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Dead-Time Effect in Multi-Channel Pulse-Height Analyzer

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The dead-time effect of multi-channel pulse-height analyzer in gamma-ray spectroscopy has been studied. The gamma-ray spectra with and without counting loss due to the dead-time effect are generated by the use of Monte Carlo simulation. Calculations are made for single-component gamma-ray emitting nuclides of different half-lives with various source intensities. The calculated resulrs are compared with each other and also with those corrected for the dead-time loss by the live-timer method and by the method of Gavron. The influence of half-life and source intensity as well as measuring period on the dead-time counting loss is discussed and the validity of the correction methods is tested.

KEY WORDS: Dead-Time Effect/ Multi-Channel Pulse-Height Analyzer/ Monte Carlo Method/

I. INTRODUCTION

In gamma-ray spectrometry, the limited time resolution of the analyzers results in counting losses of the pulses from the detectors. The pulses reaching the analyzer while it is busy with the classification of a previous pulse will be lost. It is easy to correct this counting loss, called the dead-time effect, for the single-channel pulse-height analyzer because the dead time is considered to be a function of only the number of pulses per unit time. However, in all commonly used multi-channel pulse-height analyzers (MCPHA), the dead time depends on the height of preceding pulse. Therefore, the dead time changes with the spectrum shape recorded in MCHPA as well as the number of pulses reaching it per unit time. This fact makes it difficult to estimate the dead-time effect in MCPHA.

Most modern MCPHA's have a function of automatic correction for the dead time by using a live timer. The live-time method consists of prolonging the counting period by a period equal to the time in which the MCPHA was busy. This correction is considered to be not valid for short-lived nuclides because the counting rate changes during the measurements. Several methods have been reported to correct the dead-time effect in MCPHA for short-lived nuclides.[1] However, most methods need the accurate value of live time (or dead time) during the measurements and some of them are necessary to perform a separate experiment to measure the live time. On the other hand, Gavron[2] proposed a method to correct the deadtime automatically from the measured spectrum. His simple method does not take into account the variation of the dead time during the measurements, but uses an average value determined from the observed spectrum. This method is very useful

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and especially suitable for the computer analysis of the gamma-ray spectra. However, up to the present there has been no attempt to test this method.

It is the purpose of the present work to study the counting losses due to the dead-time effect in the MCPHA and to test the validity of the correction methods for the nuclides with long and short half-lives compared with the measuring period. Since the most important quantity measured in gamma-ray spectroscopy is the intensity of the photopeak, the counting loss only for the total absorption peak is considered here. The gamma-ray spectra have been produced by computer simulation using the Monte Carlo method. The comparison was made for photopeak intensities with and without the dead-time effect, and for those corrected by the conventional live-teime method and by the method of Gavron.

II. METHOD OF CALCULATIONS

The calculations are based on the following assumptions.

1) The detector is a NaI crystal connected to a 100-channel pulse-height analyzer.
2) The photopeak is recorded at the 100-th channel.
3) A point isotropic source is placed at a distance of 30 cm on the axis of the cylindrical NaI crystal of 5.08-cm diam and 5.08-cm height.
4) The background is neglected.
5) The radioactive source consists of only one kind of nuclide with single gamma-ray component.
6) The gamma-ray energies are between 200 and 300 keV.
7) The dead time in the detector and other electronic system is neglected.
8) The pile-up effect of the pulses is not taken into account.
9) The dead time with a single count in the k-th channel in the MCPHA is a linear function of k.

Since the dead time depends on the measuring gamma-ray spectrum, the source-to-detector geometry should be fixed. In the geometry of the present work, the geometrical efficiency is equal to 1.783×10^{-3}.

From the assumption (5), the spectrum shape is considered to be the same throughout the measurement. On the other hand, the assumption (6) indicates that the influence of the spectrum shape due to the different values of photofraction on the dead-time effect is supposed to be negligible and only the difference between the nuclides used for the calculations is considered to be their half-lives.

When the analyzer is in the Wilkinson mode of operation, the dead time t_d is given by

$$t_d = a + bk , \qquad (1)$$

where k is the channel number, a is a storage time and b is an addressing time per channel. In the whole calculations, we used a and b as 20 μsec and 0.5 μsec, respectively. This value of b is equivalent to 0.05 μsec for a 1000-channel analyzer.

The following notations have been used in the calculations:

T : the real time,

T_L: the live time,
T_C: the counting-period setting of the measurement,
t_i : the time interval between two pulses reaching the MCPHA,
t_d : the dead time due to the preceding pulse,
t_f : the fractional dead time as a function of time,
N : the number of radioactive nuclides at T,
N_0 : the number of radioactive nuclides at $T=0$,
$C_T(k)$: the number of counts in the k-th channel of the real spectrum,
$C_L(k)$: the number of counts in the k-th channel of the measured spectrum,
$C'_L(k)$: the number of counts in the k-th channel of the live-time corrected spectrum,
λ : the decay constant,
γ : the branching ratio of gamma-ray emission,
G : the geometrical efficiency,
D : the detection efficiency,
P : the photopeak efficiency of the NaI crystal.

Here the detection efficiency D is the probability that the photon impinging on the NaI crystal gives rise to an output pulse, and P is defined as the product of G, D, and the photofraction of the crystal. We call the spectrum without the dead-time effect as a real spectrum, and that with the dead-time effect as a measured spectrum.

2.1. Calculation of the gamma-ray spectra with and without the dead-time effect

The flow diagram of calculating the gamma-ray spectra is shown in Fig. 1. First, the time t_i at which the next pulse reaches the MCPHA is calculated. The ordinary decay law can be written as

$$N = N_0 \exp(-\lambda T) , \qquad (2)$$

and the number of decay per unit time is equal to λN. The average time interval between two pulses is given by $1/\lambda NG\gamma$. When we consider a very short time interval, we can assume that these pulses come to the MCPHA randomly. In this case, the distribution of the time interval is given by the exponential distribution with the mean value of $1/\lambda NG\gamma$. The random sampling of the time interval t_i can be made by

$$t_i = -\ln R_i/\lambda NG\gamma , \qquad (3)$$

where R_i is a random number uniformly distributed in the interval [0, 1]. Using Eqs. (2) and (3), the value of t_i is determined and the real time T is increased by t_i.

Next, the Monte Carlo calculation is made for the incident photon. The incident direction of the photon to the crystal surface is determined and behaviour of this photon in the NaI crystal is traced by the conventional Monte Carlo method for the detector response.[3] For simplicity, the secondary electrons produced in the crystal (photoelectrons, and Compton-recoiled electrons) are not traced. The bremsstrahlung radiations from these electrons are also neglected. Both effects do not

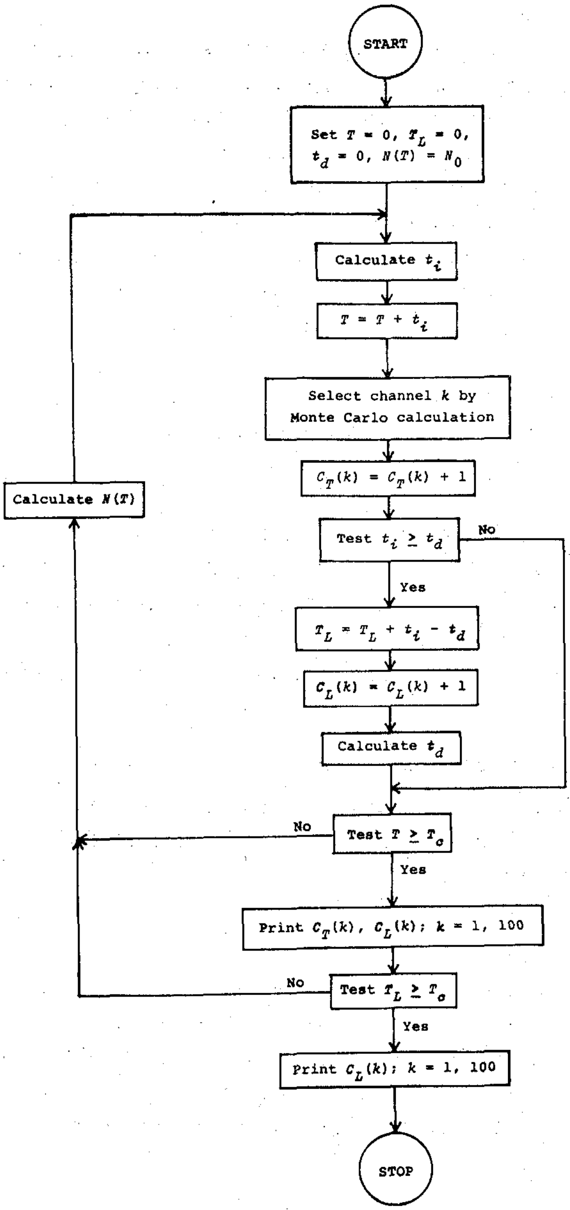

Fig. 1. Flow diagram of calculation of gamma-ray spectra.

affect so much the calculated spectrum in the energy region considered here. The channel number k at which the photon is recorded is determined by the energy absorbed in the crystal.

If t_i is larger than t_d, this pulse is considered to be measured by the MCPHA. The livetime T_L is increased by the amount of $t_i - t_d$ and the content in the k-th channel in the measured spectrum is added by one. The new dead time corresponding to this channel number is determined from Eq. (1) and the program moves to the next event.

On the other hand, if t_i is less than t_d, this pulse is not recorded by the MCPHA. Then the dead time t_d is replaced by $t_d - t_i$ and the program continues the calculation. When the real time T is equal to the measuring period T_C, the computer prints out the real spectrum without the dead-time effect and the measured spectrum with the dead-time counting loss. The whole calculation is terminated when the live time T_L becomes larger than T_C.

Thus three kinds of spectra, *i.e.* (1) the real spectrum, (2) the measured spectrum, and (3) the live-time corrected spectrum, are obtained for one calculation.

2.2. The Dead-Time Correction

Provided that the spectrum shape does not change during the measurement, the counting rate at each channel is proportional to the decay rate of the radioactive nuclide at that time. When there is no dead-time effect (the real spectrum), the number of counts in the peak channel at $T = T_C$, $C_T(100)$, can be given as

$$\begin{aligned} C_T(100) &= P\gamma \int_0^{T_C} \lambda N \, \mathrm{d}t \\ &= A_0 [1 - \exp(-\lambda T_C)] \, , \end{aligned} \tag{4}$$

where

$$A_0 = P\gamma N_0 \, .$$

On the other hand, if we take into account the dead-time effect, the pulses reaching the MCPHA are not recorded with a certain probability. We call this probability as a *fractional dead time*, $t_f(t)$. This quantity is a function of the time and corresponds to the probability of the analyzer being busy at time t. The number of peak counts at $T = T_C$ including the dead-time effect, $C_L(100)$, is given by

$$\begin{aligned} C_L(100) &= P\gamma \int_0^{T_C} \lambda N [1 - t_f(t)] \, \mathrm{d}t \\ &= A_0 \lambda \int_0^{T_C} \exp(-\lambda t) \, [1 - t_f(t)] \, \mathrm{d}t \, . \end{aligned} \tag{5}$$

Gavron[2] proposed to use an average value of $t_f(t)$ in Eq. (5). When $T = T_{C'}$ the average value of the fractional dead time during the measurement is expressed as

$$\bar{t}_f = \sum_{k=1}^{100} (a + b\,k) C_L(k) / T_C \, . \tag{6}$$

This value can be determined from the measured spectrum. Replacing $t_f(t)$ in Eq. (5) by $\bar{t}_f$, Eq. (5) is written as

$$\begin{aligned} C_L(100) &= A_0 \lambda [1 - \bar{t}_f] \int_0^{T_C} \exp(-\lambda t) \, \mathrm{d}t \\ &= A_0 [1 - \bar{t}_f] \, [1 - \exp(-\lambda T_C)] \, . \end{aligned} \tag{7}$$

Comparing Eqs. (4) and (7), the number of counts in the peak channel of the real spectrum can be estimated from the measured spectrum by the following formula:

$$C_T''(100) = C_L(100)/(1-\bar{t}_f)\,. \tag{8}$$

III. RESULTS AND DISCUSSION

The calculations of the gamma-ray spectra have been made for radioactive nuclides with different values of half-lives and with different source intensities. The measuring period T_C is chosen to be 5 min. Taking into account gamma-ray energies, three nulcides, ^{203}Hg, ^{51}Ti and ^{179m}Hf, are chosen. Their half-lives $T_{1/2}$ are longer, almost equal and shorter, as compared with T_C, respectively. The nuclear properties of these nuclides are taken from the table prepared by Lederer *et al.*[4] and listed in Table I. All the calculations in the present work have been performed on the FACOM M-200 computer in the Data Processing Center of Kyoto University.

Table I. Properties of the nuclides used for the present work (Taken from Ref. 4).

Nuclide	$T_{1/2}$	$E\gamma$ (keV)	γ (%)
^{179m}Hf	18.6 sec	217	94
^{51}Ti	5.8 min	319.8	95
^{203}Hg	46.9 day	279	77

The source intensities are determined to be between 0.5 and 500 μCi from the values of Table I and the geometrical efficiency G. By this choice of intensity, the number of histories traced is larger than 5000 for all the cases studied, except for only one case. This fact indicates that the statistical fluctuation of the number of counts at the peak channel is in general better than 1%. The exceptional case is the 1-μCi source of ^{179m}Hf, where the number of histories is about 1700. However, for such a small number of photons impinging on the detector during the measuring period, the counting loss due to the dead time is found to be practically negligible. Since we are interested in the relative change in the counting rates with and without the dead-time effect, the small number of histories, *i.e.* poor statistics of the peak counts, is not important in this case.

In order to compare the dead-time effect for various source intensities with different half-lives, it is advantageous to introduce a scaled source intensity. At the beginning of the measurement, the average time interval between two photons impinging on the detector surface is given by $1/N_0G\gamma\lambda$. We define the scaled source intensity as the ratio of the average dead time of the MCPHA to this average time interval:

$$S = N_0 G\gamma\lambda\bar{t}_d\,. \tag{9}$$

In other word, S is the product of the number of incident pohtons at the beginning of the measurement, $N_0G\gamma\lambda$, and the average dead time. For $\bar{t}_d$, we use the dead time at the middle channel of the MCPHA, *i.e.* t_d at $k=50$ in Eq. (1).

In Fig. 2, the relative photopeak intensities of the gamma-ray spectra from

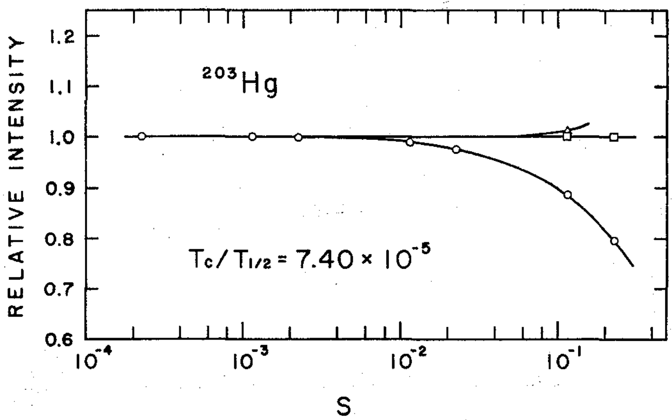

Fig. 2. Relative photopeak intensities for ^{203}Hg as a function of scaled source intensity. The measuring period is taken to be $T_C=$ 300 sec. The circles represent the peak intensities of the measured spectra, the squares are those of the live-time corrected spectra and the triangles indicate those of the Gavron correction. For small S values, all the points coincide with each other.

^{203}Hg, as the ratio to the peak intensity of the real spectrum, are plotted against the scaled source intensity. The circles represent the relative photopeak intensities of the measured spectra, $C_L(100)/C_T(100)$, and the squares are those with the live-time correction, $C'_L(100)/C_T(100)$. The triangles indicate those of the measured spectra corrected by the Gavron method, $C'_T(100)/C_T(100)$. The solid curves are drawn only to guide the eye. It can be seen that owing to the dead-time effect the measured peak intensity decreases with increasing the scaled source intensity S. The reduction becomes appreicable for $S>0.01$.

For low source intensities, these three values coincide with each other and equal to unity within the limit of statistical fluctuation. This means that the dead-time effect is negligible when the source intensity is low and all the spectra are the same as the real spectrum.

The half-life of this nuclide is very long compared with the measuring period, *i.e.* the *reduced* measuring period is $T_C/T_{1/2}\ll 1$. In this case, the change in the source intensity during the measurement is negligibly small. It is clear from the figure that for such a case the live-time correction method reproduces the real spectrum very well. The Gavron method can also yield the same result as the live-time method for $S\leqq 0.1$ and well reproduces the real spectrum. However, for higher source intensities this method tends to overcorrect the dead-time effect and gives the higher photopeak intensities.

Figure 3 shows the similar comparison of relative intensities for ^{51}Ti, where $T_C/T_{1/2}\simeq 1$. In this case, the decrease in $C_L(100)$ due to the dead-time counting loss becomes clear for $S\geqq 0.01$, similarly as in Fig. 2. The live-time correction gives a good estimate for the photopeak intensity of the real spectrum up to $S\simeq 0.1$, but underperdicts for higher source intensities. On the other hand, the Gavron method

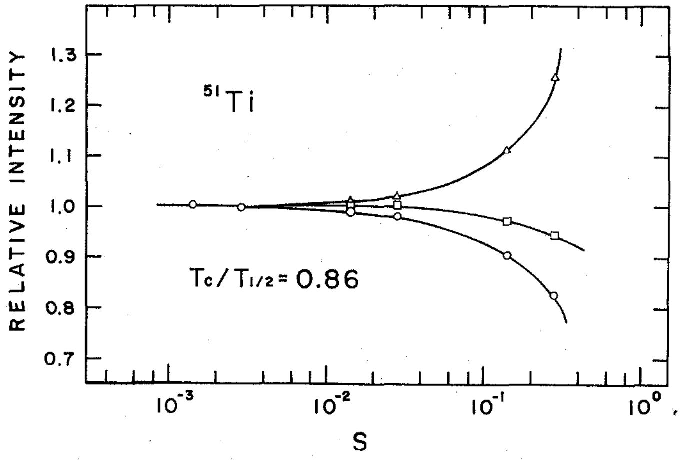

Fig. 3. Same as Fig. 2, but for ^{51}Ti.

overpredicts the peak intensities of the real spectra and this overcorrection increases rapidly for $S > 4 \times 10^{-2}$.

In Fig. 4 the comparison of the relative photopeak intensities is given for ^{179m}Hf. The half-life of this nuclide is short and $T_C/T_{1/2} \gg 1$. In this condition, the live-time method is of no use. It gives the same results as the measured spectra, because the radioactive source decays out during the measurement. It is clear from the figure that reduction of the measured source intensity takes place for $S > 0.01$. This behavior is quite similar to other cases studied above. The correction due to the Gavron method is very good up to $S \simeq 0.1$, but for higher source intensities this method overpredicts the true photopeak intensity considerably.

It can be seen that there is a large difference between Figs. 3 and 4. For $T_C/T_{1/2} \simeq 1$, the live-time method is good as long as S is smaller than 0.1, while it is

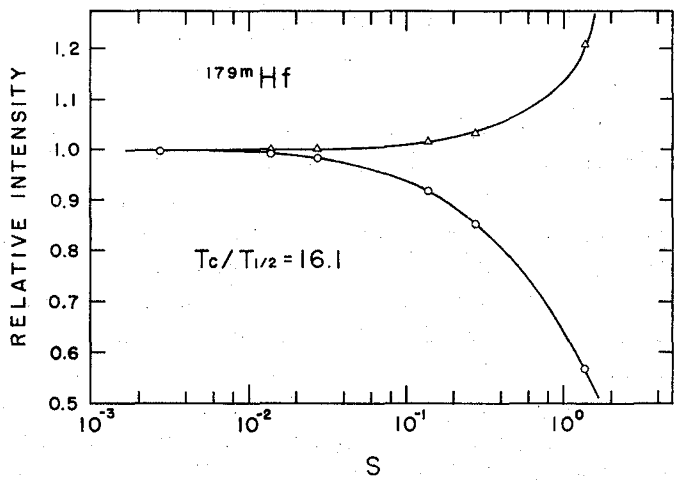

Fig. 4. Same as Fig. 2, but for ^{189m}Hf. The points for the live-time correction (□) coincide with those for the measured spectra (○).

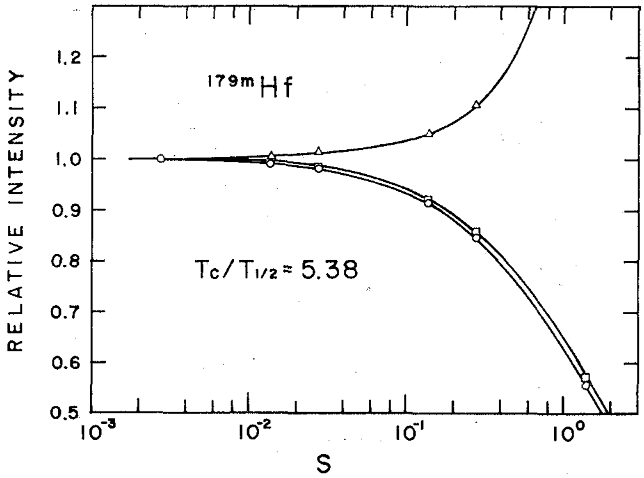

Fig. 5. Same as Fig. 2, but for ^{179m}Hf and for $T_C=100$ sec.

invalid foı $T_C/T_{1/2}\gg 1$ and gives no dead-time correction. Furthermore, the overestimation of the Gavron method is larger for $T_C/T_{1/2}\simeq 1$ than for $T_C/T_{1/2}\gg 1$. Considering these facts, it is interesting to study the dead-time effect for the intermediate $T_C/T_{1/2}$ value. For this purpose, the calculations have been made for ^{179m}Hf and for $T_C=100$ sec. This corresponds to the reduced measuring period $T_C/T_{1/2}=$ 5.38. The calculated results are shown in Fig. 5. As expected, the behaviors of the two correction methods are intermediate between Figs. 3 and 4. However, it should be noted that the live-time method only silghtly improves the measured results and still underestimates the peak intensities considerably for high S values. On the other hand, the magnitude of overcorrection due to the Gavron method does not change so much from Fig. 4 and this method gives good estimate of the true peak intensity for $S<0.1$. All the results obtained above indicate that the dead-time effect strongly depends on the measuring period as well as the source intensity.

In all the previous cases, we fixed the measuring period T_C and calculated the relative photopeak intensities as a function of the source intensity S. In order to study the dead-time effect in more detail, it is useful to calculate the relative peak intensity as a function of T_C with fixed S. Fig. 6 shows the results fcr 100-μCi source of ^{179m}Hf. This source intensity corresponds to $S=0.279$. It is interesting to note that the peak intensity of the measured spectrum increases with increasing T_C and reaches to a constant value. This can be explained as follows. For small T_C values, the source intensity during the measurement is high and the counting loss due to the dead-time effect is large. When T_C is increased, the difference of the source intensities at the beginning and at the end of the measurement becomes large. The dead-time counting loss is large at the beginning, but small at the end. Therefore, the fraction of counting loss to the total counts decreases with increasing T_C and the measured peak intensity increases with increasing T_C. When T_C becomes considerably larger than $T_{1/2}$, the radioactive source almost decay out during the measurement and the measured peak intensity remains to be constant for larger T_C.

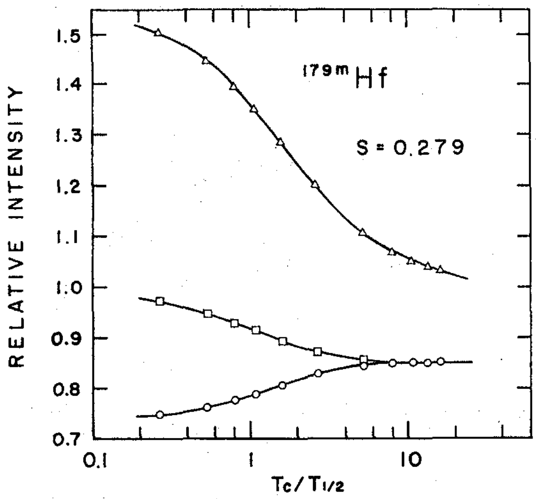

Fig. 6. Relative photopeak intensities for ^{179m}Hf as a function of the reduced measuring period $T_C/T_{1/2}$. The source intensity is 100 μCi, which corresponds to $S=0.279$.

It is clear from the figure that the live-time method gives a good estimate of the true peak intensity for small $T_C/T_{1/2}$ values. This result seems to be surprising because this method is usually believed to be invalid for short-lived nuclides. However, it should be noted that the important factor for the validity of the live-time method is the reduced measuring period, $T_C/T_{1/2}$, and not the half-life, $T_{1/2}$. Even when $T_{1/2}$ is small, we can obtain a good estimate of the peak intensity by the use of the live-time method if we choose T_C so as to be $T_C/T_{1/2}<1$ and if, even so, the counting statistics is good. The peak intensity estimated from this method decreases with increasing T_C and approaches to the value of the measured spectrum, as shown in Figs. 4 and 5.

In contrast, the Gavron method substantially overpredicts the true peak intensity for small $T_C/T_{1/2}$ values. With increasing T_C, the estimated peak intensity decreases slowly and approaches to the true value for large $T_C/T_{1/2}$. These facts indicate that this method is useful for the case of large $T_C/T_{1/2}$ values.

IV. CONCLUSION

The dead-time effect in multi-channel pulse-height analyzer has been studied by the use of the Monte Carlo method. In order to compare the various cases with different half-live nuclides, source intensities and measuring periods, the scaled source intensity and the reduced measuring period are introduced. It is found that the dead-time counting loss becomes appreciable when the sacled scurce intensity S is larger than 0.01 and the correction for the dead-time effect is necessary for such a case. The live-time correction usually underpredicts the peak intensity, while the Gavron method in general overestimates it. In the case of $T_C/T_{1/2}\ll 1$, the live-time method is very good for all S values, while the Gavron method is valid up to

$S=0.1$. When $T_C/T_{1/2}\simeq 1$, the live-time method gives a satisfactory result for the true peak intensity for the scaled source intensity less than 0.1. On the other hand, for $T_C/T_{1/2}\gg 1$ the Gavron method yields a good approximate value of the true peak intensity for $S<0.1$. When the scaled source intensity is higher than 0.1, both methods are insufficient to correct the dead-time effect for $T_C/T_{1/2}\geqq 1$.

The present calculations have been performed for NaI crystal, for fixed geometry, for limited gamma-ray energy region and for fixed values of a and b in Eq. (1). However, the results obtained above are expressed in terms of the scaled source intensity and the reduced measuring period. This fact suggests that the derived conclusion can be genrealized.

In the present work, the radioactive source is assumed to contain only one kind of nuclide. When the source consists of many kinds of nuclides with different half-lives, the spectrum shape changes with time during the measurement. It is very interesting to study the dead-time effect for such cases by the use of the present method.

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