Broad Energy Protons from the Reaction
\( \text{C}_{12}^{} + d \rightarrow p + \text{C}_{13}^{} \) (7.64 MeV and 8.33 MeV)

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Broad energy spectra of protons leading to the resonance states of \( \text{C}_{13}^{} \) were observed in the \( \text{C}_{12}^{} (d, p) \text{C}_{13}^{} \) reaction at deuteron energy of 14.60 MeV. These proton spectra are compared with cross sections based on the phase shifts of the neutron-\( \text{C}_{12}^{} \) scattering. Peak positions agree with those predicted from the phase shifts but the relative heights of the peaks disagree with the theoretical prediction. The phase shifts which can fit the proton spectra are obtained inversely from the experimental results and an anomalous behavior of the phase shifts obtained are discussed.

I. INTRODUCTION

It is well known that the final state interaction plays an important role in the direct reaction leading to the three-body final state. If two of three particles interact strongly with each other, the energy spectrum of the remaining third particle can be explained in terms of the scattering phase shifts of the interacting particles. Until now, the final state interaction has been observed mainly in a few nucleon system, but it is also expected that such interactions could be observed in the \( (d, p) \) reaction on light nuclei.

In the neutron-\( \text{C}_{12}^{} \) system, two resonance states with the same angular momentum and spin are known to exist at energies about 3 MeV higher than the threshold value. These states are the \( d_{3/2} \) resonance states at 7.64 MeV and 8.33 MeV excitation of \( \text{C}_{13}^{} \). The phase shifts indicating these resonances have been obtained from the analysis of the neutron-\( \text{C}_{12}^{} \) scattering. It is then worthwhile to examine if the final state interaction in the \( \text{C}_{12}^{} (d, p) \text{C}_{13}^{} \) reaction exhibits these resonance states.

In the present experiment, momentum spectra of protons from the \( \text{C}_{13}^{} (d, p) \text{C}_{13}^{} \) reaction at 14.60 MeV including these resonances were obtained at observation angles from 10° to 80° in 5° steps. Broad proton spectra corresponding to 7.64 MeV and 8.33 MeV states of \( \text{C}_{13}^{} \) were compared with the theoretical prediction based on the neutron-\( \text{C}_{12}^{} \) scattering.

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II. EXPERIMENTAL PROCEDURES

A deuteron beam from the Kyoto University 105 cm cyclotron was brought to a reaction chamber through a pair of quadrupole magnets and a beam analyzing magnet successively. The beam energy was defined to $14.60 \pm 0.064$ MeV with a slit system. After passing through a target, the beam was stopped in a Faraday cup and the beam current was integrated by a current integrator. The target was a self-supporting carbon film of $0.87 \text{ mg/cm}^2$ thick. Charged particles from the target were detected with a broad range magnetic analyzer followed by nuclear emulsions which were covered by aluminium foil of suitable thickness to stop particles heavier than protons. Sakura NR-EI $100 \mu$ nuclear emulsions were used and were developed with amidol developer.

III. RESULTS

Figure 1 gives the momentum spectrum of protons from the $^{12}\text{C}(d, p)^{13}\text{C}$ reaction at $20^\circ$ in the laboratory system. The ordinate shows the number of tracks per $1.6$ mm stripe. The abscissa shows the distance along the plate, and also is plotted the cor-

![Momentum spectrum of protons from the $^{12}\text{C}(d, p)^{13}\text{C}$ reaction at $20^\circ$ in the laboratory system.](image)

Fig. 1. Momentum spectrum of protons from the $^{12}\text{C}(d, p)^{13}\text{C}$ reaction at $20^\circ$ in the laboratory system.
Fig. 2. Proton spectra leading to the 7.64 MeV and 8.33 MeV states of C$^{13}$.

responding proton energy. The energy scale contains about ±40 KeV uncertainty. The energy resolution of the system is estimated to be 0.48% from the width of the sharp peaks. In the figure proton groups are designated with excitation energies of the corresponding states of C$^{13}$. Figure 2 shows the proton spectra leading to the 7.64 MeV, 3/2$^+$ and 8.33 MeV, 3/2$^+$ states of C$^{13}$ at various angles of observation. In this figure, sharp peaks seen in Fig. 1 are removed. The ordinate shows the number of tracks per 8 mm stripe corresponding to the energy width of about 60 KeV. In Fig. 3 the proton spectrum at 12° in the center of mass system is shown with a solid line. The center of mass spectrum was obtained from smoothed laboratory spectra at several angles by the interpolation method.

IV. DISCUSSION

The shape of these spectra can be represented by a stripping formula used in the analysis of the He$^4$ (d, p) He$^5$ reaction. The differential cross section of the stripping reaction is given by

\[
\frac{d^2\sigma}{d\Omega dE_p} \propto \left| \phi_s \left( \frac{\hbar}{2\pi} k_p - \frac{1}{2} \frac{\hbar}{2\pi} k_d \right) \right|^2 \sin^2 \frac{E_p}{(h \alpha)^2} \times [F_s^2(k_o \alpha) + G_s^2(k_o \alpha)] \cdot [E_p c_m(E_{max} - E_p c_m)]^{1/2},
\]

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\[
\delta_2^- = \xi_2^- + \phi_2^R.
\]

where \(\delta_2^-\) is the \(d\)-wave phase shift of the neutron-\(^{12}\)C scattering, \(\xi_2^-\) the resonant part of the phase shift and \(\phi_2^R\) the nonresonant part of the phase shift. In Eq. (1), the first term \(|\phi_2^R|^2\) represents a deuteron factor, the second and third terms the neutron-\(^{12}\)C scattering in the final state and the last term the density of final states. For other notations, see ref. 1).

The values of \(\delta_2^-\) are known in the energy range from \(E_{\text{lab}}^{14b} = 1.45\) MeV to 4.1 MeV and are shown in Fig. 4-a. Then the values of the right hand side of Eq. (1) can be calculated and the result at 12° in the center of mass system is shown in Fig. 3 with a dashed line. In the calculation, 4.8 \(F\) was taken for the channel radius, and the relation between \(E_{p}'\) and \(E_{\text{lab}}^{14b}\)

\[
(12/14)E_{p}'^{14b} - 2.226 = (12/13)E_{p}'^{14b} + [(M_\pi + M_\alpha + M_p)/(M_\pi + M_\alpha)] E_{p}'^{13n}
\]

was used, because \(\delta_2^-\) is given as a function of the incident neutron energy \(E_{\text{lab}}^{14b}\) and the proton spectrum in the \(^{12}\)C \((d, p)\) \(^{13}\)C reaction is given on the other hand as a function of the proton energy \(E_{p}'^{13n}\). The calculated curves are normalized to the peak at \(E_{p}'^{13n} = 6.45\) MeV. The fit is fairly good so it is indicated that the \((d, p)\) reaction leading to the 7.64 and 8.33 MeV states of \(^{13}\)C can be explained to proceed via a direct process affected from the final state interaction.

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Fig. 4. (a) Phase shift $\delta_{2-}$ of the neutron-$^{12}$C scattering obtained by Wills et al. (b) Approximate phase shift $\delta_{2-}$ obtained from the proton spectrum of the $^{12}$C($d,p)^{13}$ reaction. The phase shift contains uncertainty of $\pm5^\circ$ and is normalized to $90^\circ$ at $E_{p}^{cm}=6.45$ MeV. (c) Phase shift $\delta_{2-}$ of the proton-$^{12}$C scattering obtained by Barnard et al.

However, the experimental result and the calculated one do not coincide in the detailed shape of the spectrum. The experimental result gives about the same peak height at $E_{p}^{cm}=6.95$ MeV as at $E_{p}^{cm}=6.45$ MeV, but the calculated curve indicates that the peak height at $E_{p}^{cm}=6.95$ MeV should be larger than that at $E_{p}^{cm}=6.45$ MeV. If this experimental result is interpreted inversely in terms of the phase shift, the phase shift obtained at $E_{p}^{cm}=6.95$ MeV does not reach $90^\circ$. The procedure is as follows. The Eq. (1) contains the final state interaction between a neutron and a $^{12}$C nucleus on the resonance energy of $E_{p}^{cm}=6.95$ MeV. Because of the experimental fact that a peak is observed at $E_{p}^{cm}=6.95$ MeV, the validity of Eq. (1) is assumed around this peak, and then one can get the phase shifts $\xi_{2-}$ from the Eq. (1) if the experimental cross sections are substituted to the left hand side of Eq. (1).
reduction of the phase shifts, the subtraction of the background is not performed since
the amount of the background cannot be estimated exactly and the effect of the back-
ground seems to have little influence on the approximate reduction of the phase shifts.
In other words, the non-resonant part of the phase shift, $\phi_n$, is assumed to be zero. The
phase shifts thus obtained from the proton spectra at $12^\circ$ in the center of mass system is
shown in Fig. 4-b. As seen in the figure, the phase shift at $E_{p^m} = 6.95$ MeV is nearly
60° and does not reach 90°. One might question that this result is caused by the nor-
malization procedure of the phase shifts at $E_{p^m} = 6.45$ MeV, but in fact, if the nor-
malization is done at $E_{p^m} = 6.95$ MeV, the value of $\sin^2\xi_2$ at $E_{p^m} = 6.45$ MeV be-
comes larger than unity and unphysical. Thus the normalization should be done at
$E_{p^m} = 6.45$ MeV and the result in Fig. 4-b is a natural consequence of the experiment.

This result is not in accord with that of Wills et al.\cite{Footnote1} since the energy $E_{p^m} = 6.45$
MeV corresponds to the 7.64 MeV excited state of C$^{13}$ and then to the $E_{p^m\text{lab}} = 2.95$
MeV in the neutron-C$^{12}$ scattering. The reasons of this discrepancy are thought to be
twofold. First, the Eq. (1) is correct but the phase shifts in the right hand side of Eq.
(1) should be modified. Second, the Eq. (1) is inadequate to apply to the $(d, p)$
reaction.

In the first place, we assume that the Eq. (1) is correct. As is mentioned before,
the peak positions and the gross structures of these peaks are reproduced fairly well
by Eq. (1) using the phase shifts of n-C$^{13}$ scattering. Then it is concluded that the
final state interaction plays an essential role in this $(d, p)$ reaction. However, the Eq.
(1) does not mention the effect of the core excitation and assumes the residual state
consists of a neutron and a ground state C$^{13}$. In the case of n-He$^4$ scattering, Hoop
and Barschall\cite{Footnote2} observed that the phase shift is nearly 60° at the 16.7 MeV $\langle 3/2^+ \rangle$ state
of He$^4$. Akin to the present experiment, the case of p-C$^{13}$ scattering at $E_{p^m} \approx 6.7$
MeV should be inferred. The experiment was done by Barnard et al.\cite{Footnote3} and the
phase shifts obtained is cited in Fig. 4-c. The phase shift $\delta_2$ is nearly 60° at the 6.83
MeV resonance state of N$^{13}$, which is the mirror state of the 7.64 MeV state of C$^{13}$. The
phase shifts derived from the experiment on p-C$^{13}$ scattering were analyzed by Barnard\cite{Footnote4}
and he showed that the coupling between the p + C$^{13}$ ($g'$nd state) and p + C$^{13}$ ($2^+$ state)
channels is important to interpret the experimental results. Therefore, if the exci-
tation of the core nucleus C$^{12}$ is involved, the fact that the phase shift does not reach
90° is not strange and the Eq. (1) should be modified to include the case of the core
excitation. In the experiment on n-C$^{12}$ elastic scattering\cite{Footnote5}, the core excitation is
improbable energetically, but in the present experiment of the $(d, p)$ reaction, the excitation of the C$^{12}$ core is probable.

In the second place, the validity of the Eq. (1) should be examined. Eq. (1)
indicates that the final state interaction is on the energy shell. But in the $(d, p)$ re-
action, the n-C$^{12}$ scattering could occur off the energy shell, because of the fact that the
third particle, proton, could be in a common interaction volume of the n-C$^{12}$ scattering.
Since the width of the 7.64 MeV state is much smaller than that of the 8.33 MeV state,
the lifetime of the 7.64 MeV state is much longer than that of the 8.33 MeV state.
Therefore, the effect of the off shell scattering between a neutron and a C$^{12}$ nucleus
could be larger in the case of 7.64 MeV state than in the case of 8.33 MeV state. If
so, the Eq. (1) is inadequate to reduce the phase shifts from the experimental results

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around 7.64 MeV excitation. It is well known that the \((d, p)\) reaction is a very useful tool to determine the neutron orbit in the residual nucleus, but this usefulness is valid under the condition that the residual nucleus is stable or at least particle stable. No theory of the \((d, p)\) reaction has been proposed which treats both cases leading to the bound and scattering states and the anomalous behavior of the phase shifts in this experiment may be due to the misuse of the Eq. (1).

One of the authors [K. H.]° has discussed the mechanism of the \(\text{C}^{12} (d, p) \text{C}^{13}\) (7.64 MeV) reaction on the basis of the angular distribution of protons and suggested that this reaction proceeds via the core excitation process. More experimental and theoretical works should be done to interprete the structure of the 7.64 MeV state of \(\text{C}^{13}\), to treat the two step stripping reaction and to analyze the off-shell scattering.

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REFERENCES