Modified London Penetration Depth in Oxide Superconductors

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London penetration depth $\lambda_L$ was calculated for superconductors having the lifetime broadened Bardeen-Cooper-Schrieffer (BCS) density of states. With the increase of gap states, $\lambda_L$ increases and the temperature dependence deviates from the ordinary BCS form. The model gives a $T^2$ dependence of $\lambda_L$ in the case of weak-coupling BCS. The results are discussed referring some experimental results of magnetic field penetration depth of oxide superconductors.

KEY WORDS: Field Penetration Depth/ Superconductor/

Discoveries of new superconductors having higher transition temperature $T_c$ have been rewritten by pioneers1-3) very quickly since Bednorz and Muller4) found the superconductivity with $T_c=40$ K in La-Sr-Cu-O system.

Surprisingly all the new superconductors are Cu-O based materials and have the superlattice structure. However, the fundamental mechanisms in the new system have not been uncovered. Although the direct probe for the pairing mechanism is the gap parameter $\Delta$, clear-cut consensus has not been established yet. As a matter of fact, there is a big puzzle between the infrared5) and tunnel6) measurements of $\Delta$. The former suggests the weak coupling BCS, contrarily the latter appeals the strong coupling BCS. The magnetic field penetration depths $\lambda(T)$ which have been reported from several groups are also controversial. Muon-spin relaxation measurements for YBa$_2$Cu$_4$O$_8$ (YBCO) gives $\lambda(T) = \lambda(0) (1 - (T/T_c)^4)^{-1/8}$, being similar to an ordinary BCS superconductor.7,8) On the other hand, $\lambda(T)$ determined from the susceptibility measurements with powdered specimens deviates from the above relation. Cooper et al.9) derived a $T^4$ law for YBCO. However, Ishida and Mazaki reported a $T^3$ law for YBCO10) and a $T^2$ law for La$_{2-x}$Sr$_x$CuO$_4$ (LSCO).11)

As a possible explanation for the discrepancy among these experimental values of $\lambda$, we report here the calculations of the London penetration depth $\lambda_L$ in the framework of the BCS theory, where we use the broadened density of states,12) and attempt to find how the temperature dependence of $\lambda_L$ alters with the broadening parameter. Generally speaking, the London penetration depth is valid for the local limit materials which have a very short coherence length $\xi$ compared to the $\lambda$. In the case of YBCO and LSCO, the reported values of $\xi/\lambda$ are much less than...
Therefore we think the application of $\lambda_L$ to these materials is not unreasonable. Meanwhile, all the samples so far measured to determine $\lambda(T)$ are sintered polycrystalline, which are expected to have the broadened BCS density of states. Considering these features, we believe that the discussion on $\lambda_L$ which involves the broadened BCS density of states should be meaningful.

London penetration $\lambda_L$ at a temperature $T$ is expressed by\(^{35}\)

$$\lambda_L^{-2}(T) = \lambda_L^{-2}(0) \left[ 1 - 2 \int_0^\infty \left( -\partial f(E) / \partial E \right) N_s(E) dE \right], \quad (1)$$

where $N_s(E)$ is the BCS density of states, $f(E)$ is the Fermi function. Instead of $N_s(E)$ in Eq. (1), we use a broadened BCS density of states

$$N_s(E, \Gamma) = \text{Re} \left\{ \frac{E - i\Gamma}{[(E - i\Gamma)^2 - \Delta^2]^{1/2}} \right\} \quad (2)$$

where $\Gamma$ is the broadening parameter which represents the existence of energy states within the energy gap. This modified density of states has succeeded in the disordered superconductors. $\Gamma$ increases with the increase of resistivity due to the inelastic scattering processes. The density of states $N_s(E, \Gamma)$ with various $\Gamma/\Delta$ are shown in Fig. 1. Note that $N_s(E, \Gamma)$ increases with the increase of $\Gamma$. Although the gap parameter is chosen to be 13.7 meV which corresponds to the weak-coupling BCS case with $T_c=90$ K, other values also reproduce almost the same feature. The increase of the gap states which means more quasiparticle excitation results in the increase of $\lambda(T, \Gamma)$. This feature can be clearly seen in Fig. 2, where $\Gamma/\lambda(0)$ is kept in the range of granular aluminum and $T=0$. This tendency demonstrated in Fig. 2 seems to have been reflected in some experimental results. $\lambda(0)$ reported for sintered polycrystalline YBCO\(^0\) is larger than that of single crystal YBCO.\(^{35}\) The former value is 6200 Å and the latter gives 1250 Å and 260 Å which are measured in Cu-O planes parallel and perpendicular to the applied field respectively.

\[Fig. 1. \text{ Lifetime broadening BCS density of states } N_s(E, \Gamma) \text{ with various } \Gamma/\Delta.\]
Modified London Penetration Depth of Oxide Superconductors

![Graph](image)

**Fig. 2.** London penetration depth at $T=0$ as a function of the broadening parameter.

According to the work with $\text{Bi}_{0.4}\text{Pb}_{0.9}$, $\Gamma$ has an intrinsic value at low temperatures, but above a certain temperature it increases with the increase of $T$. Qualitatively speaking, $\lambda_L(T, \Gamma)$ increases rather rapidly as $\Gamma$ increases. Since there has been no information on the temperature dependence of $\Gamma$ for oxide superconductors, in the present calculation of $\lambda_L(T, \Gamma)$, $\Gamma$ is assumed to be temperature independent. Besides, $\Delta(T)$ is assumed to hold the BCS form.

In Fig. 3, we show calculated $\lambda_L(T, \Gamma)$ as well as some experimental values of $\lambda$. The results indicate that with the increase of $\Gamma$, the temperature dependence of $\lambda_L^{-2}(T, \Gamma)$ deviates more and more from the ordinary function with $\Gamma=0$. Taking into account that in the practical case $\Gamma/\Delta(0)$ may not exceed 0.5, we can see that $\lambda_L^{-2}$ for the strong coupling case $2\Delta(0)/k_B T_c = 5.1$ (corresponding to $T_c=90\ K$, $\Delta(0)=20$}

![Graph](image)

**Fig. 3.** Comparisons between experimental and theoretical temperature dependences of field penetration depth. Two sets of solid lines (SC and WC) are theoretical for the cases of strong- and weak-coupling BCS. Solid circles and triangles represent data of $\text{YBCO}^{33}$ and of $\text{LSCO}^{6}$. The dashed line is a $T^1$ law obtained for $\text{YBCO}^{33}$. (47)
meV) reproduces neither the $T^1$ nor $T^2$ dependence of $\lambda^{-2}$ found from the susceptibility measurements\textsuperscript{9-11}. On the contrary, for the weak coupling case $2d(0)/k_BT_c = 3.5$, $\lambda^{-2}$ successfully retraces the experimental $T^2$ relation in a wide range above $T/T_c = 0.1$, but a $T^1$ law of $\lambda^{-2}$ cannot be obtained.

Another modified BCS density of states which has impurity states in the energy gap for example might explain such a dependence. However, the processes of deriving $\lambda_L(T, \Gamma)$ are too simple for the oxide superconductors having the anisotropic nature in superconductivity. Further discussion therefore seems not to be worthwhile. We just suggest that the states in the gap due to any imperfection in samples gives rise to a change in $\lambda_L(T, \Gamma)$ even within the framework of the BCS theory. Note that the present discussion does not rule out other pairing mechanisms for these materials. More refined experiments of $d(T)$ are eagerly required.

In summary, we examined the effect of gap states on the London penetration depth in the framework of the BCS theory. As $\Gamma$ in the lifetime broadening BCS density of states $N_2(T, \Gamma)$ increases, $\lambda_L(T, \Gamma)$ increases and its temperature dependence deviates from the ordinary BCS form with $\Gamma=0$. For the case of weak-coupling BCS, a $T^2$ law can be reproduced by $\lambda_L(T, \Gamma)$. This suggests that the present model can partially explain the controversial experimental results of $\lambda$ in the Cu-O based oxide superconductors.

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REFERENCES


(5) See for example, G. A. Thomas, R. N. Bhatt, A. Millis, R. Cava, and E. Rietman, Proc. 18th Int. Conf. on Low Temperature Physics, Kyoto 1001 (1987).


