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<th>Four Particle Model For Three Body Breakup Reaction (Commemoration Issue Dedicated to Professor Takuji Yanabu on the Occasion of his Retirement)</th>
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Kyoto University
Four Particle Model For Three Body Breakup Reaction

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A mechanism for three body breakup reaction is studied with a four particle model neglecting the process of multiple scattering. Energy spectra of protons for the $d+d\rightarrow d+p+n$ reaction show three large enhancements corresponding to the target breakup quasi-free process, the projectile breakup quasi-free process and a new type of enhancement at low energies of protons associated with the forward deuterons. The last enhancement appears from the kinematical condition for a spectator model and that is mainly produced through the negative energy scattering off the energy shell.

I. INTRODUCTION

Breakup cross sections for three nucleon system (nucleon + deuteron) were calculated using a Faddeev theory and compared with the experimental data in good fits. Three body theory was also used to calculate the breakup cross section for the $^3H(\alpha, ap)n$ reaction at a low energy and the reasonable fit was obtained. The Faddeev theory is not yet applied to four nucleon system (deuteron + deuteron, nucleon + triton or helium 3), and the experimental energy spectra were usually analyzed using a simple theory for quasi-free scatterings. The theory however cannot explain the absolute value of the breakup cross section and also fails to reproduce the angular dependence of the enhancement maximum in the energy spectrum, although the simple theory reproduced the spectral shape of emitted particles. On the other hand a four body model calculation can explain angular correlation functions for three body breakup reaction. Then it is worthy to examine the validity of the assumptions of three body model in the case of four particle system.

The three body breakup reaction of four particle system is studied on the bases of Sloan's four particle theory neglecting multiple scattering process.

II. THEORY

I. Differential Cross Section

For the nuclear reaction as $P+T\rightarrow A+B+C$, the breakup cross section is given as

$$\frac{d\sigma}{d\Omega_d d\Omega_g dE_A} = \frac{2\pi}{\hbar\nu_{in}} \frac{1}{(2J_p+1)(2J_T+1)} \sum_j (2J+1) \rho_j U_{fi}^2 (10 \text{ mb/sr}^2 \text{ MeV}). \quad (1)$$

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with

\[
\frac{1}{\hbar \omega_{\text{in}}} = \sqrt{\frac{m_{p}(m_{p}+m_{T})}{2m_{T}E_{p}}} \gamma^{1/2},
\]

\[
\gamma = \frac{m_{0}}{\hbar} \approx 4.6 \text{ MeV} / f_{\text{m}}^{2},
\]

\(m_{p}\): mass of the projectile in AMU,
\(m_{T}\): mass of the target in AMU,
\(J_{p}\): spin of the projectile in \(\hbar\) units,
\(J_{T}\): spin of the target in \(\hbar\) units,
\(J\): total spin of the four particle system in \(\hbar\) units.

The quantity \(\rho\) is the phase space factor as,

\[
\rho = \frac{1}{(2\pi)^{6}} \gamma^{3} \rho_{\text{MeV}},
\]

\[
\rho_{\text{MeV}} = -\frac{m_{A}m_{B}m_{C}p_{A}p_{B}}{m_{C}+m_{B}-m_{A}(P_{\text{in}}-P_{A}) \cdot P_{B}/P_{B}},
\]

with

\(p_{A} = \sqrt{2m_{A}E_{A}},\)
\(p_{B} = \sqrt{2m_{B}E_{B}},\)
\(m_{A}\): mass of outgoing particle A in AMU,
\(m_{B}\): mass of outgoing particle B in AMU,
\(m_{C}\): mass of outgoing particle C in AMU,
\(E_{A}\): energy of particle A in MeV,
\(E_{B}\): energy of particle B in MeV,

where \(p_{\text{in}}, p_{A}\) and \(p_{B}\) are momenta of the projectile, particle A and particle B, respectively.

The phase space factor \(\rho_{\text{MeV}}\) is measured in MeV. The \(U\) is the transition amplitude for the three body breakup reaction and is defined as

\[
|U_{fi}|^{2} = (2\pi)^{6} \gamma^{3/2} |T_{fi}|^{2},
\]

where \(|T|^{2}\) is measured in units of \(\text{MeV}^{-3/2}\).

2. Momentum Coordinates

For the channel representation of four particle system, there are two types: \(1+3\) channel (\(ijk-l\)) and \(2+2\) channel (\(ij-kl\)). There are four \(1+3\) channels and three \(2+2\) channels. One of the \(1+3\) channel consists of three subchannels which are specified by one of three internal pairs in the three particle subsystem and one of the \(2+2\) channel consists of two subchannels specified by one of two internal pairs. The definition of the internal pair is same as used by Sloan.\(^{5}\)

For a system of four particles with masses \(m_{i}, m_{j}, m_{k}\) and \(m_{l}\), it is convenient to use the following \((p, q, s, w)\) coordinates instead of the usual relative momenta \((k_{ij}, k_{ij-kl}, k_{ijk-l}, w), (k_{ij}, k_{ij-lk}, k_{ij-kl}, w)\) or particle momenta \((k_{i}, k_{j}, k_{k}, k_{l})\) in the laboratory system.

For the \(1+3\) type,
\[ p_{i-j} = \left[ 2m_i m_j / (m_i + m_j) \right]^{-1/2} k_{i-j} \]
\[ q_{ij-k} = \left[ 2m_k (m_i + m_j) / (m_i + m_j + m_k) \right]^{-1/2} k_{ij-k} \]
\[ s_{ijkl} = \left[ 2m_i (m_j + m_k) / (m_i + m_j + m_k + m_l) \right]^{-1/2} k_{ijkl} \]
\[ w = \left[ 2(k_i + k_j + k_k + k_l) \right]^{-1/2} (k_i + k_j + k_k + k_l) \]

and for the 2+2 type,
\[ p_{i-j} = \left[ 2m_i (m_i + m_j) / (m_i + m_j) \right]^{-1/2} k_{i-j} \]
\[ q_{ij-k} = \left[ 2m_k (m_i + m_j) / (m_i + m_j + m_k) \right]^{-1/2} k_{ij-k} \]
\[ s_{ijkl} = \left[ 2(m_i + m_j) (m_k + m_l) / (m_i + m_j + m_k + m_l) \right]^{-1/2} k_{ijkl} \]
\[ w = \left[ 2(k_i + k_j + k_k + k_l) \right]^{-1/2} (k_i + k_j + k_k + k_l) \]

where \( w \) is total momentum of the four particle system. The momenta \( (p_\beta, q_\beta, s_\beta, \text{and } w) \) in the subchannel \( \beta \) is connected with the momenta \( (p_\gamma, q_\gamma, s_\gamma, \text{and } w) \) in the subchannel \( \gamma \) through the \( 4 \times 4 \) matrix \([C^\beta] \) as,
\[ \begin{pmatrix} p_\beta \\ q_\beta \\ s_\beta \\ w \end{pmatrix} = [C^\beta] \begin{pmatrix} p_\gamma \\ q_\gamma \\ s_\gamma \\ w \end{pmatrix}, \]
with
\[ [C^\beta] = [A^\beta] \cdot [D^\beta] \cdot [A^\beta]^{-1}, \]
\[ [k_\beta] = [D^\beta] [k_\gamma], \]
where \([k_\beta] \) and \([k_\gamma] \) are one column matrices of particle momenta in the subchannel \( \beta \) and \( \gamma \), respectively. The \([A^\beta]\) is the \( 4 \times 4 \) matrix which is composed of the coefficients of particle momenta in equation (4) or (5). The \([A^\beta]^{-1}\) is the inverse matrix of the \([A^\beta]\) and the \([D^\beta]\) presents the relation between the different labelings \( \gamma \) and \( \beta \) for the kinematically equivalent state. Then, the momenta \( p_\beta, q_\beta, s_\beta, \text{and } w \) are explicitly written by the momenta \( p_\gamma, q_\gamma, \text{and } s_\gamma \) as,
\[ \begin{align*}
    p_\beta &= C^\beta_{p} p_\gamma + C^\beta_{q} q_\gamma + C^\beta_{s} s_\gamma, \\
    q_\beta &= C^\beta_{p} p_\gamma + C^\beta_{q} q_\gamma + C^\beta_{s} s_\gamma, \\
    s_\beta &= C^\beta_{p} p_\gamma + C^\beta_{q} q_\gamma + C^\beta_{s} s_\gamma,
\end{align*} \]
and
\[ Z = p^2_\beta + q^2_\beta + s^2_\beta = p^2_\gamma + q^2_\gamma + s^2_\gamma, \]
where \( Z \) is the total energy in the center of mass system. In these relations, it is not
necessary that $\beta$ and $\gamma$ belong to the same channel.

When the four particles have the same mass of unity, the matrix $[A]$ is given as

$$
[A] = \begin{pmatrix}
\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\
\frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 \\
\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & -\frac{3}{2\sqrt{6}} \\
\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\
\end{pmatrix}
$$

for the 1+3 type channel,

and

$$
[A] = \begin{pmatrix}
\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\
0 & 0 & \frac{1}{2} & -\frac{1}{2} \\
\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\
\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\
\end{pmatrix}
$$

for the 2+2 type channel.

Using the new momentum coordinates, the state vectors in the relative momentum coordinates are written by using the vector $|p\rangle$, $|q\rangle$ and $|s\rangle$ which are normalized to the $\delta$-function as

$$
|k_{ij}\rangle = \frac{1}{dk_{ij}} |p_{ij}\rangle, \\
|k_{kl}\rangle = \frac{1}{dk_{kl}} |q_{kl}\rangle, \\
|k_{ij-kl}\rangle = \frac{1}{dk_{ij-kl}} |s_{ij-kl}\rangle, \\
\langle p_{ij}^\prime q_{ij}^\prime s_{ij-kl}^\prime |p_{ij} q_{ij} s_{ij-kl}\rangle = \delta^3(p_{ij}^\prime - p_{ij}) \delta^3(q_{ij}^\prime - q_{ij}) \delta^3(s_{ij-kl}^\prime - s_{ij-kl}), \\
\langle k_{ij}^\prime k_{kl}^\prime |T_{ij}|k_{ij} k_{kl}\rangle = \frac{1}{2\sqrt{2}} \langle p_{ij}^\prime q_{ij}^\prime s_{ij-kl}^\prime |T_{ij}|p_{ij} q_{ij} s_{ij-kl}\rangle.
$$

The transition amplitudes $\langle k' |T_{ij}| k\rangle$ in the relative momentum coordinates is equal to that in Eq. (3).

3. Spin Coefficients

As shown in the previous sections, the channel of the four particle state is specified by the subchannel and the interacting pair. The state vector of the subchannel is composed of two functions of the momentum state $|pq\rangle$ and the spin state $|J_s\rangle$ as,
For the $d+d\rightarrow d+p+n$ reaction, two protons are labeled as particles 1 and 3 and two neutrons are labeled as particles 2 and 4.

The antisymmetrized amplitude is written by using the amplitudes represented with the labeled four particles as,

$$
\mathfrak{T}_{12, 34}^{<12, 3, 4}|T|_{12, 34}> = \frac{1}{8} \left( (\mathfrak{T}_{<12, 3, 4} - \mathfrak{T}_{<32, 1, 4} - \mathfrak{T}_{<14, 3, 2} + \mathfrak{T}_{<34, 1, 2}) \right) T(12, 34>, 12>) .
$$

(8)

We assume that two particle interaction is not spin dependent, then the transition amplitudes are factorized to two parts as the momentum components and the spin coefficients. The spin coefficient is written for the unpolarized experiment as

$$
S = S(J_P, J_I, J_F) = <J_P|J_I, J_F>,
$$

(9)

where $J_P$, $J_I$, and $J_F$ are the spins of the subchannels and $J_P$, $J_I$, and $J_F$ are the spins of interacting pair in the final state, the intermediate state and the initial state, respectively. The spin of the subchannel is the total spin of the three particle subsystem in the $1+3$.

Table 1 Transition amplitude for $d+d\rightarrow d+p+n$

(Proton : 1, 3 and neutron : 2, 4)

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>12-34</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>p=12</td>
</tr>
<tr>
<td>$\nu$</td>
<td>14</td>
</tr>
<tr>
<td>12-34</td>
<td>$-\frac{3}{4}t^{0}\delta_{13}$</td>
</tr>
<tr>
<td></td>
<td>$-\frac{1}{2}t^{0}\delta_{13}$</td>
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<tr>
<td></td>
<td>$-\frac{1}{2}t^{0}\delta_{13}$</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{2}t^{4}\delta_{13}$</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{2}t^{4}\delta_{13}$</td>
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<tr>
<td></td>
<td>$\frac{1}{2}t^{4}\delta_{13}$</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{2}t^{4}\delta_{13}$</td>
</tr>
</tbody>
</table>

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Table II Transition amplitude for \( p^+{^3}He \rightarrow d+p+p \) (proton: 1, 2, 3 and neutron: 4)

<table>
<thead>
<tr>
<th>( J_{34} )</th>
<th>( \gamma )</th>
<th>( \nu )</th>
<th>( \lambda )</th>
<th>( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>12-34</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>134-2</td>
<td>13</td>
<td>12</td>
<td>13</td>
<td></td>
</tr>
</tbody>
</table>

| \( J=0 \) | \( \mu=14 \) | \( \frac{3}{4}t_{13} \) | \( \frac{3}{4}t_{13} + \frac{1}{4}t_{14} \) | \( \frac{\sqrt{3}}{4}t_{13} \) |
| \( J=1 \) | \( \mu=14 \) | \( \frac{1}{4}t_{13} \) | \( \frac{1}{4}t_{13} + \frac{1}{12}t_{14} \) | \( \frac{\sqrt{3}}{4}t_{13} \) |

channel or the spin of the pair coupled with the momentum \( q \) in the 2+2 channel. The overlapping of two spin state is written as, for example,

\[
\langle J_1, J_2 | J_1, J_2 \rangle = (-)^{J_1+J_2} W(J_{13} J_{13} J_{13} J_{13}) W(J_{13} J_{13} J_{13} J_{13})
\]

where \( W \) is Racah Coefficient.

For the initial d+d channel and the final d+p+n channel, the transition amplitudes with the spin coefficients are given in Table I. For the initial \( p^+{^3}He \) channel and the final d+p+p channel, the amplitudes are given in Table II.

4. Transition Amplitude

For four particle system, the transition amplitude \( T_{fi} \) in Eq. (3) is given by the solution of seven coupled integral equations studied by Sloan. To simplify the calculation, the single scattering amplitude takes in the place of the transition amplitude \( T_{fi} \) as

\[
T_{fi} = \frac{1}{n_f \pi_f} \sum_{\alpha} \sum_{\beta} \sum_{\gamma} \sum_{\nu} \sum_{\lambda} \sum_{\mu} (-)^s \sum_{\nu} \sum_{\lambda} S \cdot T_{fi},
\]

where the \( S \) is the spin coefficient defined in Eq. (9) and \((-)^s \) is the sign in Eq. (8).

For the reaction \( d+d \rightarrow d+p+n \), the initial channel is the 2+2 type and the intermediate channel is one of four 1+3 channels. The final channel is a three body channel including one bound pair. The \( n_i \) and \( n_f \) are the number of the initial subchannel and the number of the final subchannel, respectively. Then, \( n_i \cdot n_f = 8 \) as shown in Eq (8). The suffices \( \lambda, \mu, \nu, \) and \( \gamma \) of single scattering amplitude mean the interacting pairs in the initial subchannel \( \alpha \), the final subchannel \( \beta \) and the intermediate subchannel \( \gamma \), respectively.
The amplitude \( T_{\pi 2} \) is written as,

\[
T_{\pi 2} = -\frac{1}{|D_0 D_0|}\hat{\phi}_\pi(q^*_\pi) \int dq'_\pi \hat{\phi}_\pi(p'_\pi) \langle p'_\pi | T \tau (\tau - q'^*_\pi - q'^*_\pi) | q'^*_\pi \rangle \hat{\phi}_\pi(q'^*_\pi) \tag{11}
\]

where

\[
q^*_\pi = \frac{(C_\pi D_\pi - C_\pi D_\pi s_a - C_\pi D_\pi s_b) / C_\pi D_\pi}{D_\pi},
\]

\[
p'_\pi = \frac{(-D_\pi s_p - D_\pi q_s - (C_\pi D_\pi - C_\pi D_\pi q'_s)/D_\pi)}{D_\pi},
\]

\[
q'_\pi = \frac{(C_\pi D_\pi s_a - C_\pi D_\pi q'_s)}{D_\pi},
\]

\[
s'_s = C_\pi s_p / D_\pi,
\]

\[
p'_\pi = \frac{(-D_\pi s_p - D_\pi q_s)}{C_\pi D_\pi},
\]

\[
p^*_\pi = \frac{(C_\pi D_\pi s_a - C_\pi D_\pi q'_s)}{C_\pi D_\pi},
\]

The momentum \( s_a \) is chosen to relate with the momentum \( s'_s \) as,

\[
s^a = s'_s^2
\]

The \( \langle p' | T \tau | p^* \rangle \) in Eq. (11) is an off energy amplitude of nucleon-nucleon scattering.

5. Nucleon-Nucleon Scattering Amplitude

For nucleon-nucleon scattering amplitude, the separable potential is taken as

\[
V(p', p^*) = \lambda g(p') g(p^*)
\]

and S-wave amplitude is given as

\[
\langle p' | T \tau | p^* \rangle = \lambda g(p') \tau(\varepsilon) g(p^*) \tag{13}
\]

with

\[
g(p) = \frac{N}{\bar{p}^2 - \bar{p}^2},
\]

\[
\tau(\varepsilon) = \frac{J(\sqrt{\varepsilon})}{\kappa + i\sqrt{\varepsilon}},
\]

\[
J(\sqrt{\varepsilon}) = \frac{(\delta \cdot i\sqrt{\varepsilon})^2}{\kappa (\bar{\beta} - \kappa) (2\beta - \kappa - i\sqrt{\varepsilon})}.
\]

These definitions equal to those of Ebedhöf et al. and values of parameters \( \delta, \kappa \) and \( N \) are given in Table III.
Table III  Parameters for nucleon-nucleon scattering.

<table>
<thead>
<tr>
<th></th>
<th>$\delta$(MeV$^{1/2}$)</th>
<th>$\epsilon$(MeV$^{1/2}$)</th>
<th>N(MeV$^{5/4}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>triplet n-p</td>
<td>9.113</td>
<td>1.4910</td>
<td>40.630</td>
</tr>
<tr>
<td>singlet p-p</td>
<td>7.725</td>
<td>-0.7160</td>
<td>13.891 i</td>
</tr>
<tr>
<td>n-p</td>
<td>7.576</td>
<td>-0.2581</td>
<td>8.811 i</td>
</tr>
<tr>
<td>n-n</td>
<td>7.203</td>
<td>-0.3332</td>
<td>8.879 i</td>
</tr>
</tbody>
</table>

6. Singularity of $\tau$

Assuming the separable potential, the transition amplitude of Eq. (11) can be written as

$$T^*_A = \frac{1}{|D^2D^4|} \phi^*_A(q^*_2) \int dq'_2 dq' \epsilon(q'_2) (z-s^2-q'_2)$$

$$\times \int d\Omega'_q \phi(p'_2) g(p'_s) g(p'_s) \phi_s(p'_1).$$  \hspace{1cm} (14)

The $q'_s$ integral in Eq. (14) can be separated to the residue of the pole and the principal integral with the aid of the formal relation as

$$\frac{1}{q^2 - q^2 - i\epsilon} = P\left(\frac{1}{q^2 - q^2}\right) + i\pi \delta(q^2 - q^2).$$  \hspace{1cm} (15)

The principal integral can be evaluate numerically using a standard technique dividing the integration area to two regions on both sides of the pole. The angular integral in Eq. (14) has no any singularity, then the integration can be evaluate without any trouble numerically.

III. CALCULATED ENERGY SPECTRA AND DISCUSSIONS

Figure 1 shows the calculated energy spectra of protons emitted from the reaction $^2H(d, pd) n$ at 60 MeV. The proton angle is fixed at 20° and the deuteron angle are chosen in the opposit side of the proton direction in the range from 2° to 70° in the step of 4°.

Large enhancements are seen at the region for the low energy protons associated with the forward deuterons and also seen at two quasi-free scattering regions corresponding to the target breakup and the projectile breakup reactions. Figure 2 shows the angular dependence of the proton energy at the maximum enhancement. In this figure, the dashed curve shows the kinematical points corresponding to the minimum energy of the neutron. If a three body impulse approximation calculation is valid for this reaction, the enhancements will be expected at energies on the dashed curve. As shown in the figure, the dashed curve splits to two curves avoiding the area NES corresponding to the large enhancement. The enhancement in the NES region is produced from the nucleon-nucleon scattering at negative energy and then if such an enhancement is observed experimentally, it is an evidence that the negative energy scattering is important.
In conclusion, the large enhancement is predicted at the low energies of protons associated with the forward deuterons. Then it is hoped to measure the breakup cross section in this kinematical region.

Numerical calculations were made using the FACOM 160 AD at the Computer Center of the Institute for Chemical Research.

REFERENCES