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Negative Resistance and Magnetic Response of a Tunnel Junction

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The origin of negative electric resistance which appears in a tunneling experiment has been studied in accordance with the Giaever's model. The response of a superconducting tunnel junction for dc magnetic field has also been examined.

KEY WORDS: Superconductivity/ Tunnel junction/ Negative resistance/

I. INTRODUCTION

In a tunneling experiment, we often observe negative resistance. As is understood by the Giaever's model,[1] the origin of negative resistance is spurious due to the geometrical arrangement of electrodes, and depends on relative values of the total resistance of metal layers and the tunnel resistance through the insulator. However, the tunnel resistance which is an exponential function of the thickness of insulator and the work function of metals involved, may change by an order of magnitude even for 1 Å difference in the insulator thickness, and therefore, the quantitative discussion of the problem is practically so difficult. To the authors' knowledge, so far no direct examination of the model with actual data has been reported.

When the metal used is superconductive below the critical temperature, the tunnel junction becomes a Josephson junction as far as the oxide insulator between two metal layers is thin enough. In this case, it is well known that the current-voltage ($I-V$) curve of the junction shows a particular non-linearity due to the energy gap at the Fermi surface.

In the detailed measurement of $I-V$ curves, Taylor and Burstein[2] found an excess current independent of polarity, temperature, and magnetic fields at bias voltages exceeding Δ/e, where Δ is the gap parameter and e is the electronic charge.

More recently, Rajeevakumar and Chen[3] reported a new excess current of a Josephson junction with large length-to-penetration-depth ratio. From the resonance-like dependence on a magnetic field, they suggested that there exist localized vortices in the junction, and photons emitted by energy loss of a dc tunneling current are absorbed by the current loops in the vortex in a way similar to the magnetic resonance.

Another approach to the magnetic response of a Josephson junction was made by Eck *et al.*[4] In the measurements of the $I-V$ curve, they observed a temperature independent resonance-shaped peak. The essential features of their results were explained by considering the coupling of the ac Josephson current to the electromagnetic modes of

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the junction. On the basis of this coupling, they gave a simple relation $V_p = CH$, where V_p is the voltage at which the current peak occurs, H is the applied magnetic field and C is a constant.

In the present study, using an Sn-SnO_x-Sn tunnel junction prepared by an ordinary vacuum evaporation and oxidization, we measured the resistance of the junctions at 290, 77, and 4.2 K. Seven samples out of 19 showed negative resistance at 290 K, but at lower temperatures all became positive, indicating the semiconductor-like character due to the existance of a thin insulator between two metal layers. To examine the Giaever's model, analysis of the data was performed in line with the model.

Further, we made preliminary measurements of the dc magnetic response of junctions in the superconducting state, and attempted to compare our results with other works.

II. EXPERIMENTAL

A crossed-film tunnel junction of Sn-SnO_x-Sn was prepared by an ordinary vacuum evaporation and oxidization on a quartz-glass substrate. On the substrate, Sn is first evaporated (3000 Å) through a mask, where the initial pressure in the vacuum chamber is about 2×10^{-7} Torr. Then the surface of Sn film is oxidized in O_2 atmosphere (0.5 Torr) for 4–20 min. The expected oxide film of Sn is 10–20 Å. After oxidization, second vacuum evaporation of Sn is carried out through another mask. Total thickness is 6000 Å. Finally, Ag vacuum evaporation is made for protection of junction electrodes.

The resistance and the I–V characteristic curve of the junction were measured by means of the conventional four terminal method. The measuring system is shown in Fig. 1. Measurements of resistance were performed at 290, 77, and 4.2 K, while the magnetic response appeared on the I–V curve was measured at the lowest temperature available, 1.37 K. The field applied on the sample is empirically determined as H (gauss)$=0.23+0.545\, I$ (mA), and the field is applied parallel to the junction.

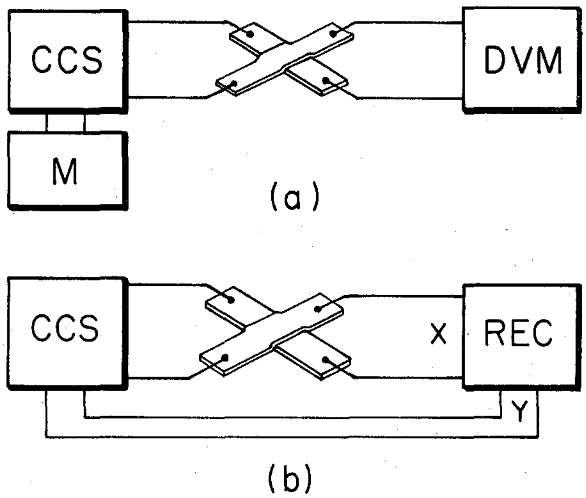

Fig. 1. A schematic diagram of (a) the resistance measurement and (b) the current-voltage measurement. CCS is the constant current source, M is the current monitor, DVM is the digital voltmeter, and REC is the recorder.

III. RESULTS AND DISCUSSION

1. Negative Resistance

In Fig. 2 are shown observed resistances of the tunnel junctions where all junctions give negative resistance at 290 K. When temperature is lowered, the resistance increases and reverses to positive at a certain temperature.

We attempt brief analysis of the data in accordance with the Giaever's model.[1] For simplicity, we treat one dimensional problem. In Fig. 3, we show a schematic arrangement of a tunnel junction, where a thin insulator is sandwiched between two metal layers, 1 and 2. The total resistance along the film is R_F, and position-dependent voltage and current are assigned as $V_1(x)$, $I_1(x)$, $V_2(x)$, $I_2(x)$. Note the position of electrodes is essential to produce a negative resistance.

When the junction is biased by a constant current I, we have $I_1(x)+I_2(x)=I$. The differential change in $V_1(x)$ and $V_2(x)$ is expressed as

$$\frac{\mathrm{d}V_1(x)}{\mathrm{d}x}=-\frac{R_F}{2s}I_1(x), \tag{1}$$

$$\frac{\mathrm{d}V_2(x)}{\mathrm{d}x}=-\frac{R_F}{2s}I_2(x), \tag{2}$$

where $2s$ is the total length of the junction.

Assigning r as the perpendicular resistance to the junction surface $\mathrm{d}x$, one gets

$$\frac{1}{R_T}=\frac{1}{r}\frac{2s}{\mathrm{d}x}. \tag{3}$$

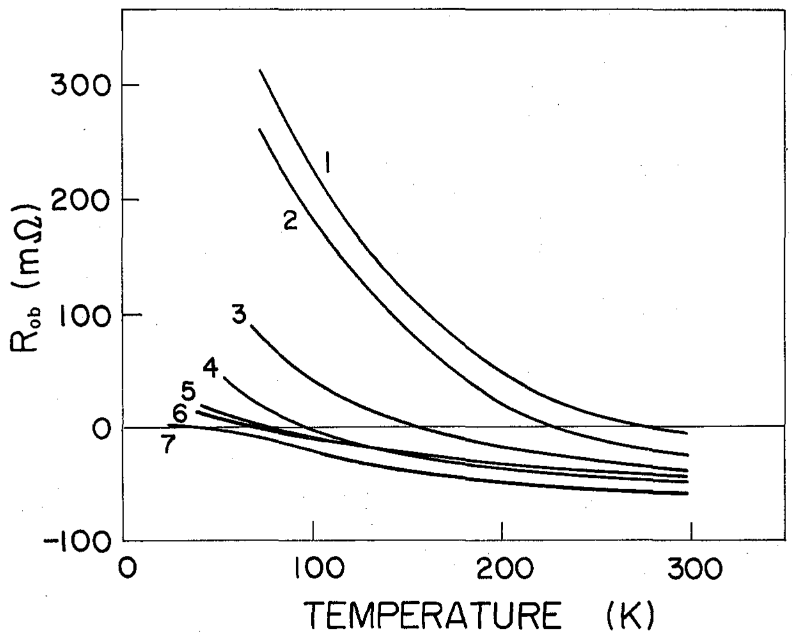

Fig. 2. Observed resistance versus temperature. Numbers on the curves correspond to 7 samples listed in Table I.

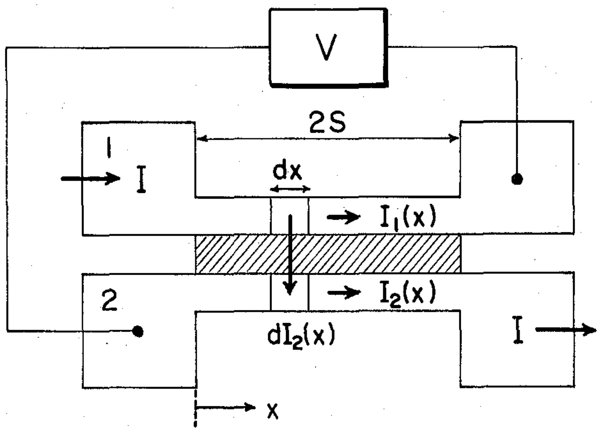

Fig. 3. A schematic arrangement of a tunnel junction. The currents as well as the potentials are functions of position x along the junction.

Therefore, the voltage difference between two layers at x is given by

$$V_1(x)-V_2(x)=2sR_T\frac{dI_2(x)}{dx}, \tag{4}$$

where R_T is the tunneling resistance of the junction.

In the arrangement shown in Fig. 3, the observed voltage is given by $V_M=V_1(2s)-V_2(0)$. From Eqs. (2) and (4), we get

$$V_2(2s)-V_2(0)=-\frac{R_F}{2s}\int_0^{2s}I_2(x)dx, \tag{5}$$

$$V_1(2s)-V_2(2s)=2sR_T\frac{dI_2(x)}{dx}\bigg|_{x=2s}. \tag{6}$$

Thus, the measured voltage is

$$V_M=-\frac{R_F}{2s}\int_0^{2s}I_2(x)dx+2sR_T\frac{dI_2(x)}{dx}\bigg|_{x=2s}. \tag{7}$$

Differentiating Eq. (4) by x and substituting Eqs. (1) and (2), one finds

$$\frac{R_F}{2s}[I_2(x)-I_1(x)]=2sR_T\frac{d^2I_2(x)}{dx^2}. \tag{8}$$

Using $I_1(x)=I-I_2(x)$, we get

$$(D^2-\alpha^2)I_2(x)=-\beta^2, \tag{9}$$

where $D=d/dx$, $\alpha^2=R_F/2s^2R_T$, $\beta^2=(R_F/4s^2R_T)I$. The special solution I_s of Eq. (9) is expressed as

$$I_s=(\beta/\alpha)^2=I/2. \tag{10}$$

On the other hand, a general solution of $(D^2-\alpha^2)I_2(x)=0$ is

$$I_g=p\sinh\alpha x+q\cosh\alpha x, \tag{11}$$

where p and q are constants. Thus,

$$\begin{aligned} I_2(x)&=I_g+I_s \\ &=p\sinh\alpha x+q\cosh\alpha x+I/2, \end{aligned} \tag{12}$$

Using the boundary conditions, $I_2(0)=0$, $I_2(2s)=I$, p and q are given by

$$p=\frac{I}{2}\cdot\frac{\cosh\alpha s}{\sinh\alpha s}, \tag{13}$$

$$q=-\frac{I}{2}. \tag{14}$$

Thus,

$$I_2(x)=\frac{I}{2}\left[\frac{\sinh\alpha(x-s)}{\sinh\alpha s}+1\right]. \tag{15}$$

Integrating and differentiating Eq. (15), one gets

$$\left.\begin{aligned} &\int_0^{2s} I_2(x)\mathrm{d}x=Is, \\ &\left.\frac{\mathrm{d}I_2(x)}{\mathrm{d}x}\right|_{x=2s}=\frac{I\alpha}{2}\,\frac{\cosh\alpha s}{\sinh\alpha s}. \end{aligned}\right\} \tag{16}$$

Substituting Eq. (16) into Eq. (7), the measured voltage V_M is finally given by

$$V_M=I\left[\alpha sR_T\frac{\cosh\alpha s}{\sinh\alpha s}-\frac{R_F}{2}\right]. \tag{17}$$

Thus, the observed resistance V_M/I is

$$R=R_T f(t), \tag{18}$$

where $f(t)=t(\cosh t/\sinh t)-t^2$ and $t=(R_F/2R_T)^{1/2}$.

The function $f(t)$ is shown in Fig. 4, where $f(t)=0$ at $t=1.2$. It is evident that the resistance of tunneling junction becomes negative when $f(t)<0$. In other words, for $R_F\gtrsim 3R_T$, R gives a negative value.

In order to apply above model to our data, it is necessary to determine temperature-independent R_T and temperature-dependence of R_F. To evaluate R_F, we measured the resistance of thin Sn strip (0.3 mm × 0.2 mm × 3000 Å). From the result, we determined R_F of the samples as 352 mΩ at 290 K, 84 mΩ at 77 K, and 12 mΩ at 4.2 K. The last value is considered to be the residual resistance. Assuming R_F is the same for all samples concerned, one can determine R_{F0}, the value of R_F at temperature which gives $R=0$.

As discussed before, $f(t)$ defined in Eq. (18) is zero at $t=1.2$. Therefore, the tunneling resistance R_T of each sample can be deduced by $R_{F0}/2.88$ from the model. Using this R_T, t at 290 and 77 K can be calculated. From Eq. (18), $f(t)$ and R at both temperatures are thus obtained. The calculated values are listed in Table I, as well as the observed ones.

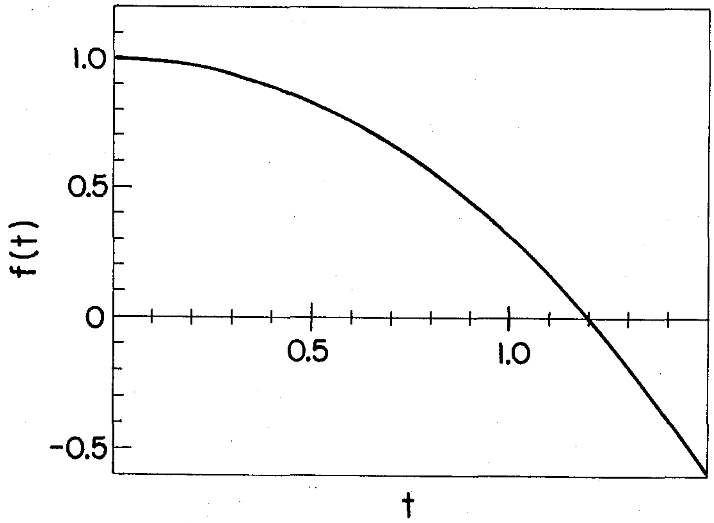

Fig. 4. Calculated $f(t)$ as a function of t.

Table I. Resistances of the tunnel junctions at 290 and 77 K. R_T is the temperature independent tunneling resistance, R is the calculated, and R_{ob} is the observed.

	R_T (mΩ)	290 K			77 K		
		$f(t)$	R (mΩ)	R_{ob} (mΩ)	$f(t)$	R (mΩ)	R_{ob} (mΩ)
1	115	−0.071	−8	−6	0.757	87	292
2	93.1	−0.318	−30	−25	0.696	65	241
3	60.1	−1.098	−66	−40	0.519	31	70
4	36.5	−2.585	−94	−50	0.266	10	15
5	27.8	−3.675	−102	−45	−0.053	−1	−4
6	23.6	−4.698	−111	−46	−0.239	−6	−7
7	13.9	−9.103	−127	−60	−1.177	−16	−11

Comparisons of calculated and experimental results indicate that the Giaever's model qualitatively explains the negative resistance, although quantitative agreement is not sufficient.

2. Magnetic Response

Measurements of the magnetic response of the junctions were carried out at the lowest temperature available, 1.37 K. We found that the I–V curve of samples with large dc Josephson current is remarkably affected by the magnetic field, while others do not give rise to an appreciable difference in the I–V curve in the magnetic field.

To see the effect on the I–V characteristics, therefore, we picked up a sample which has about 67 A/cm^2 dc Josephson current at 1.37 K. In Fig. 5 are shown the observed I–V curves in various dc magnetic field up to 14 G. From the figure, we see the following features:

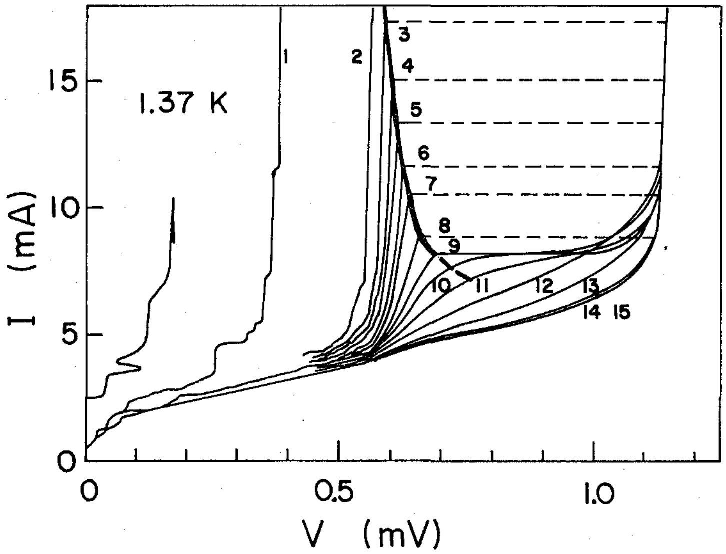

Fig. 5. The change of the current-voltage curve in the dc magnetic field. Numbers in the figure (1–15) are successively assigned as 2.73, 4.09, 4.28, 4.50, 4.77, 4.82, 5.10, 5.37, 5.45, 5.72, 6.13, 6.18, 8.18, 10.9, and 13.6 G.

(a) Since the present measurements are current biased, the voltage jump to $V=2\Delta/e$ takes place at a certain bias current, depending on the magnetic field. And the I–V curve shows hysteresis. However, for $H>\sim5.5$ G, the voltage jump does not occur and the curve becomes smooth without hysteresis.

(b) In the magnetic field larger than ~5.5 G, the excess current becomes appreciable for $V>\Delta/e$.

(c) When H exceeds 10 G, the I–V curve does not change any more, but the small deviation beyond $V=\Delta/e$ from the usual I–V curves remains.

Concerning (a), Eck *et al.*[4] have reported a temperature-independent resonance-shaped peak in the measurement of I–V curves of the Pb Josephson junctions. Their experiment was made by applying known bias voltages to the junction. From the results, they found that the voltage V_p at which the peak occurs is proportional to the magnetic field, and that the height of the peak is proportional to $\sim1/V_p^2$.

In our current biased measurements, the voltage at which the discontinuous jump takes place is reasonably considered to correspond to V_p, and one can compare the present results with those by Eck *et al.*

In Fig. 6 is shown V_p as a function of the applied magnetic field, where we get V_p/H is a constant for $H<\sim4$ G. According to Ref. 4, this constant is expressed by $(ld/\epsilon)^{1/2}$, where l is the barrier thickness, $d=2\lambda+l$, λ is the penetration depth, and ϵ is the dielectric constant of the oxide layer. Assigning $l=15$ Å and $\lambda=350$ Å, the best fit gives $\epsilon=4.5$. Besides, the peak-height (the current where the voltage jump occurs) is a strong function of voltage, which seems to be consistent with Ref. 4.

They also pointed out that as the voltage increases beyond $6\Delta/5$, absorption due to the breaking of pairs can take place and thus the resonance broadens. As seen in Fig. 5,

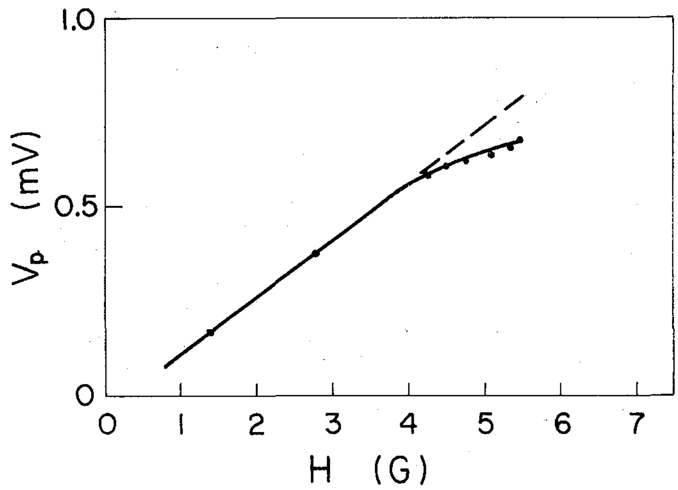

Fig. 6. The voltage V_p at which the peak occurs in the current-voltage curve versus the applied magnetic field.

similar tendency was observed in our measurements, where the I–V curves become smooth without voltage jump when the field increases beyond ~5.5 G.

Concerning (b), it is interesting to note that recently Rajeevakumar and Chen[3] have reported the excess current in the I–V curve of a long Josephson tunnel junction containing supercurrent vortices.[5] This excess current has a magnetic field dependence similar to the absorption curve in magnetic resonances. The excess current increases rapidly at bias voltage Δ/e and the magnitude is comparable to the dc Josephson current.

They pointed out that the excess current they observed is different in several aspects from those by Eck *et al*: First, the field to produce the maximum excess current is nearly the same for $V>\Delta/e$; and second, the excess current is almost independent of voltage for $V>\Delta/e$.

The present preliminary results suggest that for $H>\sim 5.5$ G, I for a fixed V seems to form a resonance like peak, but the amount of I depends on voltage in the region $V>\Delta/e$.

Concerning (c), the small deviation which onsets at $V=\Delta/e$ can evidently be attributed to the double particle tunneling.[2,6] However, it should be noted that the I–V curve in $H=5.45$ G is particular, *i.e.*, for $V=0.7$–1.0 meV, I is exactly constant and no hysteresis appears. This characteristics is difficult to understand by any possible mechanism mentioned above.

From the above discussion, we believe that the present work has revealed the coupling of the Josephson ac current to the electromagnetic modes of the junction. More refined experiment is in progress.

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