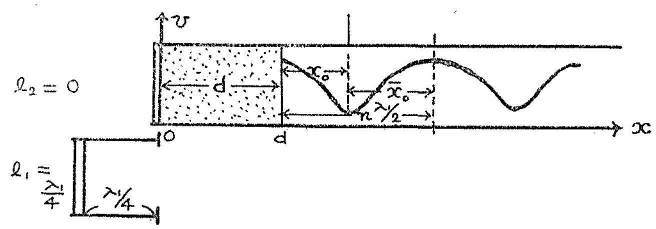

Fig 2.

Substituting (41) into (36) and combining it with (28), we obtain

$$\frac{\frac{\epsilon'}{\epsilon_0}-\left(\frac{\lambda}{\lambda_c}\right)^2-\frac{\epsilon''}{\epsilon_0}}{1-\left(\frac{\lambda}{\lambda_c}\right)^2}=\left(\frac{\Gamma_1+j\tan\theta_1}{1+j\Gamma_1\tan\theta_1}\right)\left(\frac{\Gamma_2+j\tan\theta_2}{1+j\Gamma_2\tan\theta_2}\right), \tag{42}$$

that is

$$\left.\begin{aligned}\frac{\frac{\epsilon'}{\epsilon_0}-\left(\frac{\lambda}{\lambda_c}\right)^2}{1-\left(\frac{\lambda}{\lambda_c}\right)^2}&=\frac{GH+IJ}{H^2+I^2}\\ \frac{\frac{\epsilon''}{\epsilon_0}}{1-\left(\frac{\lambda}{\lambda_c}\right)^2}&=\frac{GI-HJ}{H^2+I^2}\end{aligned}\right\}, \tag{43}$$

where

$$\left.\begin{aligned}G&\equiv\Gamma_1\Gamma_2-\tan\theta_1\tan\theta_2\\ H&\equiv1-\Gamma_1\Gamma_2\tan\theta_1\tan\theta_2\\ I&\equiv\Gamma_1\tan\theta_1+\Gamma_2\tan\theta_2\\ J&\equiv\Gamma_1\tan\theta_2+\Gamma_2\tan\theta_1\end{aligned}\right\} \tag{44}$$

These expressions (43) and (44) agree perfectly with the expressions of the shortcircuit-opencircuit method given circuit-theoretically by Surber and Crouch in 1948.

In our experiment by the method using (43) and (44), we have exprienced comparatively large errors. One of the causes of error seems to be that l_1 is not exactly situated at the point of an odd multiple of $\lambda_1/4$ in (36). Namely if l_1 has the error dl_1 at $l_1 \doteqdot \lambda_1/4$, from (13) K has the error

$$dk = j\frac{2\pi}{\lambda_1}\sec^2\frac{2\pi}{\lambda_1}l_1 \cdot dl_1, \tag{45}$$

which becomes very large because $l_1 \doteqdot \lambda_1/4$.

This naturally produces a considerable effect upon ϵ^* through (13) and (29). Therefore if the mechanical accuracy of the apparatus is not sufficient, the measured value of ϵ^* derived from (33), (34), (35) and (36) can not be expected to be very accurate.

Conclusion

The theories of the dielectric measurement in the centimeter wave region which have been published so far are not treated in the theory of the wave guide or otherwise do not give the exact explicit expression of the complex dielectric constant.

In this paper we have given the exact explicit expression of ϵ^*. Especially we have obtained the exact expression (40) which is more convenient for the experiment than any expression so far published.

Moreover, if the expression (12) is simplified by approximation, many other methods of dielectric measurement can be expected to be devised.

One of the advantages of the proposition of this paper is that the methods based on (12) replace the process which cause experimental errors in other methods with the measurement of the air column which can be performed very accurately. Another advantage is that the deformation[6], for instance the variation of the sample length, can be made unnecessary.

References

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