Gamma-Gamma Directional Correlations of Some Gamma Ray Cascades in $^{110}$Cd

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Directional correlation experiments have been performed for the 1386 keV-884 keV, 760 keV-1509 keV, 934 keV-884 keV and 884 keV-656 keV gamma ray cascades in $^{110}$Cd. The gamma rays were arranged in a plausible decay scheme. By the present measurements the spin sequences and multipolarity assignments $5(D, Q)4(Q)2$, $5(D, Q)4(Q)2$, $6(Q)4(Q)2$ and $4(Q)2(Q)0$ have been ascribed to these four gamma ray cascades, respectively.

1. INTRODUCTION

Many reports on the decay of $^{110m}$Ag (253 days) have so far been published. Siegbahn$^{13}$ studied beta and gamma rays in this decay, and first proposed the decay scheme. His results were essentially confirmed by other workers. However, Cork et al.$^{20}$ proposed a different level scheme, in which eighteen gamma rays have been assigned. The 24 s ground level, $^{110}$Ag, decays to the ground level of $^{110}$Cd by the emission of a 2860 keV beta ray$^{13,14,15}$, while the isomeric state $^{110m}$Ag, 116 keV above the ground level, has been known to decay by emitting two strong beta rays with end point energies of 89 and 530 keV$^{13,14,15}$, which lead to levels in $^{110}$Cd at 2925 and 2474 keV, respectively. Bleuler et al.$^{6}$ studied the decay of $^{110}$In and tried to make clear the level scheme of $^{110}$Cd.

Following these old works, in the period from 1957 to 1959 Sakai et al.$^{7}$, Knippere$^{9}$, Funk and Wiedenbeck$^{9}$, Cappeller and Ganssauge$^{10}$, and Taylor and Friskene$^{10}$ published, in succession, their results of gamma-gamma directional correlation measurements of some gamma ray cascades in $^{110}$Cd and made spin and parity assignments for several levels of $^{110}$Cd.

The investigation reported in this paper was undertaken to re-examine this slightly obscure point by the method of gamma-gamma angular correlation for some gamma ray cascades in $^{110}$Cd following the decay of $^{110m}$Ag.

2. EXPERIMENTAL PROCEDURE

The gamma-gamma coincidence measurements were performed by two identical scintillation detector systems and a coincidence circuit. Each detector system consisted of a 3.8 cm by 2.5 cm NaI(Tl) crystal, coupled optically to a DuMont 6292 photomultiplier, and of a single-channel pulse-height analyzer of the modified Fairstein type$^{15}$. The detector probes were shielded frontally by 2 mm of aluminium and laterally by 20 mm of lead to eliminate the false coincidences due to the scattered radiation. The output pulses from both pulse-height analyzers were fed to a conventional coincidence circuit, of which resolving time was measured

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to be $1.18 \times 10^{-7}$ s by the method using variable delay lines. The point source of $^{110\text{m}}\text{Ag}$ was prepared by evaporating a drop of AgNO$_3$ solution to dryness in a 1 mm by 1 mm hole drilled on the top surface of a 4 mm diameter polystyrene cylinder. The radioactive AgNO$_3$ solution was obtained from the Radiochemical Centre at Amersham, England. The source strength of $^{110\text{m}}\text{Ag}$ used in the present work was made to be about 30 $\mu$Cur by considering the resolving time of the coincidence circuit. The source was placed at the centre of the goniometer, consisting of two arms, one fixed and one movable. The distance between the source and front surface of each crystal was 5 cm in all measurements.

Prior to the angular correlation measurements of the gamma ray cascades in $^{110}\text{Cd}$ the single gamma ray pulse-height spectrum was measured, as shown in Fig. 1. By observing this spectrum and by considering the gamma ray cascades to be measured the positions and widths of the gate windows opened in the coincidence measurements of directional correlations were settled. The horizontal bars in the figure of the gamma ray spectrum show the discriminator settings for various correlations. Coincidence measurements were performed at angles of $90^\circ$, $105^\circ$, $120^\circ$, $135^\circ$, $150^\circ$, $165^\circ$ and $180^\circ$ between the axes of the detectors. The number of single counts in each detector was also recorded at all settings in order to estimate the accidental coincidence rate for each measurement as well as to check the over-all drift of the instrument. Each run of coincidence measure-
Gamma-Gamma Directional Correlations of Some Gamma Ray Cascades in $^{116}$Cd

Measurements for the gamma ray cascade concerned were made first in the direction of increasing angles, then in the opposite direction. The centering of the source was checked by moving the movable detector to the opposite direction providing the angle between both detectors was 90°.

The directional angular correlation of two successive gamma transitions can always be written

$$W(\theta) = \sum_{\nu} A_{\nu} P_{\nu}(\cos \theta) = 1 + A_2 P_2(\cos \theta) + A_4 P_4(\cos \theta),$$

where $P_{\nu}(\cos \theta)$ is the Legendre polynomial of order $\nu$.

The $A_2$ and $A_4$ coefficients in the correlation function were calculated by the least-squares fit using the method of Rose. Corrections for the finite solid angle of the detectors were applied using the theoretically calculated curves reported by Stanford and Rivers.

A check of the instrument used was made by measuring the well-known angular correlation of the gamma ray cascade in $^{60}$Ni, giving the satisfactory result.

3. RESULTS

The correlation measurements were performed for the four cascades of two successive gamma transitions; 1386 keV-884 keV, 760 keV-1509 keV, 934 keV-884 keV and 884 keV-656 keV cascades were measured. The values of gamma ray energies involved were adopted from the table of gamma rays emitted by radioactive nuclei prepared by der Mateosian and McKeown.

3.1. The 1386 keV-884 keV Correlation

In the case for the 1386 keV-884 keV cascade false coincidences may occur from the chance that a 1509 keV gamma ray gives a pulse in the one detector with a window for the 1386 keV gamma ray while the considerably strong 760 keV-656 keV cascade contributes an 884 keV sum peak to the other detector. However, since this probability seemed very small, in the present case the interference from any other gamma rays involved was neglected. After correction for the finite solid angle of the detectors the correlation function was obtained as

$$W(\theta) = 1 - (0.238 \pm 0.018) P_2(\cos \theta) - (0.042 \pm 0.019) P_4(\cos \theta).$$

3.2. The 760 keV-1509 keV Correlation

For the 760 keV-1509 keV correlation measurements with settings of the discriminators as shown in Fig. 1, the correction for the interfering 656 keV line due to the poorer resolutions of the detectors had to be considered. The correction for this interference could be estimated by observing the shape of the photopeak of the 661 keV ray from $^{137}$Cs. The correlation function obtained after all the corrections was

$$W(\theta) = 1 - (0.116 \pm 0.018) P_2(\cos \theta) - (0.052 \pm 0.018) P_4(\cos \theta).$$

3.3. The 934 keV-884 keV Correlation

In this case the correction for an interference from the 1386 keV-884 keV cascade should be made. If the correlation function derived directly from the
Takeshi Mukoyama, Fumiyoshi Makino and Sakae Shimizu

Experimdntal coincidence data by the least-squares analysis without correction for angular resolution is noted by $W'(\theta)$, then the uncorrected correlation function for the 934 keV-884 keV, $W'(\theta)_{934-884}$, can be given using its observed uncorrected correlation and the measured correlation of the interfering cascade of the 1386 keV-884 keV as follows:

$$W'(\theta)_{934-884} = W'(\theta)_{obs} - k W'(\theta)_{1386-884},$$

where $k$ is a constant determined from the relative intensities of the gamma rays involved. $k$ was estimated using the experimental data of the gamma ray spectra of $^{110m}$Ag reported by Dzelepov and Zhukovsky. After correction for angular resolution to $W'(\theta)_{934-884}$ the true correlation function was given by

$$W(\theta) = 1 + (0.102 \pm 0.021)P_2(\cos \theta) + (0.007 \pm 0.022)P_4(\cos \theta).$$

3.4. The 884 keV-656 keV Correlation

Since the 656 keV gamma ray in the transition from the first excited level to the ground level, in this case the interferences from the Compton degraded radia-
tions due to the 1509 keV and 1386 keV were believed to be rather small by ob-
serving the pulse-height spectrum of the 1277 keV ray from $^{22}$Na available for the present work. It was estimated that with the discriminator settings used less than about 4 percent of the total coincidence counts was due to these interfering radia-
tions. No correction was made for these interferences. However, the interfer-
ence from the 934 keV-884 keV cascade should be taken into account. The correction for this interference was made by the same procedure as in the case for the 934 keV-884 keV correlation. The result obtained after the corrections for the coincidence contribution and finite solid angle was

$$W(\theta) = 1 + (0.115 \pm 0.022)P_2(\cos \theta) + (0.013 \pm 0.023)P_4(\cos \theta).$$

4. ANALYSIS OF RESULTS

To interpret the experimental results the reasonable decay scheme of $^{110m}$Ag proposed by Funk and Wiedenbeck has been adopted in the present work. The spin assignments for levels concerned and multipolarity assignments for gamma transitions involved have been attempted by basing essentially on the present results as well as many facts confirmed by previous workers.

The ground level and first excited level at 656 keV of $^{110}$Cd have spin $0^+$ and $2^+$, respectively, being in agreement with even-even nuclei systematics. This has been definitely established by conversion coefficient measurements and the Coulomb excitation experiments.

The spin of $^{110m}$Ag has been determined as 6 by Ewbank et al. using the atomic beam technique. As the 86 keV beta decay of $^{110}$Ag has been known to be an allowed transition, the spin value of 5 or 6 or 7 may be assigned for the 2925 keV level. The $ft$ value of the 536 keV beta decay shows its transition to be a first forbidden type, then the possible spin of the 2474 keV level is re-
stricted to one of 4, 5, 6, 7 or 8. The spin difference between 2474 keV and 656 keV levels should not be larger, because the 934 keV-884 keV cascade leads from the 2474 keV level to the first excited level at 656 keV with a spin value of 2^+ and
both gamma rays are intense. By this reason the spin of the 2474 keV state may be 4 or 5 or 6. From the fact that all the gamma rays involved are relatively intense and no excited level in $^{110}$Cd has a half-life greater than $5 \times 10^{-10}$ s, multipolarities of transitions higher than quadrupole are excluded. A spin value of 7 can not be assigned for the 2925 level, since this would lead to an octupole transition for the 1386 keV gamma ray.

Bleuler et al. have concluded by their study of $^{110}$In that the 884 keV and 934 keV transitions may be assumed to be E2 and M1, respectively, or mixtures thereof, while the multipolarity of the 884 keV transition has been concluded as E2 by Dželepow and Zhukovsky from their study of the gamma rays of $^{110}$Cd. The spin value for the 1540 keV level is presumed to be either 3 or 4. However, a spin value of 4 is acceptable rationally by taking account of the fact that the second excited level of an even-even nucleus has, in general, a spin value of 0 or 2 or 4, as suggested by Scharff-Goldhaber and Weneser. According to a suggestion by Coleman, a correlation function $A_\delta$ was plotted against $A_2$ using a mixing ratio $\delta$ as a parameter by assuming the value of I in the spin sequence I(D, Q)4(Q)2 to be 4 and 5. For a dipole-quadrupole mixture, the $A_\delta$ vs. $A_2$ curves are ellipses as shown in Fig. 2. The experimental points for $A_2$ and $A_\delta$ obtained for the 1386 keV-884 keV, 934 keV-884 keV and 760 keV-1509 keV cascades are also shown in the figure. The fact that the experimental point for the 1386 keV-884 keV cascade lies near the ellipse for the 5(D, Q)4(Q)2 sequence convinced ourselves of the spin value of the 2925 keV level being 5 and the 1386 keV transition being a dipole-quadrupole mixture and the 884 keV transition being pure quadrupole.

![Diagram](https://via.placeholder.com/150)

Fig. 2. $A_\delta$-$A_2$ elliptical representations of gamma-gamma directional correlation as a function of the mixing ratio for various spin sequences.
The value of $\delta$ giving the best fit to the experimental data is +5.85. A solid curve in Fig. 3 shows the theoretical correlation calculated with the $5(D, Q)4(Q)2$ sequence for this gamma ray cascade.

![Fig. 3. Directional correlation of the 1386 keV-884 keV gamma ray cascade. A curve represents the theoretical correlation calculated for a spin sequence $5(D, Q)4(Q)2$ with a mixing ratio $\delta = +5.85$.]

In the $A_2-A_4$ diagram the proximity of the experimental point for the 934 keV-884 keV cascade to three ellipses for spin sequences $4(D, Q)4(Q)2$, $5(D, Q)4(Q)2$ and $5(Q)3(D, Q)2$ and to a point corresponding to the $6(Q)4(Q)2$ sequence makes a unique spin assignment of the 2474 keV level difficult. However, it has been reported that the measured K-conversion coefficient for the 934 keV gamma ray shows this radiation to be pure $E2$\textsuperscript{16}. And according to the theory of the collective excitations of non-axial even nuclei developed by Davydov and Chaban\textsuperscript{23}, one of excited levels of $^{110}$Cd may have a spin of 6. The theoretical coefficients for this pure $6(Q)4(Q)2$ sequence are $A_2 = -0.1020$ and $A_4 = -0.0091$, which are quite consistent with our experimental values of $A_2 = -(0.102\pm0.021)$ and $A_4 = -(0.007\pm0.022)$. Reflecting on these informations the spin value of the 2474 keV level was concluded to be 6, and a spin sequence $6(Q)4(Q)2$ has been assigned to the 934 keV-884 keV cascade. Measured values of the angular correlation for this cascade and the theoretical correlation calculated basing on the $6(Q)4(Q)2$ sequence are shown in Fig. 4.

Knipper\textsuperscript{31} has suggested that both the 760 keV and 1509 keV transitions are multipole mixtures and has also set the upper and lower limits on the mixing ratio for each. But, from the informations mentioned above, it is confirmed that the levels at 2925 and 656 keV are spins of 5 and 2, respectively. So, it seems to be more reasonable that one of them must be pure radiation and the other a dipole-quadrupole mixture. The values of the K-convresion coefficients for these
gamma rays measured by Dželepov and Zhukovsky\(^\text{15}\) seem to give no definitive suggestion for this choice. However, it seems probable that the 760 keV gamma ray is a mixture and the 1509 keV ray is pure quadrupole from the fact that our experimental data of the correlation functions for this 760 keV-1509 keV cascade lies near the ellipse for the 5(D,Q)4(Q)2 sequence, as shown in Fig. 2. The only possible choice of the sequence for this cascade was, therefore, concluded to be 5(D,Q)4(Q)2. The best fit of the mixing ratio for the dipole-quadrupole transition of the 760 keV to the experimental data was found as \(\delta = -56.4\). Measured angular correlation of this cascade and the theoretical correlation for the spin sequence 5(D,Q)4(Q)2 are shown in Fig. 5.

The multipolarity of the 656 keV transition has been determined as \(E2\) from Coulomb excitation experiments by Temmer and Heydenburg\(^\text{17}\) and Stelson and McCowan\(^\text{18}\). Taking into account of this information together with that on properties of the even-even nuclei, the spin sequence for the 884 keV-656 keV cascade was concluded to be 4(Q)2(Q)0. The theoretical correlation function for the spin sequence 4(Q)2(Q)0 is quite consistent with the experimental result, as shown in Fig. 6.

5. CONCLUSION

Analysing the experimental results and referring to the informations obtained by other workers, as described in the preceding section, all of the observed transitions and spin values of \(^{110}\text{Cd}\) levels involved can be placed in the decay scheme. In the decay scheme first proposed by Siegbahn\(^\text{15}\) and also accepted by other workers the 1509 keV transition precedes the 760 keV transition. However, the
Fig. 5. Directional correlation of the 760 keV-1509 keV gamma ray cascade. A curve represents the theoretical correlation calculated for a spin sequence $5(D, Q)4(Q)2$ with a mixing ratio $\delta = -53.4$.

Fig. 6. Directional correlation of the 884 keV-656 keV gamma ray cascade. A curve represents the theoretical correlation calculated for a spin sequence $4(Q)2(Q)0$.

slightly revised decay scheme proposed by Funk and Wiedendeck, the inverted order of these transitions is assumed. Since the later choice of the order of the transitions seems to be favourable to interpret the experimental results obtained by the present work, all transitions and spin values of $^{110}$Cd levels pertinent to the experimental data are designated in the decay scheme as shown in Fig. 7, in
a consistent manner with the analysis of the correlation data described in the preceding section. The parities of the nuclear levels of \(^{110}\text{Cd}\) could not be determined directly from the present work. However, the parities of ground and lower levels consistent with the present measurement and other experiments on multipolarity assignments of the transitions can easily be presumed. These are also in agreement with the systematics of even-even nuclei in this region.

The spins and parities of \(^{110}\text{Ag}\) and \(^{110m}\text{Ag}\) have been presumed to be 1+ and 6+, respectively, by the shell model predictions that in \(^{110}\text{Ag}\) the last proton is in a \(g_{9/2}\) state and the last neutron is in a \(g_{7/2}\) state while in \(^{110m}\text{Ag}\) the last proton and neutron occupy the \(g_{9/2}\) and \(d_{5/2}\) state, respectively. Since the 86 keV and 536 keV beta transitions from \(^{110m}\text{Ag}\) have been known to be allowed and first forbidden, respectively, the 2925 keV and 2474 keV levels in \(^{110}\text{Cd}\) should have opposite parity. If the spin of 5+ is assigned to the 2925 keV level, the 2474 keV level should have 6−. But, the internal conversion data of the 934 keV and 884 keV transitions measured by Dželepow and Zhukovsky\(^{19}\) show that both transitions are E2. From this data it becomes unavoidable to assign the spin and parity for the 2474 keV level as 6+. These conflicting possibilities of spin and parity assignments for the 2474 keV level may be explained by assuming that the 536 keV beta rays is probably an allowed transition hindered by the additional selection rules associated with the beta decay of deformed nuclei. There are some evidences which suggest that \(^{110}\text{Cd}\) may be a deformed nucleus and hindered beta tran-

Fig. 7. Simplified decay scheme for \(^{110m}\text{Ag}\) showing the gamma ray cascades which were used in the present work. All energy are in keV.
sition from $^{110m}\text{Ag}$ may be possible\textsuperscript{23}. From this standpoint the spin of 6+ has been assigned to the 2474 keV level and the spin of 5+ has also been designated to the 2927 keV level.

The results of the present work are summarized in the simplified decay scheme for $^{110m}\text{Ag}$ shown in Fig. 7. The spin assignments of the excited levels of $^{110}\text{Cd}$ involved have been found to be in agreement with those reported by Taylor and Friksen\textsuperscript{11} with an exception that the 2165 keV level was assigned to be 3+ by them while to be 4+ by our work. On this point further study seems to be preferable.

Note added in proof: After this work had been completed reports of the new extensive study on decay of $^{110m}\text{Ag}$, $^{110}\text{In}$ and $^{110m}\text{In}$ published by Yoshizawa et al.\textsuperscript{24,25} became available for us. A point of ambiguity of the present work seems to be dissolved by the new work published by them.

REFERENCES

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