A Doppler-Shift Attenuation with a Single Crystal Target

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Received December 19, 1973

It is proposed that short lifetimes around $10^{-16}$ second in $\gamma$-ray emitting nuclear reaction can be measured by Doppler-shift attenuation method using a single crystal target.

Recently it was shown that a blocking effect of fast charged particles in single crystal can be applied to the determination of short lifetimes between $10^{-15}$ and $10^{-18}$ second. This method is based on the fact that the flight distance of the nucleus-plus-incident-projectile system is measured by the regular interatomic separation in the crystal lattice as a standard. In the present paper it is proposed that the sensitivity of the Doppler-shift attenuation method can also be extended to less than $10^{-16}$ second by the use of a single crystal target.

Now it is assumed that the nucleus-plus-incident-projectile system starts from a lattice site and travels with a velocity $v$ in the direction parallel to a low index direction of a single crystal. As the interatomic separation in a single crystal is defined by the orientation of the crystal, the system collides with the nearest atom along the atomic row, which is separated at a distance $d$. If the thermal vibration of atoms in the crystal is neglected, the impact parameter of this collision is zero and the system moves with the velocity $\frac{mv}{m+2M} = \frac{\sqrt{2mE}}{(m+M)(m+2M)}$ after the collision, where $m$ and $E$ is the mass and energy of the incident projectile respectively and $M$ is the mass of the target nucleus. If the decay of the nucleus-plus-incident-projectile system by the emission of $\gamma$-ray photon is expressed by a mean lifetime $\tau$, then the probability that $\gamma$-ray photon is emitted before the system collides with the nearest atom is given by

$$1 - \exp(-d/\nu \tau).$$

Thus this is the fraction of $\gamma$-ray which has energy $h\nu(1+\nu/c)$, where $h\nu$ is the energy of $\gamma$-ray emitted from a stationary system and $c$ is the velocity of light. As the velocity $\frac{mv}{m+2M}$ of the nucleus-plus-incident-projectile system after the collision is small compared with $v$, the $\gamma$-ray energy spectrum has two peaks, i.e. one at $h\nu(1+\nu/c)$ and the other at $h\nu[1+vm/c(m+2M)]$, when observed in the direction facing to the incident projectile.

The exponential term in Eq. (1) becomes larger when $\tau$ is same in order of or less than $d/v$. In the usual experimental conditions, the velocity of the nucleus-plus-incident-projectile system is less than $10^6$ cm/s and the interatomic separation $d$ is of the order of $10^{-8}$ cm. Therefore, the lifetimes around $10^{-16}$ second can be estimated from the

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\(\gamma\)-ray energy spectrum.

However, the thermal vibration of atoms cannot be neglected in real crystals. The impact parameter of the collision between the nucleus-plus-incident-projectile system and the nearest atom depends on the amplitude of the thermal vibration of atoms. As the root-mean-square amplitude of the thermal vibration \(\sqrt{\langle \rho^2 \rangle}\) is usually less than one-tenth of the interatomic separation, \(d\), the fraction of \(\gamma\)-ray which has energy \(\hbar v(1 + v/c)\) is approximately equal to the value given by Eq. (1). The remaining fraction, \(\exp(-d/v\tau)\), is affected by the scattering of the nucleus-plus-incident-projectile system by the nearest atom. The velocity of the system has a distribution after the collision, and this gives rise to the continuous spectrum between \(\hbar v(1 + v/c)\) and \(\hbar v[1 + \frac{vm}{c(m+2M)}]\).

In order to calculate this continuous spectrum, it is assumed that the lifetime of the nucleus-plus-incident-projectile system is shorter than \(d/v\), so that the probability of emitting \(\gamma\)-ray when the system traveled the distance \(2d\) is small. The orbit of the nucleus-plus-incident-projectile system can be calculated as a function of the impact parameter of the collision by assuming a suitable interaction potential \(V(r)\). The velocity component of the system parallel to the atomic row at time \(t\) is derived and thus the energy spectrum of \(\gamma\)-ray is calculated. However, for convenience of calculations and to show the characteristic features of the energy spectrum, the “Hard-Sphere” approximation is adopted as some of the collisions we are interested are closer to head-on. The velocity component of the system parallel to the atomic row, \(v_c\), is a function of the impact parameter \(p\) and is approximately expressed as

\[
\frac{v_c}{v} = \frac{m+2M(p/r_0)^2}{m+2M},
\]

where \(r_0\) is the collision diameter defined by the scattering potential as

\[
V(r_0) = \frac{mME(m+M)(m+2M)}.\]

On the other hand, if the thermal vibration of atoms in a crystal is expressed by a Gaussian distribution, the probability of finding an atom at the distance \(r\) from the atomic row is given by

\[
P(r) = \frac{1}{\pi \langle \rho^2 \rangle} \exp\left(-\frac{r^2}{\langle \rho^2 \rangle}\right).
\]

Thus the probability that the collision impact parameter has a value \(p\) is given by

\[
P(p) = \frac{P}{\langle \sigma^2 \rangle} \exp\left(-\frac{p^2}{2 \langle \rho^2 \rangle}\right).
\]

From Eqs. (2) and (5), the continuous part of the energy spectrum of \(\gamma\)-ray between \(\hbar v(1 + v/c)\) and \(\hbar v[1 + \frac{vm}{c(m+2M)}]\) is derived. The energy spectrum of \(\gamma\)-ray normalized by its area is

\[
1 - \{1 - \exp\left(-\frac{r_0^2}{2 \langle \rho^2 \rangle}\right)\} \exp\left(-\frac{d}{v\tau}\right), \quad (E = E_{\text{max}}),
\]

\[
\frac{r_0^2(m+2M)}{4 \langle \rho^2 \rangle M} \exp\left\{\frac{mr_0^2}{4 \langle \rho^2 \rangle M} - \frac{d}{v\tau} - \frac{r_0^2v_c(m+2M)}{4M\langle \rho^2 \rangle v}\right\}, \quad (E \neq E_{\text{max}}),
\]

(9)
where \( E_{\text{max}} = h\nu(1 + v/c) \). The term \( \exp \left\{ (d/\nu v) - \left( r_0^2/2\langle \rho^2 \rangle \right) \right\} \) added to the spectrum at \( E = E_{\text{max}} \) is due to the collision in which the impact parameter is larger than the collision diameter \( r_0 \). The continuous part of the spectrum has an exponential decay with \( v_c/v \) having a maximum at \( v_c/v = m/(m + 2M) \). A schematic energy spectrum is shown in Fig. 1. Although these results are based on the hard-sphere approximation, the energy spectrum has a peak at \( h\nu(1 + v/c) \) and is continuous between \( h\nu(1 + v/c) \) and \( h\nu[1 + v_m/c(m + 2M)] \). It is expected that these characteristic features of the spectrum do not change when the accurate calculations of the orbit of the system are performed.

Fig. 1. Schematic energy spectra of \( \gamma \)-rays at the conditions:

- \( E = 4.0 \) MeV, \( m = m_p \) (mass of a proton), \( M = 29m_p \), \( d = 2.5 \times 10^{-8} \) cm, \( \sqrt{\langle \rho^2 \rangle} = 1.0 \times 10^{-9} \) cm.
- The collision diameter \( r_0 = 2.0 \times 10^{-9} \) cm (both for Coulomb potential and Bohr potential).
- The \( \gamma \)-rays emitted in the direction parallel to the direction of the incident protons. The spectra are normalized to the area of the rectangle OABC, thus the peak at \( E = h\nu (1 + v/c) \) has the area equal to the area above the exponential curve.
- a) \( \tau = 2.72 \times 10^{-16} \) sec.  
  b) \( \tau = 1.37 \times 10^{-16} \) sec.

Summarizing the results it may be concluded that the accuracy of the lifetime measurement by Doppler-shift attenuation method can be extended to less than \( 10^{-16} \) second by the use of the single crystal target. As can be seen from Eq. (6), the energy spectrum of \( \gamma \)-ray depends on the temperature of the crystal and it is expected that the experiment at lower temperatures can give better accuracy in the lifetime measurement. When the incident projectiles impinge on the crystal, the yield of \( \gamma \)-ray decreases to less than about one-tenth of that from amorphous target by the channeling effect of the incident projectile.\(^3\) Furthermore, as the direction of the motion of the nucleus-plus-incident-projectile system must be nearly parallel to one of the low index directions of the crystal, the energy of the incident projectiles should be chosen so that the reaction takes place only at the nuclei near the incident surface, in order to avoid the deflection of the projectile due to atomic potential.
REFERENCES

