The Isomeric Cross Sections of the (d, p) Reaction
on $^{130}\text{Te}$

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Both isomeric cross sections of $^{130}\text{Te}(d, p)^{131m}\text{Te}$ and $^{131g}\text{Te}$ were measured with the activation method, up to 14.1 MeV deuteron energy. The curves were explained by a new calculation method.

INTRODUCTION

Recently Otozai et al. obtained experimentally some excitation curves of the deuteron reactions such as $(d, p)$, $(d, n)$, and $(d, 2n)$ and interpreted by a simple theoretical calculation.\textsuperscript{1-4}

Many excitation curves for the production of isomers by $(d, p)$ reactions have been measured,\textsuperscript{5,6} but there is no theoretical explanation of these curves. Only the isomeric cross section ratios of $(d, p)$ reactions at 15 MeV for some doubly even target nuclei have been successfully explained by Natowitz and Wolke,\textsuperscript{5} the theoretical values were based on the distorted wave Born approximation and the shell model. But no quantitative interpretation was achieved on the change of the ratio with different deuteron energies.

Here the isomeric excitation curves of the $^{130}\text{Te}(d, p)^{131}\text{Te}$ reaction are measured. The excitation curves of $(d, p)$ and $(d, 2n)$ reaction on $^{130}\text{Te}$ have been obtained and interpreted.\textsuperscript{2} The absolute values of the isomeric cross sections can be determined accurately, because both isomers $^{131m}\text{Te}$ and $^{131g}\text{Te}$ have a common daughter $^{131}\text{I}$ of which the decay scheme is well defined and brings little ambiguity in the measurement of its radioactivities.

The interpretation of the isomeric excitation curves obtained in this work uses a new semiclassical treatment, which is consistent with the calculation of Otozai et al.\textsuperscript{1,2} In the first step of this calculation, the relative formation probabilities of excited states with different spins and energies of a residual nucleus in the stripping process are calculated by the method based on the plane wave Born approximation,\textsuperscript{7} and the corresponding absolute cross sections are determined by using parameters adopted by Otozai et al.\textsuperscript{1,2} Then these neutron capture states decay with γ-ray cascades accompanied by spin changes and finally become either the metastable state (m-state) or the ground state (g-state). In this step, the statistical method\textsuperscript{8} is employed. The consideration mentioned above corresponds to the “mixed” reaction mechanism advocated by Strutinskii.\textsuperscript{9}
Experimental

The targets and conditions of bombardments have been described elsewhere. The characteristics of the nuclei in this experiment are shown in Table I. There are three reactions by which $^{131}\text{I}$ is produced in the target, namely $^{130}\text{Te}(d, n)^{131}\text{I}$, $^{130}\text{Te}(d, p)^{131}\text{mTe} ightarrow ^{131}\text{I}$, and $^{130}\text{Te}(d, p)^{131}\text{gTe} ightarrow ^{131}\text{I}$. Each cross section of the latter two reactions can be obtained by counting $^{131}\text{I}$ activities grown in two samples of tellurium isolated from the target at different times after bombardment. The number of $^{131}\text{I}$ nuclei in the tellurium sample which was isolated at $t_1$ soon after bombardment has grown to $N_1(t)$ at time $t$.

Table I. Characteristics of the Nuclei in the Present Experiment.

<table>
<thead>
<tr>
<th>Nuclide</th>
<th>Spin</th>
<th>Half-life</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target</td>
<td>$^{130}\text{Te}$</td>
<td>0</td>
</tr>
<tr>
<td>Product</td>
<td>$^{131}\text{mTe}$</td>
<td>11/2</td>
</tr>
<tr>
<td>Daughter</td>
<td>$^{131}\text{I}$</td>
<td>3/2</td>
</tr>
</tbody>
</table>

$N_1(t) \approx \sigma_n n Q_g \exp (-\lambda_g t_1) \exp [-\lambda_d (t-t_1)]$

$+ \sigma_m n Q_m [\lambda_m/ (\lambda_m - \lambda_d)] \exp (-\lambda_m t_1) \exp [-\lambda_d (t-t_1)]$. (1)

In the sample isolated at $t_2$, after $^{131}\text{gTe}$ has decayed,

$N_2(t) \approx \sigma_n n Q_m [\lambda_m/ (\lambda_m - \lambda_d)] \exp (-\lambda_m t_2) \exp [-\lambda_d (t-t_2)]$. (2)

Here, $\sigma$ is the cross section, $n$ the number of target nuclei in a unit area of a target, $Q$ the equivalent number of incident deuterons at the end of bombardment ($t=0$), $\lambda$ the decay constant and suffixes $m$, $g$, and $d$ the m-state, g-state, and daughter, respectively. Thus the isomeric cross sections can easily be obtained by using Eqs. (1) and (2).

After bombardment, the target was dissolved with aqua regia, the solution was dried, and the residue was redissolved into hydrochloric acid solution. Then the solution was divided into two parts, and the respective parts were treated at different times with the following procedures. Sulphur dioxide saturated water and hydrazine were added into the solution. Black precipitate of tellurium was filtered onto the filter glass, which had been weighed previously. Then the chemical yield was determined, and the activities of $^{131}\text{I}$ were counted ten days later. The isolation time was determined by the instant of the precipitation of black tellurium. The photopeak counts of the 364 KeV gamma ray of $^{131}\text{I}$ were counted with a 4.4 cm $\times$ 5.1 cm NaI(Tl) scintillator. The counting efficiency was determined by means of the $\beta-\gamma$ coincidence method.

Results

Experimental results are shown in Table II where the errors indicated mean the 90% confidence estimated from three runs.

(171)
Table II. The Experimental Results.

<table>
<thead>
<tr>
<th>$E_d$ (MeV)</th>
<th>Cross section (mb)</th>
<th>m-state</th>
<th>g-state</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.3</td>
<td>1.0</td>
<td>4.3</td>
<td></td>
</tr>
<tr>
<td>6.5</td>
<td>16.3</td>
<td>58</td>
<td></td>
</tr>
<tr>
<td>8.2</td>
<td>$56\pm11$</td>
<td>$134\pm26$</td>
<td></td>
</tr>
<tr>
<td>10.2</td>
<td>$71\pm11$</td>
<td>$130\pm17$</td>
<td></td>
</tr>
<tr>
<td>12.0</td>
<td>59</td>
<td>111</td>
<td></td>
</tr>
<tr>
<td>14.1</td>
<td>$60\pm24$</td>
<td>$88\pm30$</td>
<td></td>
</tr>
</tbody>
</table>

The cross sections for the high spin isomer take a constant value in the high energy region of incident deuteron, while the cross sections for the low spin isomer make a broad peak in the vicinity of 9 MeV of deuteron energy.

THEORETICAL CALCULATION

The average density of neutrons at the surface of the nucleus supplied by deuterons whose protons are at $r_p$ from the center of the nucleus is approximated by the following relation:

$$D(E_d, r_p) dr_p \approx r_p^2 dr_p |C(k_d, r_p)|^2 \int |\chi(R-r_p)|^2 d\Omega_n,$$

(3)

where $E_d$ and $k_d$ are the kinetic energy and momentum of the incident deuterons, respectively. The wave function $C(k_d, r_p)$ represents the deuteron plane wave in the Coulomb field and was evaluated with the WKB approximation. The wave function $\chi$ is of the incident deuteron. The neutron stripping cross section when the proton is at $r_p$ is

$$\sigma_{n\text{-}strip}(E_d, r_p) dr_p = \xi_n R^2 D(E_d, r_p) dr_p.$$

(4)

In this case, the neutrons are captured by the target nucleus with variable angular momenta. The parameter $\xi_n$ represents the average sticking probability of these neutrons against the nucleus, and the value is taken as unity in the calculation of some excitation curves of the deuteron reactions including the $^{130}\text{Te}$ reaction.

In order to calculate the isomeric cross sections, it is necessary to estimate formation cross sections for the individual neutron capture state of the residual nucleus with different energies and spins. The probability that the residual nucleus is excited to $E_f$ by capture of a neutron with an angular momentum $l_n$ is given by the plane wave Born approximation for (d, p) scattering

$$P(E_d, E_f, l_n) dE_p \propto \chi^2 |J_{l_n}(k_n R)|^2 W_{j_f}(E_f) dE_p d\Omega,$$

(5)

where

$$\begin{align*}
\alpha^2 &= M_n B_d h^2, \\
E_f &= E_d - B_d - E_p + B_n.
\end{align*}$$

(6) (7)
Here \( j_m \) is the spherical Bessel function, \( M_n \) the nucleon mass and \( E_p, B_d, \) and \( B_n \) the kinetic energy of the scattered proton, the deuteron binding energy and the additional neutron binding energy of the target nucleus, respectively. The angular momentum of the capture state is expressed by \( J_f \), and \( W_{J_f}(E_f) \) is the level density of the state at \( E_f \). The absolute values of the momentum \( k_p \) and \( k_d \) are

\[
k_d^2 = k_p^2 + k_n^2 - 2k_pk_n\cos\theta
\]  

(8)

where \( \theta \) denotes the angle between the deuteron and the proton. The matrix element \( A_{1n} \) can be estimated from following assumption. The partial width for the neutron emission of the residual nucleus which leaves the target nucleus in its initial state can be expressed by perturbation theory. If the width is equal to the neutron capture probability, \( A_{1n} \) is expressed as follows according to the principle of detailed balance:

\[
A_{1n} \propto R(2J_f+1)/W_{J_f}(E_f).
\]

(9)

Substituting expression (9) in Eq. (5), the probability distribution for an \((E_f, J_f)\) state can be obtained by using the vector addition of angular momenta

\[
P(E_d, E_f, J_f) dE_p \propto \frac{(2J_f+1)}{[(2J_f+1)(2s+1)]} \sum_{S=|J_f-s|}^{J_f+s} \sum_{I=|I-J_f|}^{I+s} \int \left(\frac{k_p}{k_d}\right)^\frac{1}{2} \frac{\alpha^2}{2} + \frac{1}{2k_p^2} \left[ j_{I-n}(R) \right]^2 d\Omega dE_p.
\]

(10)

where \( I, s, \) and \( S \) are the spins of the target nucleus, the neutron and the channel, respectively. It is necessary to take into consideration the correction for the kinetic energies of the deuteron and of the proton in the Coulomb field. For simplicity, \( E_d \) and \( E_p \) in (10) were replaced by \( E_d' \) and \( E_p' \), which are defined by the following relations:

\[
E_d' = E_d - B(r_p), \quad E_p' = E_p - B(r_p), \quad (11)
\]

where \( B(r_p) \) is the Coulomb barrier energy retained by the proton at \( r_p \). We obtain the formation cross section of any \((E_f, J_f)\) state at a given \( r_p \) by using Eqs. (4) and (10)

\[
\sigma_{n\text{-strip}}(E_d, E_f, r_p, J_f)dr_p dE_f = \sigma_{n\text{-strip}}(E_d, r_p) \times \left[ P(E_d', E_f, J_f) dE_p' \right] \sum_{J_f} \sum_{E_d'} P(E_d', E_f, J_f) dE_p' dr_p.
\]

(12)

The absolute formation cross section of any \((E_f, J_f)\) state can be given by integrating Eq. (12) with \( r_p \). Otozai et al. gave the range of \( r_p \) for the integration to be \( R + \rho \) to infinitive, where \( \rho \) was the parameter managing the contribution of entire absorption of deuterons. Thus we obtain

\[
\sigma(E_d, E_f, J_f) dE_f = \int_{R+\rho}^{\infty} \sigma_{n\text{-strip}}(E_d, E_f, r_p, J_f) dr_p dE_f.
\]

(13)

All of these excited \((E_f, J_f)\) states decay to either the m-state or the g-state with \( \gamma \)-ray cascades. Here it is assumed that the single neutron excited state with \((E_f, J_f)\) decays through compound states. Then the spin transitions in \( \gamma \)-cascade can be calculated by the statistical method. The spin-dependent level density formula is given by

\[
(173)
\]
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\[ W(E, J) = W(E, 0)(2J+1)\exp\left[-\frac{(J+1/2)^2}{2\sigma^2}\right], \]

(14)

where the spin cut-off factor \( \sigma \) is expressed by the nuclear moment of inertia \( I \) and the thermodynamic temperature \( T \)

\[ \sigma^2 = \frac{IT}{\hbar^2}. \]

(15)

And if one takes \( I_{\text{rigid}} \) as that of the rigid rotor,

\[ I_{\text{rigid}} = \frac{2}{5}mAR^2. \]

(16)

The practical calculation method in this step followed that of Bishop et al.\textsuperscript{1, 2) The spin shedding ridge was given by the average spin of those of the m- and g-states.

The HITAC-5020 computer at University of Tokyo was used for the present calculation. An effective upper limit of \( r_p \) were divided into 20 divisions, \( E_f \) into 10 divisions from 0 to \( B_n \), and the adopted values for \( l_n \) were 0 to 19. The parameters used in this calculation are shown in Table III. The nuclear radius \( R \) was given by using the parameter \( r_0 \) in the following relation:

<table>
<thead>
<tr>
<th>Process</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( r_0 )</td>
<td>1.6 fm</td>
</tr>
<tr>
<td>Stripping</td>
<td>( \rho )</td>
<td>2.2 fm</td>
</tr>
<tr>
<td></td>
<td>( \xi_n )</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>( a )</td>
<td>( \frac{1}{10} ) A MeV\textsuperscript{-1}</td>
</tr>
<tr>
<td>( \gamma )-disintegration</td>
<td>( r_6 )</td>
<td>1.22 fm</td>
</tr>
</tbody>
</table>

(17)

\[ R = r_0A^{1/3}. \]

The nuclear-spin cut-off factor was evaluated as that of the rigid rotor. The parameters in the stripping process were the same as those which had been used to interpret\textsuperscript{2) the excitation curves of (d, p), (d, n), and (d, 2n) reactions on \( {}^{130}\text{Te} \), and the parameters in the \( \gamma \)-disintegration process were the usual ones in statistical calculations. Figure 1 shows the absolute cross section producing every \( (E_f, J_f) \) state for the \( {}^{130}\text{Te}(d, p){}^{131}\text{Te} \) reaction at 14 MeV deuteron energy. The spin distribution before the last \( \gamma \)-ray emission is shown in Fig. 2, where a dash-and-dot line represents the position of the spin

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Fig. 1. The cross section producing every \( (E_f, J_f) \) state for the \( {}^{130}\text{Te}(d, p){}^{131}\text{Te} \) reaction at 14 MeV deuteron energy.

Fig. 2. The spin distribution before the last \( \gamma \)-ray emission.
(d, p) Isomer Production Reaction on $^{130}$Te

shedding ridge. The sum of the cross sections for the lower spin states than the ridge is assumed\cite{12} to be of the low spin isomer and that for the higher of the high spin isomer, respectively.

DISCUSSION

The experimental isomeric cross sections of the $^{130}$Te(d, p)$^{131}$Te reaction obtained in this work are shown in Fig. 3, where solid circles are for the g-state and open circles for the m-state. The solid line and the dotted line are the results from the present calculation for the g-state and the m-state, respectively. It should be noted that this simple calculation should interpret very well the experimental isomeric excitation curves of the $^{130}$Te(d, p)$^{131}$Te reaction.

![Fig. 3. Experimental and calculated isomeric excitation curves of the $^{130}$Te(d, p)$^{131}$Te reaction. The experimental values are the present data.](image)

In this connection, the same calculation was applied to the $^{89}$Y(d, p)$^{90}$Y reaction and is compared in Fig. 4 with the experimental isomeric cross sections which Riley and Linder\cite{6} and Natowitz and Wolke\cite{5} obtained consistently.

The shape and quantity of the excitation cross section curves of the (d, p) isomer production as well as those of the (d, p), (d, n), and (d, 2n) reactions\cite{1-4} can be reproduced by the simple and semiclassical treatment within the experimental error. The good reproducibility written above is matchless in the experiences of the ones who have worked in the research for the excitation cross section curves, though there may be the criticism that the interpretation should adopt the modern treatment of the DWBA and the shell model.

(175)
Fig. 4. Experimental and calculated isomeric excitation curves of the $^{89}$Y ($d$, $p$) $^{90}$Y reaction. The experimental values are the data of Natowitz and Wolke\textsuperscript{5} and of Riley and Linder.\textsuperscript{6}

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REFERENCES