3. Study on Surface Electricity. (XIV)

Several Characters of U-effect

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In the previous papers we have made descriptions of U-effect[1] and its various applications which involved the measurements of electrokinetic potential[2] and interfacial electrical capacity[3] as well as the devices of mechano-electrical conversions[4]. In the last case, its efficiency and characters are subject to the device of the conversion system, e. g. armature of the pick-up or diaphragm of the microphone *etc.*, as well as to the characters of the element itself. As the former factor is the same in every transducers, our present purpose is to grasp the latter ones. The most important of them are the inner impedance of the element, its frequency and amplitude haracters.

As, comparing the efficiencies of U-effect I with II, it is easily seen that the former is far smaller than the latter[4], the use of U-effect II in practice is recommended. Hence, all of the experiments in this article were done with the elements containing mercury-solution interfaces (U-effect II). Their characters depend chiefly upon their sizes, species and concentrations of the solution and the numbers of interfaces.

I. Methods

(1) Measurement of the inner impedance[5]

The usual methods of impedance measurement cannot be recommended here because of the electrochemical reactions at interfaces by outer (alternating) electromotive force. A new method without outer electromotive force was deviced by us and called "Impedance matching method", the outline of which was as follows:

When, generating an alternating current by the mechanical vibration of the element (U-effect II), we change the load resistance, we can observe a peak in the "power-load curve", where "power" means that which is supplied to the load by the vibrating element. According to the maxium power

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transfer theorem applied to the special case where the phase angle of load impedance is constant, the inner impedance of the element and the outer impedance of load (here resistance only) have the same moduli at this maximum. So, this load resistance at maximum point is equal to the inner impedance of the element at that frequency.

(2) Measurement of minute amplitude of vibration[6]

The frequency f of a self oscillator is given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

where L and C are the inductance and capacitance of its resonating circuit. When the capacitance is changed to $C+\Delta C$, the frequency change Δf is given by

$$\Delta f = \frac{\partial f}{\partial C}\,\Delta C = -\frac{1}{2}\,f\frac{\Delta C}{C}\,.$$

When $\Delta C \ll f$, C, Δf is proportional to ΔC. Accordingly we can convert the capacitance change into frequency change, which is one of the general modes of frequency modulation. The frequency change can be converted into d. c. voltage by a proper detecting circuit and a cathode ray oscilloscope.

In applying this principle to the amplitude measurement of vibration, we construct an electrical condenser with a movable electrode attached to the vibrating body and a fixed one connected parallell to the resonating circuit of a self oscillator. We can observe a wave of the same frequency and amplitude characters as the mechanical vibration on the screen of the oscilloscope connnected to the detector. With the calibration of the oscilloscope readings we can measure the minute amplitude of vibration.

II. Experimental

(1) Apparatus

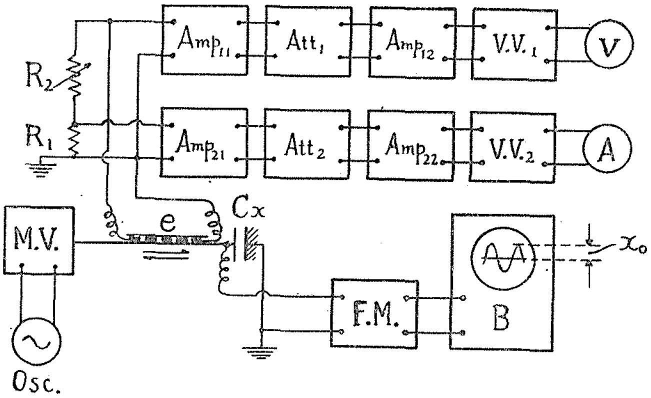

Fig. 1.

The whole system of the measuring device is schematically shown in Fig. 1.

(i) Measurement of the gain of the element.

When the moving coil vibrator (M. V.) driven by an oscillator (Osc.) vibrates the element (e) attached to it, an alternating potential drop appears at the load resistance (R_1+R_2). This output voltage is measured through an amplifier–attenuator system (Amp_{11}-Att_1-Amp_{12}) by a valve voltmeter ($\mathrm{V.\,V.}_1$-V).

(ii) Measurement of the inner impedance

The potential drop at constant resistance R_1 is a measure of the current of the circuit, which is measured through an amplifier–attenuator system (Amp_{21}–Att_2–Amp_{22}) by a valve voltmeter ($\mathrm{V.\,V.}_2$–A). The potential drop at (R_1+R_2) is measured by V as before. Now, putting the power supplied to load P, we obtain the following formula,

$$\begin{aligned}\log P &= \log (\mu_1 \mathrm{V} \cdot \mu_2 \mathrm{A}) \\ &= \log \mu_1 + \log \mu_2 + \mathrm{const} \\ &= Db_1 + Db_2 + \mathrm{const}\end{aligned}$$

for constant V and A. e. g. 1 Volt each, where μ_1 and μ_2 are proper attenuation factors and Db_1 and Db_2 are the readings of the attenuators Att_1 and Att_2 in decibels. When the load resistance is changed by varaiable resistance R_2, a maximum of ($Db_1 + Db_2$) is detected, in which case ($R_1 + R_2$) is equal to the inner impedance of the element at the frequency used.

(iii) Measurement of the amplitude

The condenser C_x in Fig. 1 is the frequency modulator ($C + \Delta C$). F. M. includes self oscillator and detecting circuit. The amplitude of vibation is measured by the height x_0 of the wave on the screen of the oscilloscope B.

(2) **Element**

(i) Solution

It is desirable to use the solution with low viscosity and high conductivity. We used 1 N. HCl aq.

(ii) Size

The most decisive factor on the characters of the element is its size. We used three elements of different diameters, (I) 0.76, (II) 0.49 and (III) 0.37 mm.

(iii) Number of interfaces

The first three of the following experiments were performed with elements with forty interfaces, but in the last ones, where the relation between output voltage and the number of interfaces was examined, we short-circuited the mercury phases four by four.

(3) Results

(i) Inner impedance

Table 1. Inner impedance.
Frequency=1,000c/s.

Diameter mm.	(Diameter)^2 mm^2.	Impedance ohms
0.76	0.5776	5,000
0.49	0.2401	20,000
0.37	0.1368	30,000

The experiments gave the results as shown in Table 1 and Fig. 2 at 1,000 c/s, where the inner impedances were given against the diameter and (diameter)^2, the latter of which represents the cross section of the element.

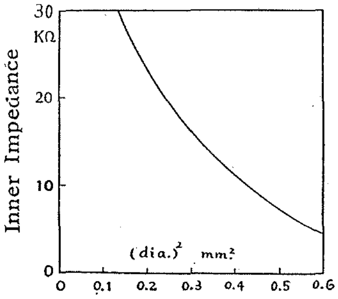

Fig. 2

As the solution resistance is very small, the inner impedance Z_g resides chiefly in the interfacial capacitance C. Hence

$$|Z_g| = \sqrt{R_i^2 + \frac{1}{\omega^2 C^2}} \fallingdotseq \frac{1}{\omega C_0 s}$$

where C_0 is the capacity per unit area, ω the angular frequency $(2\pi f)$, s the cross section and R_i the solution resitance. As s is nearly proportional to the square of the diameter d,

$$|Z_g| \propto 1/d^2$$

That is, $|Z_g| - \frac{1}{d^2}$ curve is a hyperbola.

(ii) Frequency character

The relations between output voltage and frequency of vibration at constant amplitude are given in Table 2 and Fig. 3. Here the load resistances

Table 2. Frequency character.
Amplitude of mechancal vibration=$1.2 \cdot 10^{-3}$ mm. Output voltage (mV.).

Element / Frequency (c/s)	I	II	III
2,000	7.9	16.8	21.1
1,000	7.1	20.0	23.7
500	5.6	15.9	23.7
100	2.7	1.8	0.5
Load (ohms)	5,000	20,000	30,000

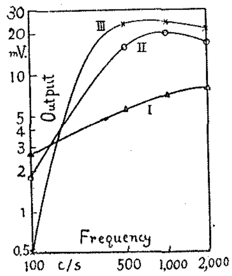

Fig. 3

were matched at 1,000 c/s. Evidently, the larger the diameter, the higher the response at lower frequencies was, and *vice versa*. Such peaks are reasonable in the mode of free end type of vibration as used here, those being the natural peaks of the elements. If piston type of element are used, we can obtain a flat character curve.

(iii) Amplitude character

The relations between the output voltage and the amplitude of vibration at 1,000 c/s are given in Table 3 and Fig. 4.

Table 3. Amplitude character.
Frequency=1,000 c/s.

Element / Amplitude of vibration (mm.)	I	II	III
$0.3 \cdot 10^{-3}$	2.4 mV.	5.0	6.0
0.7	5.0	11.2	11.9
1.2	9.4	22.4	23.7
1.7	13.3	29.9	35.5
Load (ohms)	5,000	20,000	30,000

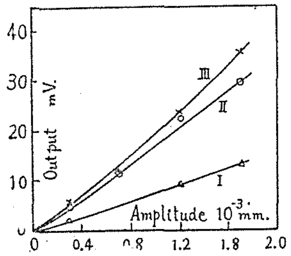

Fig. 4.

As U-effect II is a phenomenon of an alternating capacity current of mercury-solution interfaces, the output voltage is of course proportional to the change of the interfacial capacity, which is again proportional to the change of the interfacial area. As the amplitude of vibration calculated from x_0 is that of the one dimensional variation, the area change must be proportional to the square of this amplitude. This explains the non-linearity of the curve in Fig. 4. (see also reference[6]).

(iv) Number of interfaces

The output voltage increased linearly with the number of interfaces. This is shown in Table 4 and Fig. 5. Here, the load was matched to the element in every case. The reason of this was discussed in other place[3].

Table 4. Relation between output voltage and the number of interfaces.
Element: II Amplitude of vibration$=2.6 \cdot 10^{-3}$ mm. Frequency=500 c/s.

Number of interfaces	Output voltage (mV.)	Load (ohms)
4	4.5	2,000

8	10.6	4,000
12	15.8	6,000
16	22.4	8,000
20	26.6	10,000
24	33.5	12,000
28	37.6	14,000
32	42.2	16,000
36	50.1	18,000
40	59.6	20,000

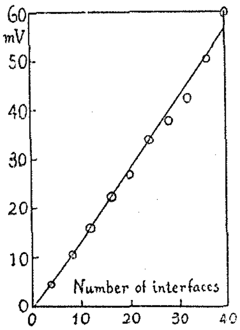

Fig. 5.

Summary

Several important characters of U-effect II were experimentally determined.

(1) The inner impedances of the elements were measured by "Impedance matching method". They were inversely proportional to the cross section of the elements.

(2) The frequency character curves showed natural peaks at lower frequencies with the element of larger cross section and at higher frequencies with the one of smaller cross section in the mode of free end vibration.

(3) The amplitude character curves were non-linear which could be explained from the dimension of vibrational amplitude.

(4) The output voltage increased linearly with the number of mercury-solution interfaces.

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References and Notes

(1) Ueda, Watanabe and Tsuji, *J. electrochem. Soc. Japan*, **19**, 142 (1951); **19**, 193 (1951); *Chem. and chem. Ind.*, **4**, 316 (1951); *Mem. Coll. Agr.. Kyoto Univ.*, **57**, 22 (1950); *This Bulletin*, **18**, 108 (1949); **19**, 44 (1949); **20**, 28 (1950).

(2) Ueda, Watanabe and Tsuji, *Mem. Coll. Agr., Kyoto Univ.*, **60**, 1 (1951); **60**, 8 (1951); *This Bulletin*, **22**, 31 (1950); **23**, 23 (1950).

(3) Ueda, Watanabe and Tsuji, *Mem. Coll. Agr., Kyoto Univ.*, 60, 13 (1951); *This Bulletin*, **24**, 12 (1951); **25**, 30 (1951); **29**, 32 (1952).

(4) Ueda, Watanabe and Tsuji, *Mem. Coll. Agr., Kyoto Univ.*, **57**, 22 (1950); *This Bulletin*, **20**, 28 (1950).

(5) The method used here is quite the same in principle as that of the capacity measurement, the details of which were described in the papers on it. See references in (3).

(6) Ueda, Watanabe and Tsuji, *This Bulletin*, **28**, 47 (1952).