Bull. Inst. Chem. Res., Kyoto Univ., Vol. 64, No. 1, 1986

Study of Permanent Magnet Quadrupole Lens For Proton Linac

H. OKAMOTO and Y. IWASHITA

Received February **4, 1986**

Three types of permanent magnet quadrupole lens were fabracated and the harmonic fields in the aperture were measured. Two of them are the trapezoidally segmented geometries and another has rectangular segments. The bore field was measured with a rotating coil and the harmonic field components were obtained.

KEY WORDS: Permanent magnet quadrupole (PMQ)/ Proton drift tube linear accelerator (proton DTL)/ Harmonic field components/

1. INTRODUCTION

In general, the operating frequency of proton linear accelerator is about 200 MHz and conventional electromagnets have been used as the focusing system of proton beam. But in proton drift tube linear accelerator (proton DTL) operated at higher frequency to get higher shunt impedance, the size of a drift tube is so small that electromagnet is not suitable for practical use. Recently K. Halbach proposed the very compact focusing lens using rare earth cobalt permanent magnet called permanent magnet quadrupole lens (PMQ)[1]. By using PMQ it is possible to create a very strong quadrupole field in the small aperture and its outer size is much smaller than that of

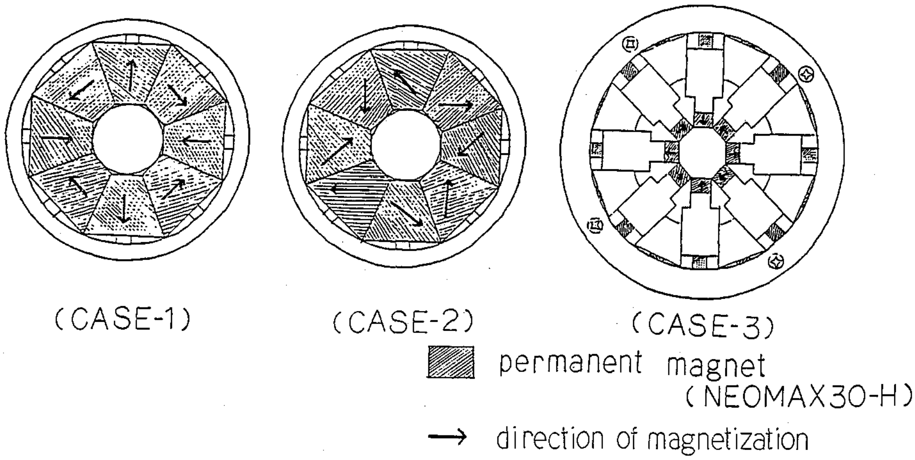

Fig. 1. Cross sections of three types of PMQ models. (a), (b) trapezoidally segmented PMQ, (c) rectangular-segment PMQ.

岡本宏己: Facility of Nuclear Science Research, Institute for Chemical Research, Kyoto University.
岩下芳久: Laboratory of Nuclear Reaction, Institute for Chemical Research, Kyoto University.

the electromagnet of conventional type. Furthermore it is necessary to supply the electric power for the electromagnet and the cooling system for the coil is required but they are not necessary for PMQ, so the structure of the drift tube becomes simple.

Three types of PMQs shown in Fig. 1 were fabricated and their bore fields were measured by the rotating coil to obtain the harmonic field components of them[3]. Each of three PMQs is consisted of eight pieces. PMQ named CASE-1 and CASE-2 are trapezoidally segmented geometries. The difference between them is 45 degree rotation of the easy axis of each piece. The advantage of CASE-2 is that only one kind of segment is needed in terms of easy axis. PMQ named CASE-3 has the rectangularly segmented geometry and the direction of magnetization is the same as CASE-1 and the position of each piece is adjustable by movable holding system. The permanent magnet material of these PMQs called NEOMAX30-H, which consists mainly of neodymium, iron and boron, was developed by Sumitomo Special Metals co., LTD. NEOMAX30-H has a very strong remanent field (11.2–11.9 kG) and its B–H curve is very linear within the desired range[4].

2. HARMONIC FIELD COMPONENTS OF PMQ

The bore fields of three kinds of PMQs given by Halbach are as follows ($n \geq 2$)

(a) trapezoidally segmented PMQ

$$\underline{B}^*(\underline{z}_0)=\underline{Br}\sum_{\nu=0}^{\infty}\left(\frac{\underline{z}_0}{r_1}\right)^{n-1}\frac{n}{n-1}\left[1-\left(\frac{r_1}{r_2}\right)^{n-1}\right]\cos^n\left(\frac{\pi}{M}\right)\frac{\sin(n\pi/M)}{n\pi/M} \tag{1}$$

(b) rectangularly segmented PMQ

$$\underline{B}^*(\underline{z}_0)=\underline{Br}\sum_{\nu=0}^{\infty}\left(\frac{\underline{z}_0}{r_1}\right)^{n-1}\frac{n}{n-1}\cdot$$

$$\frac{1}{n\pi/M}\left[\sin(n-1)\alpha\cdot\cos^{n-1}\alpha-\left(\frac{r_1}{r_2}\right)^{n-1}\sin(n-1)\beta\cdot\cos^{n-1}\beta\right] \tag{2}$$

$$\alpha=\frac{\pi}{M},\quad \beta=\frac{r_1}{r_2}\tan\alpha$$

$$(n=N+\nu M,\ \nu=0,\ 1,\ 2,\ \ldots)$$

where r_1 is the bore radius, r_2 is outer radius of PMQ, M is the number of segments, $\underline{Br}$ is the remanent field of the permanent magnet material and $\underline{z}_0$ is a point on the complex plane defined by $\underline{z}_0=x_0+iy_0=re^{i\phi}$, with $i^2=-1$. Complex numbers are identified by underlining the symbols and the complex conjugate of a quantity is indicated by an asterisk. N is the order of the desired multipole such as $N=2$ for quadrupole, $N=3$ for sextupole and so on. The quadrupole field gradient of CASE-1, 2, 3 can be calculated by using equation (1), (2). Putting Br=11.9 kG and suppose the slope of B–H curve is 1.1, the following values are obtained.

<CASE-1, CASE-2> ……24 kG/cm

<CASE-3> ……12 kG/cm

3. MEASUREMENT METHOD OF HARMONIC FIELD

The harmonic fields in the aperture of PMQs were measured by the rotating coil method. The outline of this method is as follows: The search coil is rotated in the aperture of PMQ and the induced voltage from the coil is measured and A–D converted to record. Then these data are analyzed by Fast Fourier Transformation (FFT) to obtain the amplitudes of the harmonic field components.

The data are transformed into complex Fourier components by FFT, so the amplitudes of n-th harmonic field component have the real part Fcn and the imaginary part Fsn. The center of the quadrupole field where the dipole component vanishes is calculated numerically by the following equation.

$$\underline{\Delta R}=\Delta x+i\Delta y=-r_{\mathrm{c}}\left(\frac{\underline{F}_1^*}{\underline{F}_2^*}+\sum_{\mathrm{n}=1}^{\infty}\frac{\underline{F}_{\mathrm{n}1}^*}{\underline{F}_{\mathrm{n}2}^*}\right) \tag{3}$$

$$\underline{F}_{\mathrm{nm}}^*=\left(\frac{r_{\mathrm{c}}}{\underline{z_{\mathrm{n}}}}\right)^{\mathrm{m}-1}\sum_{l=\mathrm{m}}^{\infty}\underline{F}_l^*\ {}_{l-1}C_{\mathrm{m}-1}\left(\frac{\underline{z_{\mathrm{n}}}}{r_{\mathrm{c}}}\right)^{l-1}$$

$$\left(\underline{F}_{\mathrm{n}}=F_{\mathrm{cn}}+iF_{\mathrm{sn}},\ \underline{z_{\mathrm{n}}}=-r_{\mathrm{c}}\sum_{\mathrm{k}=1}^{\mathrm{n}}\frac{\underline{F}_{\mathrm{k}-1,\,1}^*}{\underline{F}_{\mathrm{k}-1,\,2}^*},\ \underline{F}_{\mathrm{on}}=\underline{F}_{\mathrm{n}}\right)$$

where $\underline{\Delta R}$ is the position of quadrupole field center on the complex plane whose origin is the rotating axis of the search coil, and r_c is the rotating radius of search coil, which is equal to the width of coil.

4. MEASUREMENTS

The percentage values of harmonic field amplitudes $|\underline{F}\mathrm{n}|/|\underline{F}_2|$ are shown in Fig. 2 and Fig. 3. The values shown in Fig. 2 come from measured data by search coil

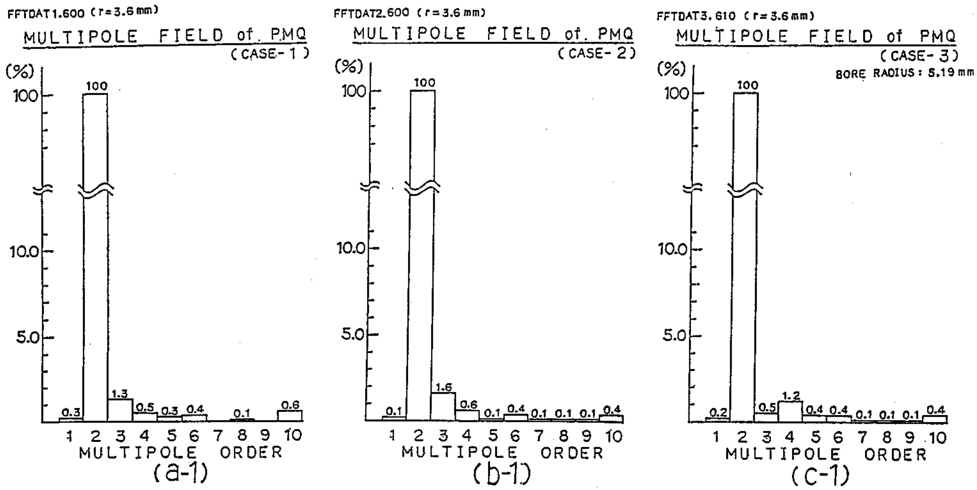

Fig. 2. Measured harmonic field companents in PMQ aperture. (a–1) CASE–1, (b–1) CASE–2, (c–1) CASE–3.

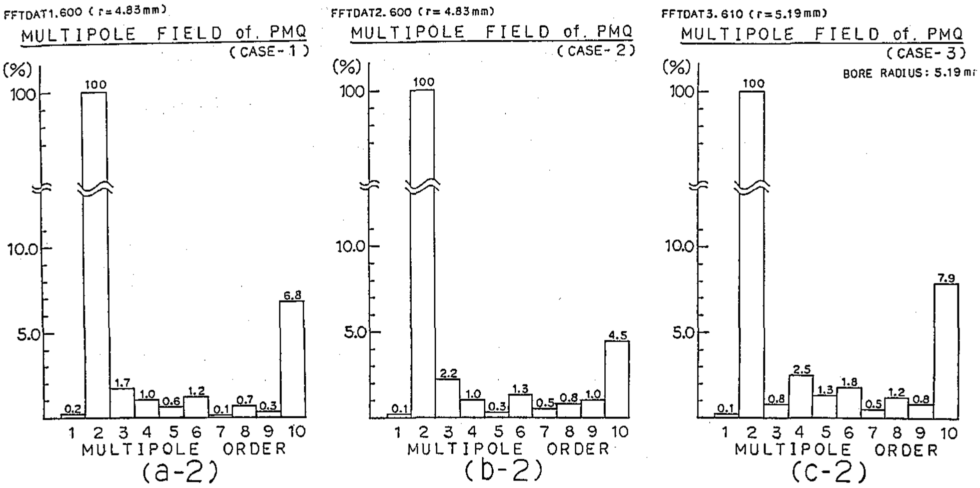

Fig. 3. Converted harmonic field components at the bore radius (a–2) CASE–1, (b–2) CASE–2, (c–2) CASE–3.

when the coil position is adjusted to minimize the dipole component. Fig. 3 shows the converted values at the bore radius. The amplituds of harmonic field components higher than quadrupole are very small.

To minimize the dipole field amplitude, measurements had to be repeated many times at the various coil position, but by using equation (3) quadrupole field center can be obtained from only one measurement. The values obtained from the calculation at geometrical center are as follows:

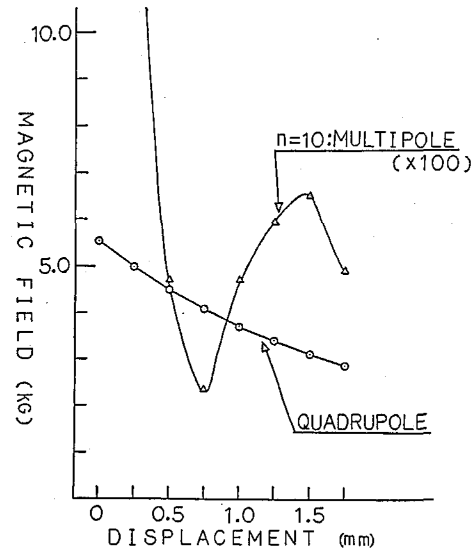

Fig. 4

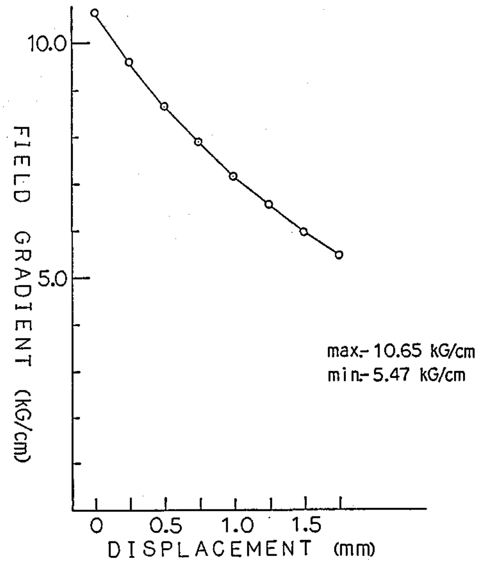

Fig. 5

Fig. 4. Magnetic field strengths of quadrupole component and 10-th harmonic component in CASE–3 aperture vs. the displacements of eight pieces.

Fig. 5. Quadrupole field gradient of CASE–3 vs. the displacements of eight pieces.

$$<\text{CASE-1}> \quad \Delta x=41.4\ \mu\text{m} \quad \Delta y=15.6\ \mu\text{m} \quad (|\underline{\Delta R}|=44.2\ \mu\text{m})$$
$$<\text{CASE-2}> \quad \Delta x=89.2\ \mu\text{m} \quad \Delta y=63.1\ \mu\text{m} \quad (|\underline{\Delta R}|=109.3\ \mu\text{m})$$

These values of $|\underline{\Delta R}|$ almost agree with the result obtained by measurements. Quadrupole field gradients obtained by measurements are as follows:

$$<\text{CASE-1}> \quad 22.1\ \text{kG/cm}, \quad <\text{CASE-2}> \ 21.1\ \text{kG/cm}$$

The positions of all pieces are adjustable in CASE-3. The change of the field strength to the displacements of eight pieces are shown in Fig. 4. In this figure 10-th harmonic field strength is 100 times magnified compared to quadrupole. Fig. 5 shows the change of quadrupole field gradient. It turns out that quadrupole field reduces gradually and the strength of 10-th harmonic field has minimum value. The quadrupole field gradient could be changed in wide range. These results of measurements agree with the theorical calculations.

5. DISCUSSION

According to the computer simulation of beam dynamics, required quadrupole field gradient to focus the proton beam is less then about 20 kG/cm.[2] CASE-1 and CASE-2, whose sizes are practical for proton DTL operated at higher frequency, have achieved more than 20 kG/cm of field gradient and the harmonic fields of higher order are very small. Also CASE-3 is promising where the required quadrupole field is not strong because its quadrupole field gradient is adjustable and harmonic field components of higher order can be optimized. Consequently PMQ like CASE-1, 2, 3 is quite useful for proton DTL to focus the beam.

REFERENCES

(1) K. Halback, *Nucl. Instr. Meth.*, **169** (1980) 1.
(2) L. D. Hansbrough, Compiler, "PIGMI: A Design Report for a Pion Generator for Medical Irradiations", LANL report LA-8880 (1981).
(3) Y. Ikeda, Y. Katayama, Y. Iwashita, *Bull. Inst. Chem. Res. Kyoto Univ.* **62**, No. 1 (1984).
(4) Sumitomo Special Metals Co., LTD. Data of NEOMAX.
(5) L. Smith, R. Gluckstern, *Rev. Sci. Instr.* vol. **26**, No. 2 (1955).
(6) J. E. Stovall, MP. Division Internal Report, MP-3-38, 3 (1968) LANL.