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A Method for Estimating Residual Inductance in High Frequency A. C. Measurements

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A new method was proposed to estimate residual inductance inherent in a specimen and its terminal leads in dielectric measurements. For a specimen specified by a capacitance and a resistance which are independent of measuring frequency, the product of capacitance and resistance as the readings of an a.c. bridge is to show a linear relation to the squared frequency. The residual inductance can be calculated from the slope and the intercept of the linear relation. By the application of the method to parallel networks of a capacitor and a resistor with varied length of the terminal leads, reasonable results are obtained concerning the impedance per unit length of the leads. The method is also applied to a parallel plate cell system filled with salt solutions. The inductance evaluated for the cell system is seen to be varied with the conductance of the salt solution used. Comparison of the method is made with the Schwan method which was used to estimate the residual inductance.

I. INTRODUCTION

In dielectric measurements by means of an a.c. bridge over a several-hundred megahertz range, the bridge readings of the capacitance and the conductance are seriously affected by residual inductance arising from the terminal leads and from the measuring cell itself in such a way that negative values of capacitance are obtained even for the specimen with virtually positive capacitance. In order to derive correct values of capacitance and conductance for a specimen from the bridge readings, it is essential to obtain reliable values of the residual inductance inherent in the measured system.

Schwan[1] described a method for the determination of such residual inductances on the assumption that the dielectric constants of aqueous KCl solutions are virtually independent of salt concentration at least up to 0.1 mole/l. This assumption is, however, not fully verified or supported by experimental evidence.[2]

This paper is concerned with a new method to estimate a value of residual inductance for any single dielectric specimen without recourse to the assumption adopted by Schwan. The utility of the method developed here is discussed by demonstrating its application to a dummy system consisting of pure capacitor and resistor as well as to a measuring cell system filled with salt solutions.

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II. THEORETICAL

A. Relations for Equivalent Network

When a residual inductance L is assumed to be in series with an equivalent parallel network composed of capacitance C and conductance G or resistance R specifying a dielectric specimen as shown in Fig. 1, the equivalent parallel capacitance C_x and conductance G_x or resistance R_x as the bridge readings are given by the following relations:

$$C = \frac{C_x(1+\omega^2 LC_x) + LG_x^2}{(1+\omega^2 LC_x)^2 + (\omega LG_x)^2}, \tag{1}$$

$$G = \frac{G_x}{(1+\omega^2 LC_x)^2 + (\omega LG_x)^2}, \tag{2}$$

and

$$R = R_x\left[(1+\omega^2 LC_x)^2 + \left(\omega\frac{L}{R_x}\right)^2\right], \tag{3}$$

or alternatively

$$C_x = \frac{C(1-\omega^2 LC) - LG^2}{(1-\omega^2 LC)^2 + (\omega LG)^2}, \tag{4}$$

$$G_x = \frac{G}{(1-\omega^2 LC)^2 + (\omega LG)^2}, \tag{5}$$

and

$$R_x = R\left[(1-\omega^2 LC)^2 + \left(\omega\frac{L}{R}\right)^2\right], \tag{6}$$

where the angular frequency ω is the measuring frequency f multiplied by 2π.

As an example, Fig. 2 shows the frequency dependence of C_x and R_x calculated from Eqs. 4 and 6 for a specimen specified by $C=10$ pF and $G=10$ mS (or $R=100\ \Omega$) in series with varied inductances. As seen in Fig. 2, the limiting value of C_x at low frequencies decreased with the increase of L, whereas the limiting value of R_x at

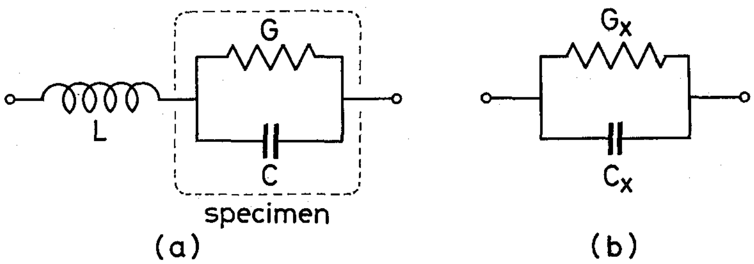
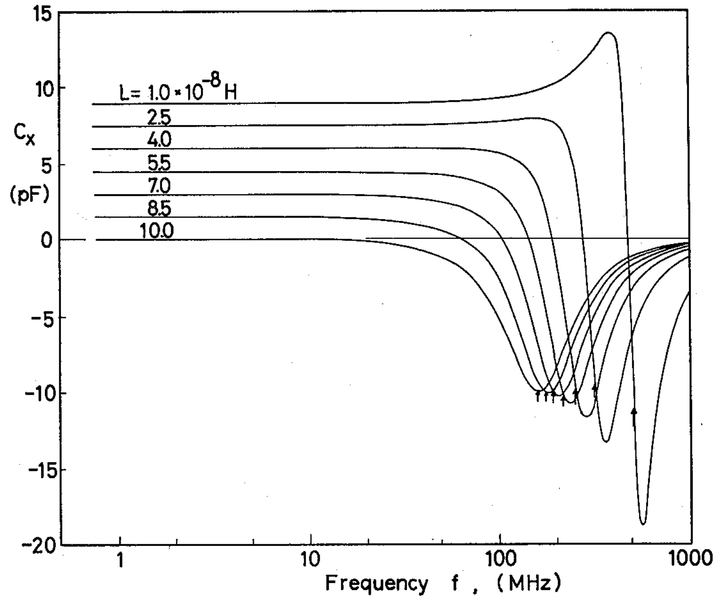
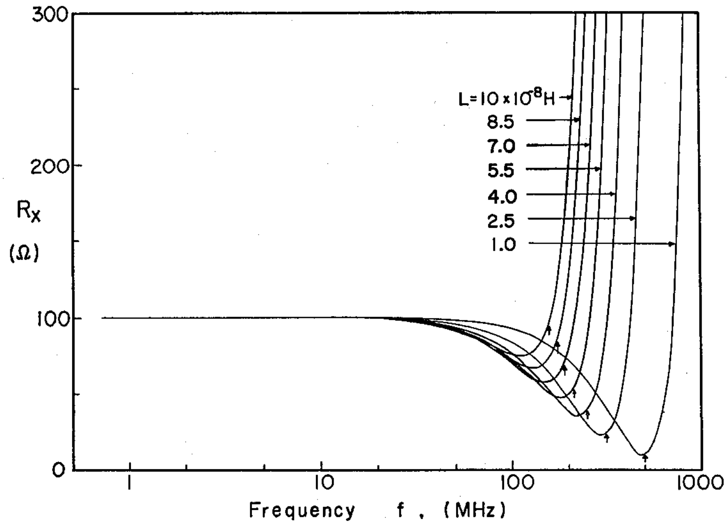


Fig. 1. (a) Equivalent circuit including residual inductance L in series with a parallel combination of conductance G and capacitance C specifying the specimen.
(b) Equivalent parallel conductance G_x and parallel capacitance C_x as the bridge readings.

low frequencies was equal to R irrespective of L values. Near and above the L-C resonance frequency, C_x and R_x showed steep descent and ascent with frequency in remarkable contrast with C and R which are both independent of frequency. As readily seen from these examples, the behavior of C, G and R was very different from



(a)



(b)

Fig. 2. Frequency dependence of (a) C_x and (b) R_x calculated from Eqs. 4 and 6 for a specimen specified by C=10 pF and R=100 Ω (or G=10 mS) with varied inductance L. Short arrows (↑) beside the curves in the Figure denote the L-C resonance frequency given by $2\pi f_r=\omega_r=1/\sqrt{LC}$.

the frequency dependence of C_x, G_x and R_x. It is, therefore, important to make reliable assessment of L for calculating C, G and R from Eqs. 1, 2 and 3.

In the following sections, methods are discussed to evaluate L by the analysis of dielectric behavior of C_x and G_x for specimens possessing fixed values of C and G.

B. Schwan's Method of Estimating Residual Inductance

We henceforth confine ourselves to simple specimens specified by C and G which are independent of frequency. At low frequency limit, Eqs. 4 and 5 are reduced to

$$C_{x0} = C - LG^2, \tag{7}$$

and

$$G_{x0} = G, \tag{8}$$

where C_{x0} and G_{x0} denote the limiting values of C_x and G_x at low frequencies respectively. Insertion of Eq. 8 to Eq. 7 gives

$$C_{x0} = C - LG_{x0}^2. \tag{9}$$

According to Schwan a measuring cell is filled with an electrolyte solution, whose salt concentration is varied so that the capacitance C_{x0} may be plotted against squared conductance G_{x0}^2. If the capacitance C and the inductance L of the cell system containing the electrolyte solution are independent of G_{x0}, then the plots of C_{x0} against G_{x0}^2 are to fit a straight line whose negative slope is equal to L and intercept is given by C.

The values of L may thus be determined by the use of only the values of C_{x0} at lower frequencies, the measurement at varied frequencies being unnecessary. In this instance, a series of specimens have to be prepared so that the values of C and L may be kept unchanged with varied G_{x0}.

C. New Method of Estimating Residual Inductance

i) Conductive capacitor

Division of Eq. 4 by Eq. 5 gives

$$\frac{C_x}{G_x} = -\frac{LC^2}{G}\omega^2 + \frac{C - LG^2}{G}. \tag{10}$$

Equation 10 states that C_x/G_x shows a linear relation to the squared angular frequency ω^2 provided L, C and G are all independent of frequency, and that the negative slope α and the intercept β are given by

$$\alpha = \frac{LC^2}{G}, \tag{11}$$

and

$$\beta = \frac{C - LG^2}{G}, \tag{12}$$

respectively. Substituting Eq. 12 to Eq. 11 to eliminate C, we have

$$G^3 L^3 + 2\beta G^2 L^2 + \beta^2 GL - \alpha = 0. \tag{13}$$

Rearrangement of Eq. 12 gives

$$C = \beta G + LG^2. \tag{14}$$

For numerical calculation by means of Eqs. 13 and 14, G_{x0} may be used in place of G following Eq. 8.

The values of L may thus be calculated from Eq. 13 by use of measured values of α, β and G_{x0}. Substituting the values of β, G_{x0} and L to Eq. 14, one can calculate the values of C.

ii) **Non-conductive capacitor**

When the values of G are infinitely small as in the case of no resistor, Eq. 4 is reduced to

$$\frac{1}{C_x} = - L\omega^2 + \frac{1}{C}. \qquad (15)$$

Following Eq. 15 the reciprocal of C_x shows a linear relation to the squared frequency ω^2 with a negative slope L and an intercept $1/C$.

III. EXPERIMENTAL

The a.c. bridge used was a Boonton RX-Meter Type 250-A, the frequency range being from 0.5 to 250 MHz. The bridge was designed to show the equivalent parallel combination of capacitance C_x in pF and resistance R_x in ohm.

Model systems used for the test of the method to determine the residual inductance L were composed of a parallel network of a resistor and a capacitor with terminal leads of varied length as shown in Fig. 3. The capacitors were tubular polystyrene

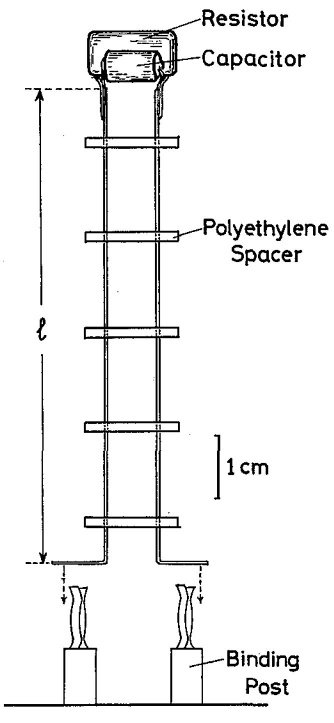


Fig. 3. Illustration of parallel network of a resistor and a capacitor with two terminal leads in varied length l.

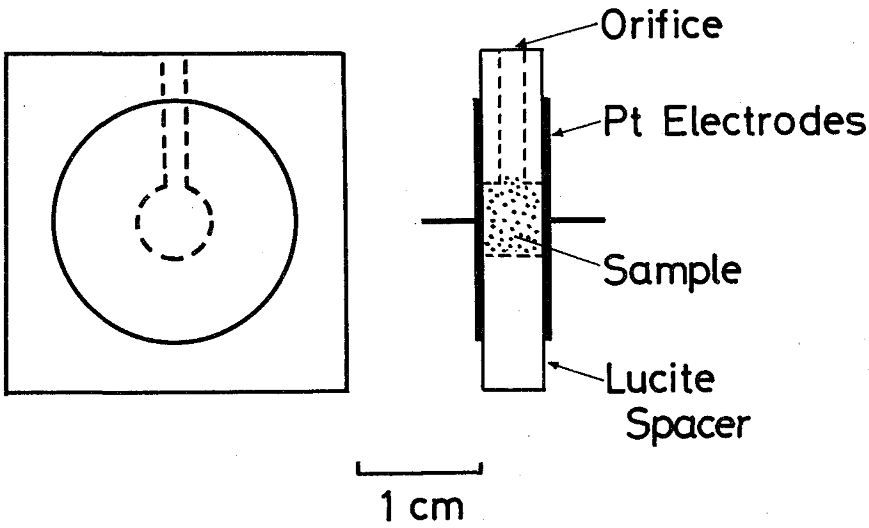


Fig. 4. A parallel plate cell used in the test for the case of salt solutions.

condensers. The resistors were of a metal film type. In order to obtain stable and reproducible data, the two terminal leads were kept parallel with each other by fine polyethylene rod spacers, the use of which was confirmed by experiment to give rise to no serious errors and faults regarding later discussion. Six specimens for the parallel networks, referred to as Specimens A to F, are listed in Table I.

A measuring cell used in the test for the case of salt solutions was a parallel plate condenser of platinum discs which were coated with platinum black and separated by a Lucite spacer as shown in Fig. 4.

IV. RESULTS AND DISCUSSION

A. Estimation of Residual Inductance for a Combined System of a Capacitor and a Resistor

Figure 5 shows the frequency dependence of C_x and R_x for Specimen B (100 Ω and 10 pF in parallel) at varied length l of the parallel leads. General features seen in Fig. 5 are very similar to those shown in Fig. 2, suggesting that an equivalent circuit for Specimen B may be given by a network shown in Fig. 1-(a). Similar results were found for Specimens C, D and E.

i) **Analysis by the new method**

Figure 6 shows the relation of C_xR_x to squared frequency f^2 for Specimens B and C with varied length of the leads. As readily seen in Fig. 6, the plots of C_xR_x against f^2 for l=7.5 cm were given by a straight line, whereas the plots were composed of two successive straight lines for shorter length of the leads. Similar results on the plots of C_xR_x against f^2 were obtained also for Specimen D. The reason for this sudden change of the slopes at higher frequency side is not known and is left out of present consideration.

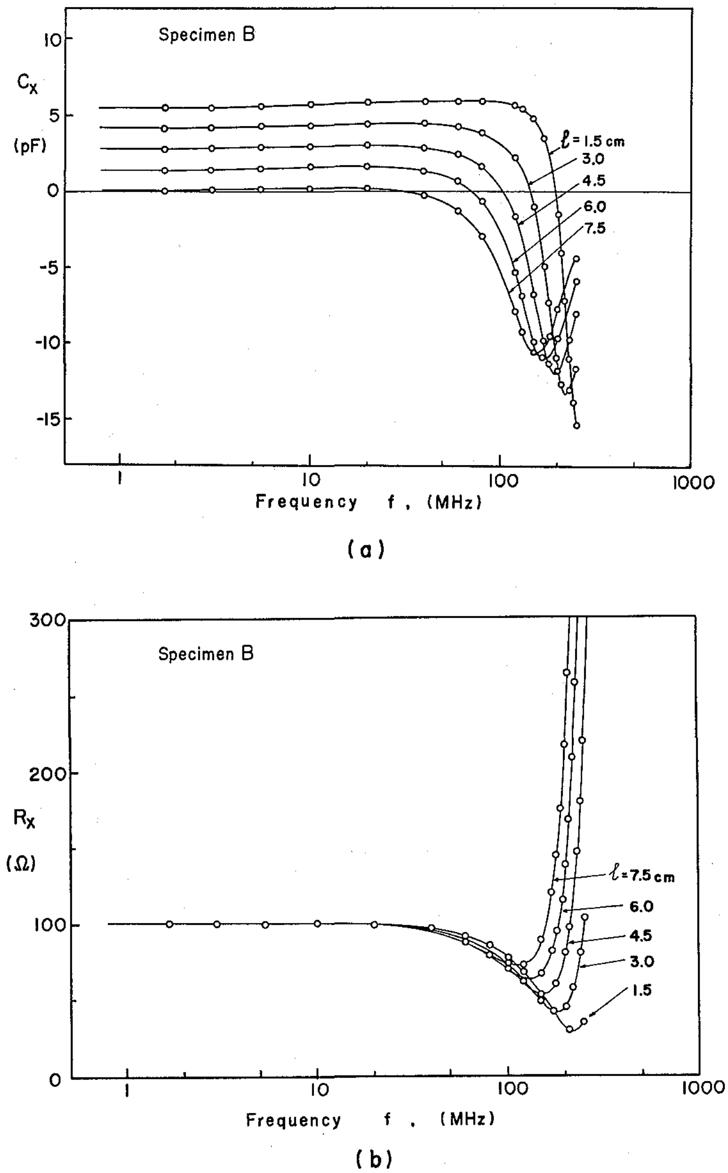


Fig. 5. Frequency dependence of (a) C_x and (b) R_x for Specimen B with varied length l of the two terminal leads. Specimen B is a parallel combination of a 100 Ω resistor and a 10 pF capacitor.

The negative slope α and the intercept β were obtained from the straight line in the lower frequency side in Fig. 6, so that the values of L and C were calculated by means of Eqs. 13 and 14 respectively. The values of L and C thus obtained are plotted against the length of leads in Figs. 7 and 8.

The data of frequency dependence for Specimen E, which is composed of a capacitor alone, and Specimen F, the parallel leads alone, can be treated with Eq. 15 which is applicable to non-conductive capacitors. The plots of $1/C_x$ against f^2 for

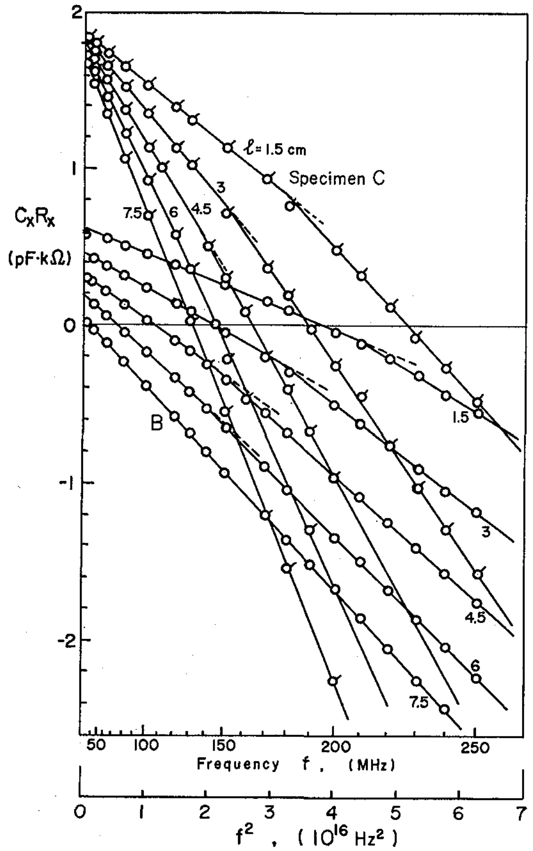


Fig. 6. Plots of C_xR_x against squared frequency f^2 for Specimens B and C with varied length l of the terminal leads.

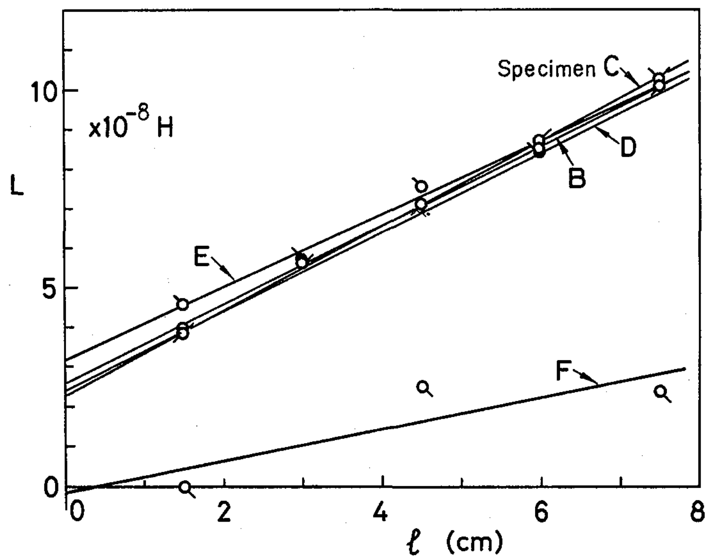


Fig. 7. Dependence of residual inductance L on the length l of the terminal leads for Specimens B, C, D, E and F.

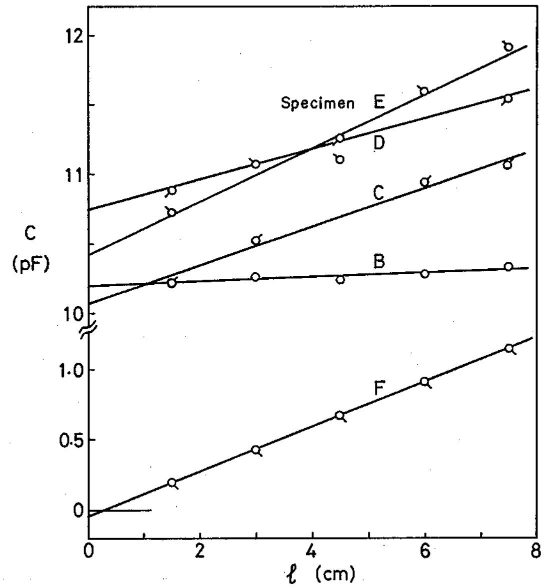


Fig. 8. Dependence of capacitance C on the length l of the terminal leads for Specimens B, C, D, E and F.

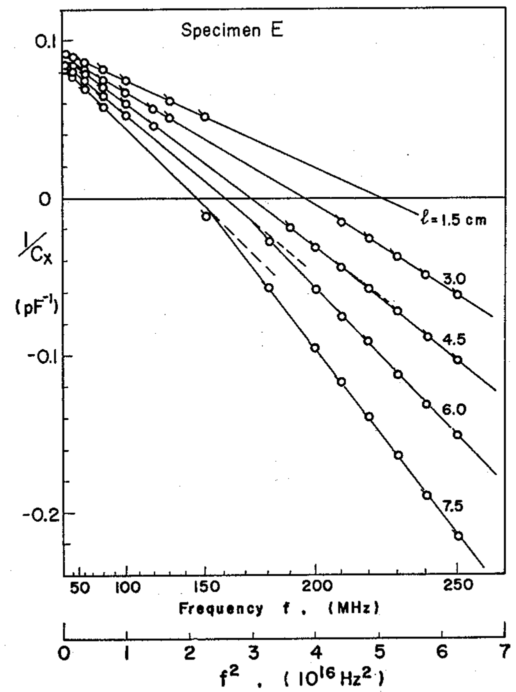


Fig. 9. Plots of the reciprocal of C_x against squared frequency f^2 for Specimen E, composed only of a capacitor, with varied length l of the terminal leads.

Specimen E are shown in Fig. 9. In this case the plots showed two successive straight lines for longer length of the leads in a manner similar to the case of Fig. 6. Values of C and L thus obtained from the straight lines in lower frequency side in Fig. 9 are also shown in Figs. 7 and 8.

The results are expressed by linear functions of the lead length l as

$$L = L_0 + L_1 l. \tag{16}$$

and

$$C = C_0 + C_1 l. \tag{17}$$

Here L_0 and C_0 are the inductance and the capacitance at $l=0$ respectively, and are thought to represent the impedance properties inherent in the lumped elements used such as the resistor and the capacitor. The values of L_1 and C_1 are the contributions from unit length of the parallel leads. The values of L_0, L_1, C_0 and C_1 are summarized in Table I. It is found from Table I that L_1 and C_1 show the values common to Specimens B, C, D and E except for C_1 of Specimen B. This fact suggests that L_1 and C_1 are attributed to the unit length impedance inherent in the parallel leads.

For Specimen F, which has no lumped element, C_0 showed almost naught, and C_1 took a value similar to those for Specimens C to E. The values of L_0 and L_1 for Specimen F showed different values from other Specimens presumably owing to the path of electric current which is not concentrated at the upper end of the parallel

Table I. Characteristic Constants Determined for Specimens Consisting of Parallel Combination of a Resistor and a Capacitor

Specimen	A	B	C	D	E	F
Constitution[a]						
resistor (Ω)	100	100	200	1000	none	none
capacitor (pF)	none	10	10	10	10	none
			New method[b]			
$L_0(10^{-8}\text{H})$	−0.554	2.56	2.28	2.38	3.13	−0.153
L_1 (10^{-8} H/cm)	−0.081	1.01	1.07	1.00	0.930	0.394
C_0 (pF)	−4.47	10.2	10.1	10.1	10.4	−0.044
C_1 (pF/cm)	−0.818	0.016	0.139	0.109	0.192	0.160
			New method[c]			
$C_0-L_0G_{x0}^2$ (pF)	−3.93	7.65	9.50	10.7	10.4	−0.044
$C_1-L_1G_{x0}^2$ (pF/cm)	−0.737	−0.989	−0.129	0.0989	0.192	0.160
			Schwan's method[d]			
$C_0-L_0G_{x0}^2$ (pF)	−3.45	6.79	9.37	10.6	10.6	−0.044
$C_1-L_1G_{x0}^2$ (pF/cm)	−0.947	−0.889	−0.115	0.150	0.169	0.160

a Constituent resistor and capacitor of the Specimen.

b Estimated from Figs. 7 and 8.

c Calculated by use of the numerical values of L_0, L_1, C_0 and C_1 estimated with the new method.

d Obtained as the intercept and the slope of straight lines in Fig. 11 following Schwan's method.

leads. For Specimen A, which is composed of the resistor alone, L_0, L_1, C_0 and C_1 showed very unreasonable values with negative sign.

By the use of respective values of L thus obtained for Specimen B, the capacitance

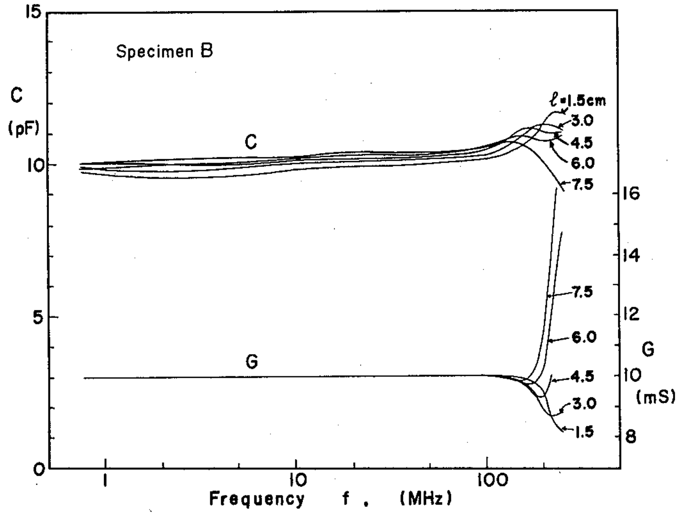


Fig. 10. Capacitance C and conductance G subjected to the correction of residual inductance by means of Eqs. 1 and 2 for the data of Specimen B shown in Figs. 5-(a) and 5-(b).

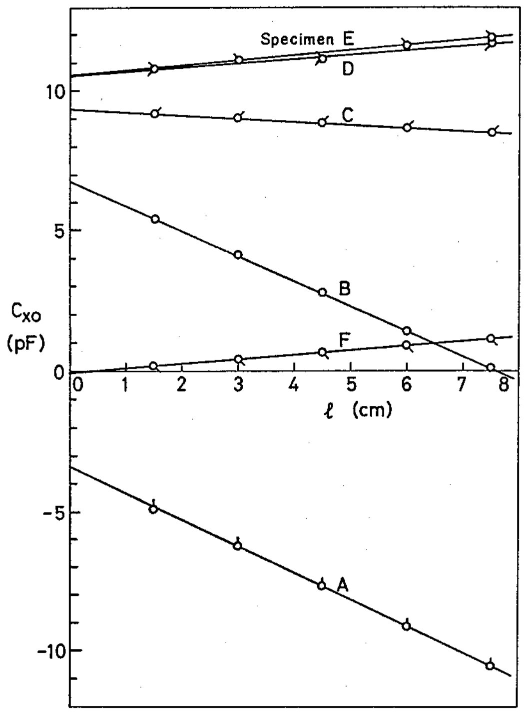


Fig. 11. Dependence of limiting capacitance C_{x0} at low frequencies on the length l of the terminal leads for Specimens A, B, C, D, E and F.

C and the conductance G associated with the measured C_x and R_x shown in Fig. 5 were calculated by means of Eqs. 1 and 2, the results being shown in Fig. 10. As seen in Fig. 10, the capacitance C is kept constant up to about 100 MHz within the accuracy of about 4%, whereas the conductance G is remained constant up to about 150 MHz within the accuracy of 1%.

ii) **Analysis by Schwan's method**

The original method proposed by Schwan to determine the value of L is to measure C_{x0} for a set of specimens with the same capacitance and inductance and with varied conductance. In the present discussions, however, his method is applied to a set of specimens with varied inductance by changing the length of parallel leads. Figure 11 shows the dependence of C_{x0} on the length l of parallel leads for Specimens A, B, C, D, E and F. As readily seen in the Figure, the plots of C_{x0} against l showed straight lines with high accuracy.

On the other hand, substitution of Eqs. 16 and 17 to Eq. 9 gives

$$C_{x0} = (C_0 - L_0 G_{x0}^2) + (C_1 - L_1 G_{x0}^2) l. \tag{18}$$

The linear relation of C_{x0} to the length l found in Fig. 11 can thus be interpreted in terms of Eq. 18, the intercept and the slope being expressed by $(C_0 - L_0 G_{x0}^2)$ and $(C_1 - L_1 G_{x0}^2)$ respectively.

In Table I the intercept $(C_0 - L_0 G_{x0}^2)$ and the slope $(C_1 - L_1 G_{x0}^2)$ obtained from Fig. 11 following Schwan's method are compared with those calculated from L_0, L_1, C_0 and C_1 which were evaluated by means of the new method. It is seen that the agreement is qualitatively satisfactory between the values by Schwan's and those by the new method.

B. Estimation of Residual Inductance for a Parallel Plate Cell

An example of practical importance is the determination of residual inductance for a cell system which is filled with liquid specimens. The dielectric constant and the conductivity of the aqueous KCl solution used are assumed to be specified by the concentration of the solute and to be independent of the frequency within the frequency range used in the present experiment.

The frequency dependence of the capacitance and conductance was observed for a parallel plate cell shown in Fig. 4 filled with aqueous KCl solutions in varied concentrations.

i) ***Analysis by the new method***

Figure 12 shows the plots of $C_x R_x$ against squared frequency f^2 obtained from the data in varied concentrations of KCl. In contrast to the case of the parallel combination of a capacitor and a resistor shown in Fig. 6, the plots of $C_x R_x$ against f^2 for the cell system showed a rapid increase of $C_x R_x$ below 10 MHz owing to the increase of capacitance due to electrode polarization. For the cases of dilute solutions of KCl, the plots showed deviation from the straight lines at higher frequencies. The linear parts of the plots at medium frequencies, approximately from 20 to 180 MHz, may be used to evaluate L and C by means of Eqs. 13 and 14. The results obtained are summarized in Table II. The value of L is seen to decrease with the increase in the conductance of the cell system. The values of C or dielectric constant ϵ increased with increasing salt concentration. Further discussions, however, appear

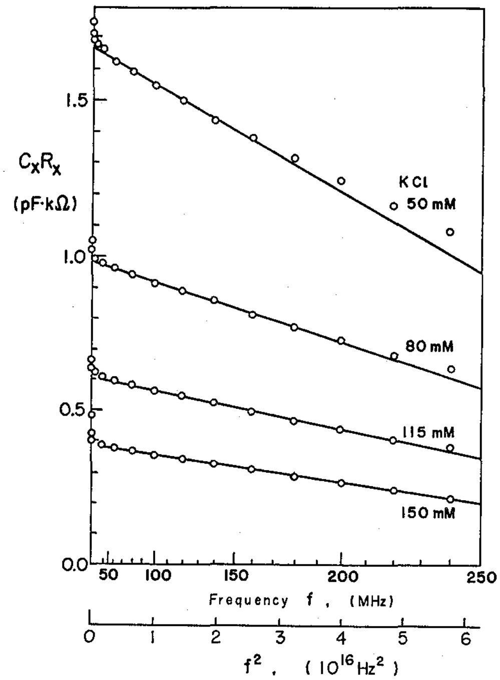


Fig. 12. Plots of C_xR_x against squared frequency f^2 for a cell system containing aqueous KCl solutions in varied concentrations.

Table II. Characteristic Constants Determined for a Parallel Plate Cell Filled with Potassium Chloride Solutions in Varied Concentrations

	G_{x0} (mS)	L (10^{-8} H)	C (pF)	Dielectric constant ϵ
		New method		
KCl concentration (mM)				
20	1.46	2.96	5.51	68.0
50	3.28	2.90	5.62	70.3
80	5.21	2.54	5.65	70.9
115	7.32	2.39	5.72	72.3
150	9.48	2.37	5.79	73.8
distilled water[a]	0.007	1.52	5.78	73.6
		Schwan's method		
		2.27	5.82	74.5

a The values were determined by means of Eq. 15 because of the very low conductance.

to be difficult on the dependence of C or ϵ upon the salt concentration, because the values of C thus estimated are somewhat less reproducible in contrast with the high accuracy on the estimation of L.

ii) **Analysis by Schwan's method**

In order to confirm the linear relation of C_{x0} to G_{x0}^2, the plots of C_{x0} against

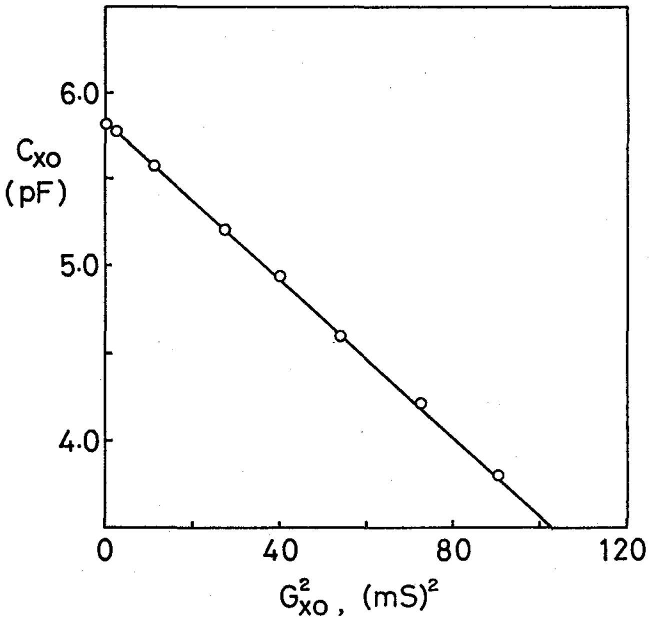


Fig. 13. Plots of limiting capacitance C_{x0} at low frequencies against squared limiting conductance G_{x0}^2 at low frequencies for a cell system containing aqueous KCl solutions in varied concentrations.

G_{x0}^2 are shown in Fig. 13, thus the results enabling us to evaluate L by means of Eq. 9. The value of L calculated from the slope of the straight line in Fig. 13 is listed in Table II. The values of L estimated by Schwan's method seem to be smaller than those by the new method.

V. SUMMARY

1. According to the results shown in Table I as applied to a parallel network of a capacitor and a resistor, satisfactory agreements were obtained between the new method and Schwan's method. In particular, the analysis by means of the new method showed systematic results on the dependence of L and C upon the length l of the parallel leads.
2. From Table II, which summarizes the results for a parallel plate cell, the value of L obtained by means of the new method is seen to be varied slightly with conductance of the specimen. Since the values of C estimated by the new method were less reproducible in respect of the change in the salt concentration, it was difficult to discuss the dependence of dielectric constant ϵ for the aqueous solution upon its salt concentration. Schwan's method gave a set of values of L and C which were slightly different from those by the new method.
3. It is unnecessary for Schwan's method to obtain the data at varied frequencies, as the method is applied only to limiting values of capacitance and conductance at low frequencies. The method is not applicable to calculate the value of L from the data of only one specimen, and requires the data on a series of specimens with varied

conductance or varied length of the leads and with unvaried capacitance.

4. The new method requires the data of capacitance and conductance at varied frequencies, being usable to determine the value of L and C from the data for only one specimen.

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