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Magnetic Susceptibility of a Microbridge-Coupled Superconductor in Non-Sinusoidal Periodic Fields

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Using a weakly-connected loop model, the profile of superconducting transition of a microbridge-coupled superconductor has been generated. Applied magnetic fields to the specimen are supposed as periodic but non-sinusoidal.

KEY WORDS: Superconductivity/ ac susceptibility/ Higher-harmonic susceptibility/ Microbridge-coupled superconductor/

INTRODUCTION

The Hartshorn-type ac bridge has been widely used in the field of low-temperature physics. This method has benefited from the development of a lock-in technique and has been extensively modified from the original. The fundamental susceptibility $\chi_1'-i\chi_1''$ is conventionally measured by using a two-phase lock-in analyzer, *i.e.*, we need not make an elaborate effort to balance the bridge. In the case that a substance behaves in a non-linear manner against an external magnetic field, the higher-harmonic susceptibility $|\chi_n|$ can be obtained by applying our modification to the bridge.[1] As to an exciting magnetic field, however, the operation of the bridge has so far been limited to a sinusoidal one.

Recently, we investigated a microbridge-coupled superconductor by means of the fundamental and higher-harmonic susceptibilities.[1,2] We found the temperature-dependent and amplitude-sensitive properties of the superconducting transition. And the odd-harmonic susceptibilities were also observed to appear in the ac response of microbridge-coupled superconductor. A fine explanation of these properties was given from the standpoint of weakly-connected loop model, which implicates a non-linear process in the response. The results previously obtained lead to a further interest how the ac response would be in various kinds of non-sinusoidal periodic fields.

In the present work, we attempt to generate a superconductive profile of microbridge-coupled superconductor against nonsinusoidal exciting field with the aid of weakly-connected loop model.

CALCULATION

1. Sinusoidal Magnetic Field

In our previous works, we investigated the response of superconducting Tc with

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multiply-connected structure against a small ac magnetic field, which can be explained by the weakly-connected loop model.[1,2] There, we considered that the specimen can be viewed as a microbridge-coupled network of superconductors and that a number of shielding loops induced in the network behave like a single loop as a whole due to the coherent nature of the specimen. Assigning J_0 as the critical current of the equivalent loop, h_0 as the amplitude of the field, and h_m as the magnetic field which iuduces a supercurrent J_0 in the loop, we define $\sin\theta = h_m/h_0$. From the model, χ_1' and χ_1'' are analytically expressed by

$$\chi_1' = -\frac{1}{4\pi^2}\left(\alpha - \frac{1}{2}\sin 2\alpha\right), \quad \chi_1'' = \frac{1}{4\pi^2}\sin^2\alpha, \tag{1}$$

where $\alpha = 2\sin^{-1}(\sin\theta)^{1/2}$. The odd-harmonic susceptibility $\chi_n = \chi_n' - i\chi_n''$ is also given by

$$\left.\begin{aligned} \chi_n' &= \frac{1}{4n\pi^2}\left[\frac{\sin(n+1)\alpha}{n+1} - \frac{\sin(n-1)\alpha}{n-1}\right] \\ \chi_n'' &= -\frac{1}{4n\pi^2}\left[\frac{\cos(n+1)\alpha - 1}{n+1} - \frac{\cos(n-1)\alpha - 1}{n-1}\right]. \end{aligned}\right\} \tag{2}$$

By the change in parameter θ from 0 to $\pi/2$, the profile of superconducting transition can be generated. Here we assumed that J_0 is proportional to $(1-T/T_0)^{3/2}$, which is a microbridge-type temperature dependence. T_0 is the onset temperature of superconductivity.

2. Non-Sinusoidal Magnetic Field

We consider the cases that the bridge is excited by three kinds of non-sinusoidal magnetic fields, *i.e.*, a triangular-wave magnetic field $h_1(t)$, a square-wave field $h_2(t)$, and a trapezoidal-wave field $h_3(t)$. The amplitudes of fields are always chosen as h_0. The fields $h_1(t)$, $h_2(t)$, and $h_3(t)$ are expanded by Fourier series as

$$\left.\begin{aligned} h_1(t) &= \frac{4h_0}{\pi}\sum_{n=1}^{\infty}(2n-1)^{-1}\sin(2n-1)\omega t, \\ h_2(t) &= \frac{8h_0}{\pi}\sum_{n=1}^{\infty}(-)^{n-1}(2n-1)^{-2}\sin(2n-1)\omega t, \\ h_3(t) &= \frac{4h_0}{\pi}\sum_{n=1}^{\infty}(2n-1)^{-2}\sin(2n-1)\omega s\sin(2n-1)\omega t, \end{aligned}\right\} \tag{3}$$

where s is a rise time of trapezoidal wave. They contain higher harmonics in themselves, which complicate the problem by analytical reason. In the next section, we develop the numerical evaluation for the response of microbridge-coupled superconductor.

3. Numerical Method

In Fig. 1, the procedure of present calculation is summarized as a flow chart. The field $h(t)$ is supposed to have a period τ. The critical current of the equivalent loop $J_0(T)$ is assumed to be proportonal to $(1-T/T_0)^{3/2}$ at temperatures near T_0. At each specified temperature, we subsequently generate $h(t_i)$ from $i=1$ to 1024, where $t_i=(i-1)\tau/1024$. Details of the generation of sample magnetization $m(t)$ are shown in Fig. 2. For each t_i, the magnetic field $b(t_i)$ in the loop is determined as

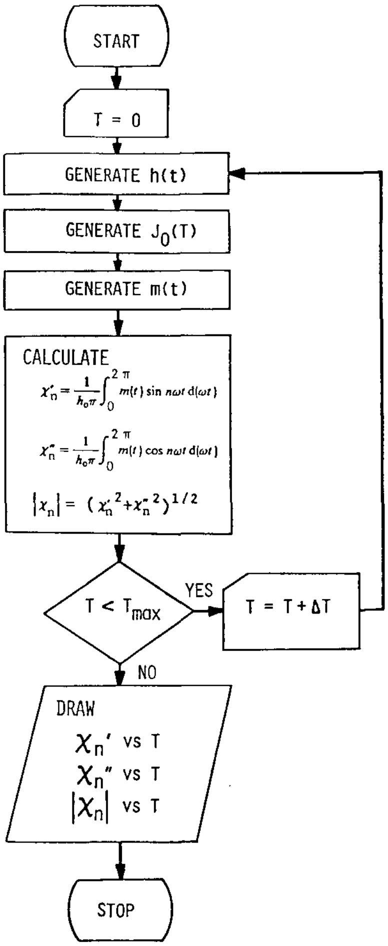

Fig. 1. Schematic flow diagram of the program.

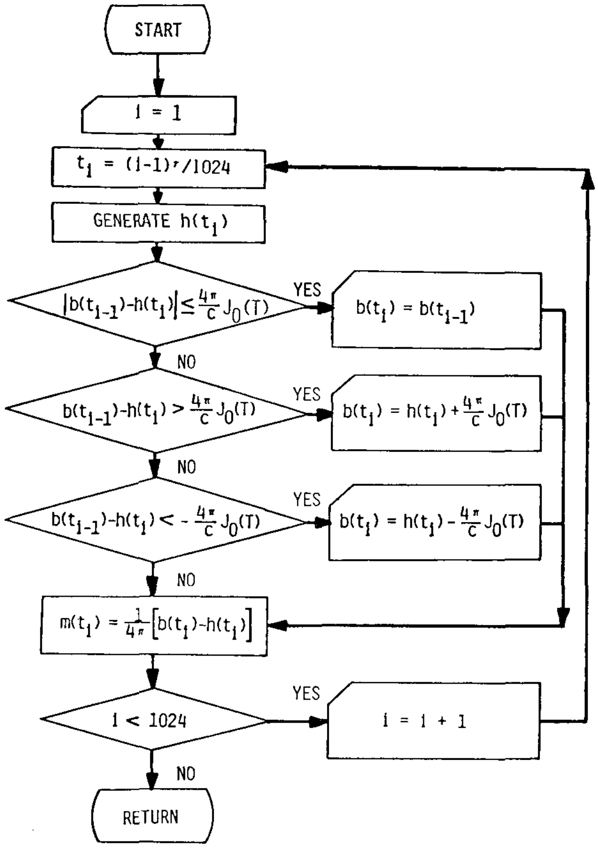

Fig. 2. The computational flow diagram of the sample magnetization $m(t)$.

$$\left.\begin{aligned} b(t_i) &= b(t_{i-1}) \text{ for } |b(t_{i-1}) - h(t_i)| \leq 4\pi J_0(T)/c, \\ b(t_i) &= h(t_i) + 4\pi J_0(T)/c \text{ for } b(t_{i-1}) - h(t_i) > 4\pi J_0(T)/c, \\ b(t_i) &= h(t_i) - 4\pi J_0(T)/c \text{ for } b(t_{i-1}) - h(t_i) < -4\pi J_0(T)/c. \end{aligned}\right\} \quad (4)$$

The magnetization $m(t_i)$ of the specimen is obtained by

$$m(t_i) = \frac{1}{4\pi}[b(t_i) - h(t_i)]. \quad (5)$$

This process is repeated for another period τ. This is for the sake of obtaining a stationary behavior with respect to time. One gets Fourier expansion of stationary $m(t)$ by

$$m(t) = \sum_{k=1}^{\infty} h_0[\chi_k' \sin k\omega t + \chi_k'' \cos k\omega t]. \quad (6)$$

From this expression, χ_k' and χ_k'' are given by

$$\left.\begin{aligned}\chi_k' &= \frac{1}{h_0\pi}\int_0^{2\pi} m(x/\omega)\ \sin kx \mathrm{d}x,\\ \chi_k'' &= \frac{1}{h_0\pi}\int_0^{2\pi} m(x/\omega)\ \cos kx \mathrm{d}x,\\ &(k=1,\ 3,\ 5,\ 7,\ \cdots\cdots).\end{aligned}\right\} \tag{7}$$

At 100 points of temperature during superconducting transition, we calculate $m(t)$ and analyze it by means of Fourier expansion. The transition curves thus generated are programmed to be drawn by the preview routines.

RESULTS AND DISCUSSION

First, we apply the numerical method to the sinusoidal exciting field $h(t) = h_0 \sin \omega t$, for which the analytical expression of susceptibilities are available [see Eqs. (1) and (2)]. In Fig. 3 is shown the generated transition curve in terms of fundamental susceptibility. The higher-harmonic susceptibilities χ_n' and χ_n'' (n=3, 5, 7, 9, 11) are also drawn in

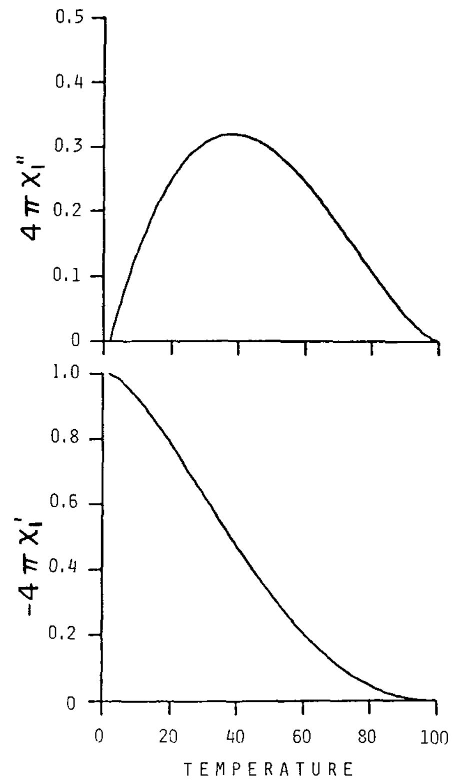

Fig. 3. Numerically generated transition curve in terms of fundamental susceptibility χ_1' and χ_1''. The sinusoidal exciting field is applied to a microbridge-coupled superconductor.

Fig. 4. The curves of Figs. 3 and 4 show an excellent agreement with those expected from Eqs. (1) and (2). This confirms that the present method is reliable.

Next, the responses of the microbridge-coupled superconductor against non-sinusoidal periodic magnetic fields are examined. The trapezoidal magnetic fields, which contain

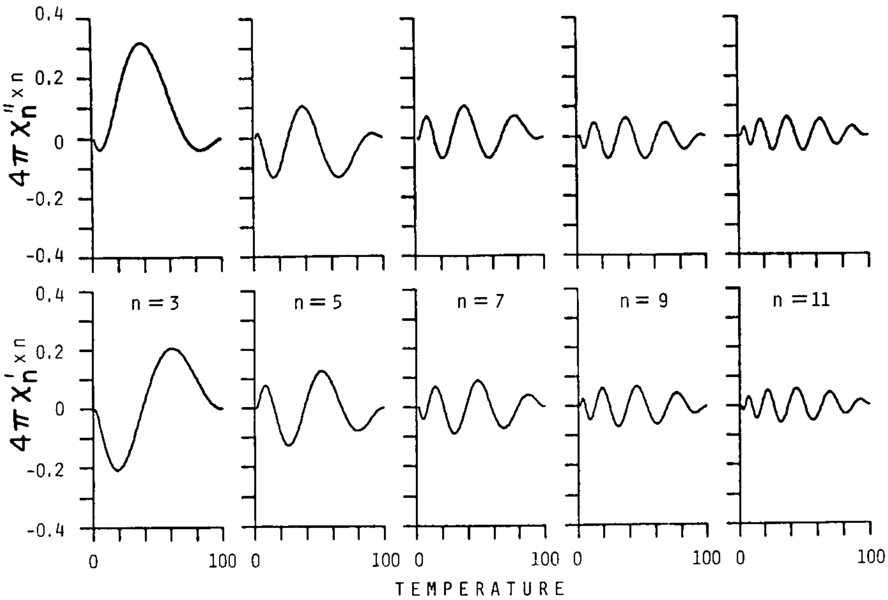

Fig. 4. Generated higher-harmonic susceptibilities χ_n' and χ_n'' *vs* temperature. The exciting field of the bridge is sinusoidal.

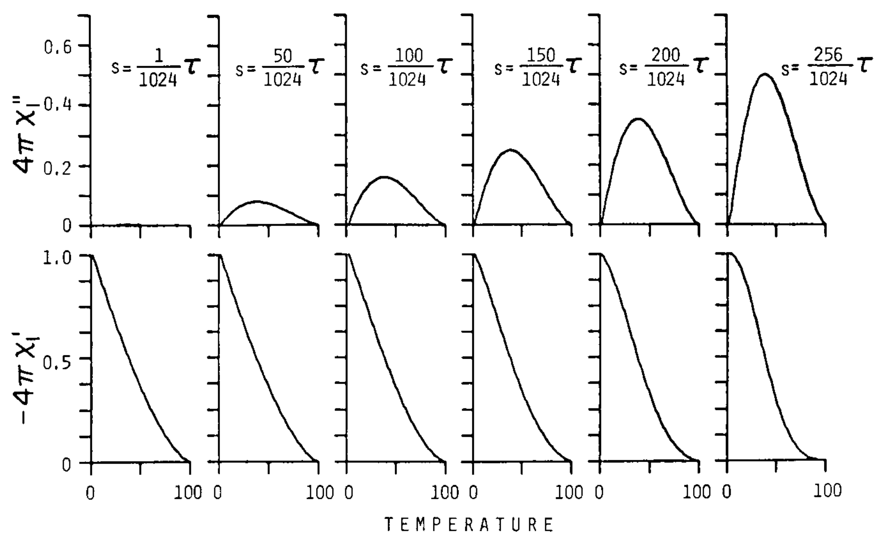

Fig. 5. Generated fundamental susceptibilities χ_1' and χ_1'' vs temperature. The exciting fields are trapezoidal. The rise time s of the field is shown in each figure, where τ is a period. $s=\tau/1024$ and $256\tau/1024$ correspond to square-wave and triangular-wave fields.

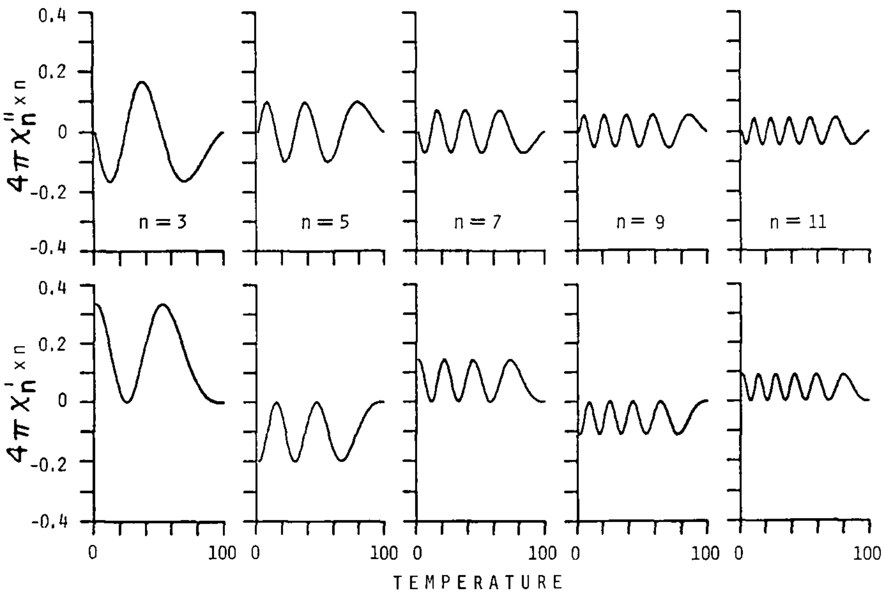

Fig. 6. Generated higher-harmonic susceptibilities χ_n' and χ_n'' *vs* temperature. The exciting field is triangular.

triangular and square wave fields as a special case, are used to excite the system. In Fig. 5 are shown the transition curves of fundamental susceptibility for rise times 1/1034, 50/1024, 100/1024, 150/1024, 200/1024, and 256/1024 of a period τ. The rise time 1/1024 corresponds to square wave and 256/1024 correponds to triangular wave. Surprisingly, the square-wave exciting field does not cause any dissipation in the superconductor. The gradual growth in χ_1'' is found as rise time increases. The superconductor in the triangular field is more dissipative than in the sinusoidal field (see Figs. 3 and 5). In Fig. 6, an example of odd-harmonic susceptibilities are also shown for triangular wave. The differences between Figs. 4 and 5 are apparent.

In conclusion, non-sinusoidally excited magnetic substances have a possibility to cause a versatile feature of the response. It is hoped that the method will be widely examined in various kinds of materials such as spin glass.

REFERENCES

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