New Approximate Methods of Dielectric Measurement in the Centimeter Wave Region I*

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In the previous paper, starting from Maxwell's equations in the wave guide system and taking into consideration the conditions at the boundaries between the sample and the air columns, we introduced the fundamental equation for the dielectric measurement and from this equation, we derived several new methods of measurement, one of which especially was simpler in practical calculation than any method proposed by other researchers.

In the present paper, we introduce the form of fundamental equation which is convenient to make approximation and show the simplest methd out of the various methods which can be obtained by approximation from that equation for low loss sample. This method is also simple in procedure and calculation of mesurement, compared with the approximate methods reported up to the present.

Furthermore, we describe the experiments which have been performed on two low loss samples and shown a good agreement with the theory.

INTRODUCTION

The approximate methods of the microwave dielectric measurement using the wave guide have been reported by many authors. These methods are principally based on approximate forms of a complicated implicit function which contains the propagation constant $\gamma_d = \alpha_d + j\beta_d$ of the sample.

So far as we know, in 1946, Roberts and von Hippel proposed for the first time an approximate method for low loss materials utilizing the standing wave in the rectangular wave guide.[1]

This method is based on the approximate expansion of the transcendental equation containing $\gamma_d d$, which was introduced by them for the case that the back face of the sample was terminated by a perfectly reflecting plane, as shown in the figure. This method is valid only when the sample length d is near to $3/4\ \lambda_d$ and the VSWR Γ at the front face of the sample is very large, where λ_d is the wavelength in the sample column.

Afterward, Dakin and Works[2] introduced a more convenient approximate me-

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thod by a modification of this laborious method by Roberts and von Hippel.

In this method, the procedure of measurement is fairly easy and the calculations for both α_d and β_d are also comparatively simple, because terms of $1/\Gamma$ and $\alpha_d d$ are neglected here. The validity of this approximation is generally restricted by the sample length. They verified empirically by a numerical check of the results of many measurements, that their method could be applied to the materials having tan δ less than 0.1 independently of the sample length.

Surber referred to an approximate method too in his paper in which the measurement for medium loss and high loss liquids was chiefly treated.[3]

In his general method, he introduced especially a parameter, D which he called the "dissipation factor" of the guide system, The use of D instead of tan δ resulted in a simplification of the analysis, that is, for low loss materials, the validity and the percent error of the approximation could be discussed with more generality in terms of D than those of tan δ.

He determined the value of ε' by measuring λ_d and for the determination of ε'', he showed an exact expression of $\alpha_d \lambda_d$, which holds only for the case where the sample length is equal to an integral multiple of $\lambda_d/4$ and the sample column has a shortcircuit or an opencircuit termination. From this expression, he derived a comparatively simple form by neglecting terms of D for the low loss materials having D less than 0.1 and moreover for $0.3 \geqq D \geqq 0.1$, he used the first approximation of that expression.

However, this method requires the continuous variation of the sample length and so the experimental procedure seems to be rather complicated.

Furthermore, Surber and Crouch proposed an approximate method applicable when the sample is exactly an integral multiple of a quater wave length long and has a shortcircuit or an opencircuit termination.[4]

This method has the mathematical advantage that the equation fundamental for the measurement reduces to a very simple form since the impedance of the sample column is approximately a pure resistance for these resonant lengths and also the experimental advantage that the over-all sensitivity for loss measurement is increased since the power absorption occurs in the neighbourhood of its maximum and so the dielectric loss relative to the guide losses increases. This method also, however, requires either a continuous variation of the sample length or a frequency variation. But the frequency variation method as well as the sample length variation has not been so simple in our experience. These types of resonance method are not applicable to the materials for which tan $\delta > 0.1$, as Surber and Crouch have said.

Besides the methods mentioned above, several approximate methods have been used in practical measurements. But the most these methods[5],[6] are principally the same as those of Surber and Surber-Crouch.

In the present paper, we derive the fundamental equation for the approximate method from the theory of dielectric measurement introduced in the previous paper[7] and describe the most convenient method out of the various methods which can be devised from this fundamental equation. The experiments performed by this method on benzene (C_6H_6) and carbon tetrachloride (CCl_4) are also described.

Furthermore, we investigate mathematically the accuracy of our approximate method and discuss the corrections for the approximation.

In the dielectric measurement for low loss materials, the losses of the guide wall must be, of course, considered as important factors, but are neglected within the scope of the present paper. About these losses, however, we will discuss in the next paper on this work.
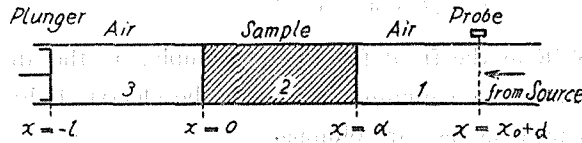

Fig 1.

1. THEORY OF MEASUREMENT

From the boundary conditions at the two faces, $x = 0$ and $x = d$ of the sample, which is inserted in the rectangular wave guide section excited by H_{01}-wave as shown in the figure, we have previously obtained the following equation[7]:

$$T = \frac{\gamma_g}{\gamma_d} \cdot \frac{e^{2\gamma_d d} + r_2}{e^{2\gamma_d d} - r_2} , \tag{1}$$

with

$$T \equiv \frac{e^{2\gamma_g d} + r_1}{e^{2\gamma_g d} - r_1} \tag{2}$$

and

$$r_2 = \frac{\dfrac{\gamma_d}{\gamma_g} \tanh \gamma_g l - 1}{\dfrac{\gamma_d}{\gamma_g} \tanh \gamma_g l + 1} , \tag{3}$$

where γ_i and r_i $(i = 1, 2, 3)$ represent the propagation constants and the complex reflection coefficients in the respective columns. In the above equations γ_1 and γ_3 are replaced by γ_g, as $\gamma_1 = \gamma_3$ and γ_2 by γ_d and moreover, the guide is assumed to be terminated by a perfectly reflecting plane at $x = - l$.

Substituting (3) into (1), the following equation is derived:

$$T = \frac{\gamma_g}{\gamma_d} \tanh (\varphi + \gamma_d d), \tag{4}$$

where

$$\varphi = \tanh^{-1}\left(\frac{\gamma_d}{\gamma_g} \tanh \gamma_g l \right).$$

Now, neglecting the loss of the guide wall α_g against β_d, (4) reduces to the following form :

$$T = \frac{j\beta_g}{\alpha_d + j\beta_d} \tanh\left[\alpha_d d + j\beta_d d + \tanh^{-1}\left(\frac{\alpha_d + j\beta_d}{\beta_q}K\right)\right], \qquad (5)$$

where $K \equiv \tan \beta_g l$.

On the other hand, we have shown in the previous paper that T defined by (2) is transformed into the convenient form for the measurement : [7]

$$T = \frac{\Gamma + j \cot \beta_g x_0}{1 + j\Gamma \cot \beta_g x_0}, \qquad (6)$$

where Γ is the VSWR at the front face of the sample, x_0 the distance from the front face of the sample to a minimum point of the electric field and $\beta_g = 2\pi_1/\lambda_g$, λ_g being the wavelength in the air columns.

The value of this T reduces to a real number for such l that x_0 is equal to

$$(2n + 1) \lambda_g/4 \qquad (7)$$

and

$$n\lambda_g/2, \qquad (8)$$

where n is a positive integer. Accordingly, the right-hand side of (5) must be real for these two cases.

Assuming that $\alpha_d \ll \beta_d$ and expanding the term $\tanh^{-1}[(\alpha_d + j\beta_d) K/\beta_q]$ in the right-hand side of (5) by Taylor's theorem, (5) can be, to the first approximation, expressed as follows :

$$T = \frac{j\beta_d}{\alpha_d + j\beta_d} \tanh\left[(1 + F) \alpha_d d + j\left(\beta_d d + \tan^{-1}\frac{\beta_d}{\beta_q}K\right)\right], \qquad (9)$$

where

$$F \equiv \frac{\beta_g K}{\beta_g^2 d + \beta_d^2 K^2 d}. \qquad (10)$$

Eq. (9) is the fundamental equation for the approximate method of dielectric measurement.

Now, by separating the right-hand side of (9) into the real and imaginary parts and letting the latter equal to zero for the above mentioned l's, the following is obtained :

$$T\{I\} = \frac{\beta_g\,[\tan B\,(\beta_d + \alpha_d \tanh A\, \tan B) + \tanh A\,(\alpha_d - \beta_d \tanh A\, \tan B)]}{\beta_d^2\,[1 + (\tanh A\, \tan B)^2]} = 0,$$

where

$$A \equiv (1 + F)\, \alpha_d d \qquad\qquad (11)$$

and

$$B \equiv \beta_d d + \tan^{-1}\frac{\beta_q}{\beta_a}\, K. \qquad\qquad (12)$$

This equation can be satisfied for either of the following cases:

$$\cot B \simeq -\frac{\alpha_d}{\beta_d}\tanh A \qquad\qquad (13)$$

and

$$\tan B \simeq 0. \qquad\qquad (14)$$

Then, the real part of the right-hand side of (9) is expressed as follows:

$$T\{R\} = \frac{\beta_q\,[\beta_d\,\tanh A\,(1 + \tan^2 B) - \alpha_d\,\tan B\,(1 - \tanh^2 A)]}{\beta_a\,[1 + (\tanh A\,\tan B)^2]}$$

Substituting (13) into (12) and this expression of $T\{R\}$,

$$\frac{\beta_d}{\beta_q}\, K_1 = \cot (\beta_d d + \xi) \qquad\qquad (15)$$

and

$$T_1\{R\} = \frac{\beta_q}{\beta_a}\, \coth A_1 \qquad\qquad (16)$$

are derived respectively, and substituting (14) into (12) and the $T\{R\}$,

$$\frac{\beta_d}{\beta_g}\, K_2 = -\tan \beta_d d \qquad\qquad (17)$$

and

$$T_2\{R\} = \frac{\beta_q}{\beta_d}\, \tanh A_2 \qquad\qquad (18)$$

respectively follow, where

$$\xi = -\frac{\alpha_d}{\beta_d}\, \tanh A_1.$$

The suffixes 1 and 2 in the above equations correspond to the cases shown by (7) and (8) respectively, as evidently seen from (16) and (18). So the relations $T_1\{R\} = \Gamma_1$ and $T_2\{R\} = 1/\Gamma_2$ hold.

Expanding (15) by Taylor's theorem, the following is obtained to the first approximation:

$$\frac{\beta_d}{\beta_g}\, K_1 = \cot \beta_d d - \frac{\xi}{\sin^2 \beta_d d} \quad .$$

(67)

This equation and (17) yield

$$\left(\frac{\beta_a}{\beta_g}\right)^2 = -\frac{1}{K_1 K^2}\left(1 + \frac{2\xi}{\sin 2 \beta_a d}\right) \tag{19}$$

and furthermore, (16) and (18) reduce to the following forms respectively:

and
$$\left.\begin{array}{l}\coth (1 + F_1)\, a_a d = \left(\dfrac{\beta_a}{\beta_g}\right)\Gamma_1 \\[3mm] \tanh (1 + F_2)\, a_a d = \left(\dfrac{\beta_a}{\beta_g}\right)\dfrac{1}{\Gamma_2}\end{array}\right\}. \tag{20}$$

In the case where $2\,\xi/\sin 2\,\beta_a d \ll 1$ and F's $\ll 1$, which occurs depending chiefly on the sample length, (19) and (20) reduce to the following simple forms:

$$\left(\frac{\beta_a}{\beta_g}\right)^2 = -\frac{1}{K_1 K_2} \tag{21}$$

and

$$\tanh a_a d = \frac{1}{\sqrt{\Gamma_1 \Gamma_2}}. \tag{22}$$

Accordingly, by measuring only K's and Γ's, $\varepsilon^* = \varepsilon' - j\varepsilon''$ of the sample can be determined by the following formulae:

$$\left.\begin{array}{l}\varepsilon' = \left[1 - \left(\dfrac{\lambda}{\lambda_c}\right)^2\right]\left(\dfrac{\beta_a}{\beta_g}\right)^2 + \left(\dfrac{\lambda}{\lambda_c}\right)^2 \\[3mm] \varepsilon'' = \left[1 - \left(\dfrac{\lambda}{\lambda_c}\right)^2\right]\dfrac{a_a}{\beta_a}\left(\dfrac{\beta_a}{\beta_g}\right)^2\end{array}\right\}, \tag{23}$$

where λ and λ_c are the free space wavelength and the cutoff wavelength respectively and in the first equation of (23) the term of $a_a{}^2$ is neglected.

2. EXPERIMENT

The procedure is as follows. After determining the constants λ, λ_g and λ_c of the apparatus, the probe is placed at the points $(2n + 1)\lambda_g/4$ and $n\lambda_g/2$ apart from the front face of the sample. Then, adjusting the plunger so that the electric field may be minimum at each position of the probe, Γ and l are measured for each position of the plunger respectively. Using these observed values of l's and Γ's, $(\beta_a/\beta_g)^2$ and a_a are determined by (21) and (22).

In practice, sample lengths at which $\sin 2\,\beta_a d$ is small must be avoided and furthermore, if F's can not be neglected against unity, (20) must be used instead of (22) to determine a_a, though in the present experiment, values of a_a were computed by (22). About these points we will discuss in the next paragraph.

The apparatus used in the present experiment is almost the same as that men-

tioned in the previous paper.[8] Thin layers of mica (the thickness is 0.003 cm.) were inserted between the air columns and the sample column.

The used wavelengths are

λ = 3.392 cm. (8845 Mc/sec), λ_g = 5.072 cm. and λ_c = 4.572 cm.

As the samples for the examination of the method mentioned in the preceding paragraph, benzene and carbon tetrachloride were used. In the following tables the experimental results on these samples are shown.

Table 1.

d (cm.)	K_1	K_2	Γ_1	Γ_2	ε'	ε''	Method 6 ε'
			Benzene			20°C	
12.00	0.2863	-0.9033	18.7	74.3	2.287	0.0032	2.283
9.68	-0.08177	3.8237	16.6	111.4	1.986	0.0034*	2.005
5.64	0.1663	-1.5527	31.0	109.4	2.289	0.0043	2.285

Table 2.

d (cm.)	K_1	K_2	Γ_1	Γ_2	ε'	ε''	Method 6 ε'
		Carbon tetrachloride				20°C	
12.00	0.6530	-0.4094	19.2	73.1	2.232	0.0031	2.235
9.68	0.1245	-2.1455	20.6	74.3	2.235	0.0037	2.238
5.64	0.3174	-0.8433	24.8	94.9	2.230	0.0057	2.234

* This value was computed with the value of β_d obtained in the case where d=12.00 cm.

3. DISCUSSION

(1) On the Measurement of ε'

As shown in the tables in the preceding paragraph, the values of ε' obtained by the approximate method proposed in the present paper agree with those obtained by the method 6 which is non-approximate and was proposed in the previous paper.[8] This good agreement is obiously explained from the fact that the expression for ε' of the method 6 reduces to the form (21) of this approximate method by neglecting the terms containing the factor $1/\Gamma''$s.

The measured values of ε' of benzene agree with the values obtained by Bleaney, Loubser and Penrose : 2.2835 and 2.2850 (at λ = 3.2cm. and 20°C),[9] and these

values have been also confirmed by Heston and Smyth, and others.[6],[10],[11] The value for d = 9.68 cm., however, is comparatively different from the other values by us. This disagreement may result from the fact that 9.68 cm. is about 15 times as large as $\lambda_d/4$ = 0.645 cm. and therefore, sin 2 $\beta_a d$ is nearly equal to zero, that is, the correction term becomes appreciable in (19). In practice, the sample lengths which are near $n\lambda_a/4$ must be avoided. It has been empirically made clear that this circumstance may also hold in method 6.

In the experiment on carbontetrachloride, our result shows a good agreement for each sample length, since it is not an unsuitable length as mentioned above, and moreover, our measured values are very near to the value by Bleaney, Loubser and Penrose: 2.2386 (at λ = 3.2 cm. and 20°C)[9] and to the value by Heston and Smyth: 2.240 (at λ = 10.0 cm. and 1.277 cm. and 20°C).[10]

(2) On the Measurement of ε''

We have experienced that the measurement of ε'' is very troublesome in method 6 and have not been able to obtain good results in the experiment on cetyl alcohol.[8] Also, in this approximate method, good results are not obtained compared with those about ε', as shown in the tables, that is, the obtained values of ε'' of the both samples have shown remarkable deviations with different sample lengths. The values for d = 5.64 cm. exceptionally deviated from the others.

For the determination of the values of ε'' shown in the tables, we have computed the values of a_d, by the use of (22) in which F's are not considered, but we have obtained by using (21) almost the same values of ε'' as the above. $|Fd|$ can be shown to be maximum for $K = \pm \beta_a/\beta_d$, and even then $|F|$ is less than 3 percent, if the sample length is sufficiently long. Accordingly, errors due to the neglect of F may affect at most the third significant figure of the measured value.

Therefore, the neglect of F can not explain the deviation of the measured values of ε'', so we must search for the cause for it in the following points.

i) The VSWR becomes very large and so it is generally difficult to measure its value accurately.

ii) Eqs. (13) and (14) can not satisfy the condition that $T\{I\} = 0$, for small sample lengths.

But we may not be able to conclude that the cause for the deviation of the measured values of ε'' results only from these points.

On the materials used in the present experiment, a few results of the loss measurement in the centimeter wave region have been reported,[1],[9],[12],[13] Among these, the tan δ of benzene obtained by Roberts and von Hippel agrees with our results in the first significant figure, but we can not compare the values obtained in the present experiment with those by other reseachers because of differences in the purity of the sample, the used wavelength and the temperature.

The values obtained by method 6 have shown extreme deviations and have not agreed with the results by this approximate method, for the respective sample lengths except in the case $d = 12.00$ cm. of benzene. The calculation in method 6 is simpler than in other non-approximate methods proposed so far, but it is considerably complicated compared with that in this approximate method. Then we have experienced that a little error of Γ affects seriousl the result of the calculation. This fact, together with the observed values of the VSWR having generally low accuracy, seems to be a cause for these deviations. On the contrary, in this approximate method a_a can be computed using the observed values of the VSWR only and then the error attending the calculation may be smaller. By this reason, this approximate method seems to be better than the method 6 for the measurement of ε'' of low loss samples such as used in the present experiment.

CONCLUSION

The approximate method described in the present paper possesses the merits that both the sample deformation and the frequency variation are unnecessary while in most of the approximate methods by others, at least one or the other of them must be used. Then the procedure of ours is extremely convenient and moreover, the calculation is also very simple.

So far as ε' is considered, our experimental results show a good agreement with the theory and therefore, it is confirmed that for low loss materials such as used in this experiment, our approximate method can be used to determine the value of ε' with a sufficient accuracy, instead of non-approximate methods, in which complicated calculations are necessary.

The measured values of ε'' have considerable deviation with the different sample lengths and the cause of this deviation has not been cleared up. Furthermore, it is difficult to compare indiscriminately our results with those reported so far by other researchers, because the conditions of measurement are generally different. Therefore, the validity of our method of loss measurement is not experimentally confirmed, though it seems to be better than other approximate methods in point of procedure and calculation of measurement. We will hereafter investigate this problem.

This approximate method may also be utilized for the measusement of medium loss and high loss materials, if the sample length is adequately chosen and F taken into consideration. About these we will report in the later paper.

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