Nucleon Polarization Effect on Spectrum of μ-Mesonic Hydrogen Atom

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The experimental accuracy in spectra of μ-mesonic atoms is limited due to the weakness of meson beams and the poor resolving power of detecting instruments at the present stage, but it seems significant to analyze theoretically various effects about the μ-mesonic hydrogen atom which is the most fundamental μ-mesonic atom. We want to suggest that the polarizability of the pion cloud of the nucleon gives rise to the energy shifts of the μ-mesonic hydrogen atom.

The polarizability of the pion cloud by external electric field is a very important problem in the meson theory of the nucleon structure. Though this effect can be analyzed from scattering data, there are some ambiguous points. That is, the value of the polarizability obtained from the low energy neutron scattering by nuclei[1] and that obtained from the Compton effect of the proton[2] are different from each other by one or two orders of magnitude. Then we are investigating whether this polarization effect may be detected from the 2p→1s transition spectrum of the μ-mesonic hydrogen atom.

The pion cloud of the proton will be polarized by the Coulomb field of the μ-meson, and the interaction between the μ-meson and this induced electric dipole moment of the proton will give rise to the energy shifts.

Assuming that the polarization constant of the pion cloud of the nucleon is spin independent and consequently may be expressed by a scalar α, then we have $-\alpha e^2 r^{-4}$ as the perturbation Hamiltonian inducing the pion cloud polarization effect on the spectrta of the μ-mesonic hydrogen atom, where e is the unit of charge and r is the radial coordinate of the μ-meson with respect to the nucleore.

Although we have not yet obtained final results, the outline of our calculation will be shown. First of all, the energy shift of the 1s state is given by the following expression :

$$<n=1,\ l=0\left|-\frac{\alpha e^2}{r^4}\right|n=1,\ l=0> = -4\alpha e^2\int_0^\infty \frac{\exp(-2r)}{r^2}dr\ \mu\text{.a.u.,} \qquad (1)$$

where μ.a.u. denotes the μ-mesonic atomic unit of energy. This is a divergent integral, but the physical meaning of the polarization of the pion cloud will be lost as r approaches to zero. Therefore, we should perform cutting off conveniently. We introduce $[1-\exp(-r/D)]^2$ as a cutting off factor, where D is a

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cutting off radius. Then the integral in Eq. (1) will be replaced by the following expression :

$$\mathrm{I}=\int_0^\infty \frac{\exp(-2r)}{r^2}[1-\exp(-r/D)]^2 dr. \tag{2}$$

By partial integration, Eq. (2) may be written as follows:

$$\left.\begin{aligned}\mathrm{I}=\lim_{\varepsilon\to 0}&\Big\{\frac{1}{\varepsilon}\exp(-2\varepsilon)[1-2\exp(-\varepsilon/D)+\exp(-2\varepsilon/D)]\\&+\log\varepsilon\Big[2\exp(-2\varepsilon)-2(2+1/D)\exp[-(2+1/D)\varepsilon]\\&+(2+2/D)\exp[-(2+2/D)\varepsilon]\Big]\Big\}-4\int_0^\infty\exp(-2r)\log r\,dr\\&+2(2+1/D)^2\int_0^\infty\exp[-(2+1/D)r]\log r\,dr\\&-(2+2/D)^2\int_0^\infty\exp[-(2+2/D)r]\log r\,dr\end{aligned}\right\} \tag{3}$$

The divergent parts of Eq. (3) being cancelled, we get the matrix element

$$\left.\begin{aligned}&<n=1,\ l=0\left|-\frac{\alpha e^2}{r^4}\right|n=1,\ l=0>\\&=-4\alpha e^2[2\log 2-2(2+1/D)\log(2+1/D)+(2+2/D)\log(2+2/D)]\ \mu.\text{a.u.}\end{aligned}\right\} \tag{4}$$

Next, we shall consider the level shift of the 2p state. We can easily obtain

$$<n=2,\ l=1\left|-\frac{\alpha e^2}{r^4}\right|n=2,\ l=1>=-\frac{\alpha e^2}{24}\ \mu.\text{a.u.} \tag{5}$$

It is reasonable to assume that the cutting off radius D is of the order of the π-meson Compton wave length. Then we obtain 433 as the value of Eq. (2), and it is easily shown that the energy shift of the 1s state is several thousand times larger than that of the 2p state.

From the simple static approximation of meson theory[3], where the π-meson field is assumed to be described by the static equation under the action of electric field of strength E:

$$(\nabla^2+e^2E^2z^2)\varphi(r)-\kappa^2\varphi(r)=4\pi g\delta(r), \tag{6}$$

α is shown to be about $2\times10^{-41}\,\mathrm{cm}^3$ ($1.3\times10^{-9}\,\mu$.a.u.). In Eq. (6), $\varphi(r)$ is the π-meson field, ∇^2 is the Laplace operator, δ is Dirac' delta function, κ is the inverced Compton wave length of the π-meson, z is the Z-comonent of the pion coordinates, and g is the pion-nucleon coupling constant.
Thus the shift of the 2p→1s transition energy becomes

$$-2\times10^{-6}\,\mu.\text{a.u.}\sim-10^{-2}\,\text{eV}. \tag{7}$$

On the other hand, if we use the pion cloud polarization constant of the nucleon obtained by applying the Chew theory[4], the shift of the 2p→1s transition energy becomes about one order of magnitude smaller than the above. At any rate, the pion cloud polarization effect of the nucleon in the μ-mesonic hydrogen atom is two or three orders of magnitude larger than the natural line

* The unit of the third power of length in μ-mesonic atomic unit is $1.66\times10^{-32}\,\mathrm{cm}^3$.

breadth of the 2p state. Therefore, we have a possibility of detecting this effect in principle. Of course, we should calculate other offects, that is, those of vacuum polarization, hyperfine structure, the interaction of the μ-meson and atomic electrons, and the extended charge distribution of the nucleon core, etc.

Whereas the measurement of the μ-mesonic hydrogen spectrum is now very difficult and this difficulty is concerned with the problem of X-ray deficiency[5,6] which is also a fundamental research object from the stand point of the quantum theory of matter, we hope that these kinds of experiments will be performed in near future.

In conclusion, the present effect which we are interesting in this note is observable in principle, and it is expected that the future improvement of experimental instruments would make possible the study of the nucleon structure using the spectrum of the μ-mesonic hydrogen atom.

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