For the calibration curve to measure the amounts of potassium by the radioactivity of $^{40}$K we determined the proportionality existing between the counting rates and the percentages of potassium, using the standard sample. The standard samples used were the potassium compounds ($\text{KC}$, $\text{KNO}_3$, $\text{K}_2\text{Cr}_2\text{O}_7$) mixed with the sodium chloride or calcium carbonate, and the percentages of potassium were from 5 percent to 28 percent. We evaluated the self-absorption curve of the beta-rays from $^{40}$K in the samples. The counting rates became constant in the range of thickness of samples greater than 0.35 g./cm$^2$. Therefore, we took the thickness of samples as 0.4 g./cm$^2$. The samples were generally placed 3 mm. under the window of the G-M counter throughout these measurements.

Using the above mentioned calibration curve, we determined the percentages of potassium content from the counting rates of radioactivity in plant ashes. Plant ashes used were as follows: $\text{Sasa paniculata}$, $\text{Miscanthus sinensis}$, $\text{Artemisia vulgaris L}$, $\text{Pteridium aquilinum}$ $\text{Kuhn}$, $\text{Cyperus serotinus}$ $\text{Rottb}$, $\text{Serrulata spontanea}$ $\text{Makino}$, $\text{Petasites japonicus}$. To ascertain the amounts of potassium in plant ashes, we quantified them by the chemical chloroplatinate method. There were differences of about 3 percent between the values estimated from counting rates and those determined by chemical method. Further researches are now in progress.

7. Study on High Dielectric Constant Ceramics. (XVII)

Coupled Vibrations in Electrostrictive Vibrators

Kiyoshi Abe, Tetsuro Tanaka and Koji Uo

(Abe Laboratory)

Theoretical analysis of coupled vibration, which can be seen in an electrostrictive vibrator having the shape of rectangular plate, cylinder or hollow cylinder, were considered in previous report and it was verified that such theory agrees quite well with the experimental results in case of $\text{BaTiO}_3$ ceramic vibrator. Higher harmonics of vibration, which have been neglected previously and must be taken into consideration in actual case, will be discussed in this report.

The resonant angular frequencies (r.a.f.) of two independent vibrations must be described as follows considering higher harmonic vibrations:

$$w_{a2} = \frac{n^2E}{a_n^2\rho}, \quad w_{b2} = \frac{n^2E}{b_n^2\rho}$$ (1)
If coupling effects exist between these two vibrations, the r.a.f. will be given in next equations substituting \( p = \frac{a_m}{d_n} \),

\[
\begin{align*}
\omega_1^2 &= \frac{\pi^2 E}{b_n^2 \rho} \frac{(p^2 + 1) - \sqrt{(p^2 + 1)^2 - 4p^2(1 - \kappa^2)}}{2p^2(1 - \kappa^2)} \\
\omega_2^2 &= \frac{\pi^2 E}{b_n^2 \rho} \frac{(p^2 + 1) + \sqrt{(p^2 + 1)^2 - 4p^2(1 - \kappa^2)}}{2p^2(1 - \kappa^2)}
\end{align*}
\]

(2)

where \( x \) is the coupling coefficient, which is equal to Poisson's ratio \( \sigma \) in case of two dimensional coupling and to \( \{2\sigma^2/(1 - \sigma)\}^{1/2} \) in case of three dimensional coupling.

In rectangular plate having length \( a \) and width \( b \)

\[
a_m = a/m, \quad b_n = b/n
\]

(3)

so the two r.a.f. of coupled vibrations will be obtained by substitution of eq. (3) in eq. (2), where \( m \) and \( n \) are positive integer and take odd values in an electrostrictive vibration.

In a thin hollow cylinder having axial length \( a \) and radius \( r \), \( w_a \) is the same to that is given in eq. (1), but \( w_b \) must be described as follows:

\[
w_b^2 = \frac{\pi^2 E}{b_n^2 \rho} \frac{E}{r^2 \rho} (1 + n^2)
\]

(4)

and so,

\[
b_n = \frac{\pi r}{\sqrt{1 + n^2}}
\]

(5)

where \( n = 0, 1, 2 \ldots \). The r. a. f. \( w_1 \) and \( w_2 \) of coupled vibrations can be obtained by substitution \( a_m = \frac{a}{m} \) and eq. (5) in eq. (2).

As to the vibration of circular disc or cylinder having axial length \( a \) and \( r \), \( w_a \) is the same to that is given in eq. (1) and \( w_b \) is described as follows.

\[
w_b^2 = \frac{\zeta_n^2 E}{r^2 \rho (1 - a^2)}
\]

(6)

and so,

\[
b_n = \pi r \sqrt{1 - a^2} / \zeta_n
\]

(7)

where \( \zeta_n \) is the roots of the next formula.

\[
\zeta_n J_0(\zeta_n) - (1 - \sigma)J_1(\zeta_n) = 0
\]

(8)
The values of $\zeta_n$ are listed on the Table. The r.a.f. $w_1$ and $w_2$ of coupled vibrations can be obtained by substitution of $a_m = a/m$ and eq. (7) in eq. (2).

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\zeta_1$</th>
<th>$\zeta_2$</th>
<th>$\zeta_3$</th>
<th>$\zeta_4$</th>
<th>$\zeta_5$</th>
<th>$\zeta_6$</th>
<th>$\zeta_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.26</td>
<td>2.0235</td>
<td>5.3816</td>
<td>8.5672</td>
<td>11.728</td>
<td>14.881</td>
<td>18.030</td>
<td>21.177</td>
</tr>
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<td>0.28</td>
<td>2.0362</td>
<td>5.3965</td>
<td>8.5696</td>
<td>11.730</td>
<td>14.883</td>
<td>18.031</td>
<td>21.178</td>
</tr>
<tr>
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<td>5.3894</td>
<td>8.5719</td>
<td>11.732</td>
<td>14.884</td>
<td>18.032</td>
<td>21.179</td>
</tr>
<tr>
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<td>5.3930</td>
<td>8.5742</td>
<td>11.734</td>
<td>14.885</td>
<td>18.033</td>
<td>21.180</td>
</tr>
<tr>
<td>0.34</td>
<td>2.0735</td>
<td>5.3970</td>
<td>8.5766</td>
<td>11.735</td>
<td>14.887</td>
<td>18.034</td>
<td>21.181</td>
</tr>
</tbody>
</table>

To obtain the material constant $E/\rho$ or coupling coefficient $\kappa$ by experiments, it is convenient to use the next relations deduced from eq. (2).

$$\frac{E}{4\rho} = \frac{(a_m^2 + b_m^2)f_1f_2}{f_1^2 + f_2^2} \quad (9)$$

$$\sqrt{1 - \kappa^2} = \frac{f^2 + 1}{\rho} \cdot \frac{c}{c^2 + 1} \quad (10)$$

where $f = w/2\pi$ and $c = f_1/f_2$. Fig. 1 shows one of the experimental results carried out about a hollow cylindrical vibrator consisting of BaTiO$_3$ ceramics; dotted lines
shows the theoretical values deduced from eq. (1) and real curves the experimental results. Dotted line at 3.7 MC shows the resonant frequency of longitudinal thickness mode.

8. Study on High Dielectric Constant Ceramics. (XVIII)

BaTiO₃ Ceramic Vibrator as a Filter Element

Kiyoshi ABE, Tetsuro TANAKA, Shigeru MIURA and Akira MURATA
(Abe Laboratory)

The characteristics of BaTiO₃ ceramic vibrator is not good for the application as a filter element because of its ferroelectric properties and especially of its temperature coefficient of frequency constant. Authors investigated a method for improving these undesirable characteristics, and a fairly good result could be obtained. The principle of the procedure for improving the characteristics of ceramic vibrator is as follows. A BaTiO₃ ceramic vibrator has a large positive temperature coefficient of frequency constant amount to 2000~3000 × 10⁻⁶ at room temperature, but ordinary elastic materials such as metal or glass, on the other hand, have negative temperature coefficient. And so, it is able to compensate the temperature coefficient by conjoining the latter to the former. The kind of material of the latter and volume percentage, in which the latter occupies in a combined vibrator, will determine the degree of compensation of the temperature coefficient. The adhesion of the other material to ceramics will be of use, at the same time, in reducing the other unstable characteristics due to the ferroelectric properties of BaTiO₃ ceramics.