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Asymmetry Parameters for the First Forbidden β-Decay in ^{170}Tm

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Asymmetry parameters in the beta-ray angular distributions for the first forbidden beta-decay of ^{170}Tm have been evaluated with the numerical values of relevant nuclear matrix elements, which were obtained from the experimental data on the longitudinal beta-ray polarization and the beta-gamma directional correlation.

With the asymmetry parameters evaluated, effective asymmetry parameter taking into account the effect of beta-decays branching are discussed in relation to our recent experiment (β-RADOPS) on the detection of beta-ray asymmetry from polarized ^{170}Tm.

KEY WORDS: First-forbidden β-decay/ Asymmetry parameter/ polarized ^{170}Tm/

1. INTRODUCTION

Magnetic moment is one of the most essential quantities to understand nuclear structure. It gives useful information in the determination of single-particle properties of nuclei. In the case of deformed nuclei it can help to give an idea of deformation.

Various methods have been used to measure magnetic moments of nuclei. In the case of stable and long-lived unstable nuclei, for example, atomic beam and magnetic resonance (NMR, ESR, ENDOR, etc) methods have been ordinary used. But in the case of short-lived unstable nuclei which are rarely produced, more sensitive detection techniques are demanded. Observation of magnetic resonance with beta- or gamma-ray radiation detection from nuclei oriented by any method (recoil angle selection following nuclear reactions, tilted foil, dynamic polarization, optical pumping, etc) is suitable for this case from its high-sensitivity.

We describe briefly now how to measure magnetic moments of β-unstable nuclei by beta-ray radiation detection method[1]. Due to the parity nonconservation in weak interactions, the beta-rays are emitted asymmetrically with respect to the nuclear polarization axis. We can find the degree of nuclear polarization by measuring this asymmetry. And we can observe magnetic resonance by means of this beta asymmetry, that is, the enhancement and/or the destruction of the nuclear polarization with the application of rf magnetic field. With such observations of magnetic resonance, the hyperfine constants and/or the nuclear g factors are determined, thus getting nuclear magnetic moments.

The beta-ray angular distribution function from a polarized nuclei is described by[2]

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$$W(\theta)=1+A\rho_1\cdot\frac{P}{E}\cos\theta. \tag{1}$$

Here θ is the angle of the beta-ray emission with respect to the nuclear polarization axis. ρ_1 is the degree of nuclear polarization ($\rho_1=\Sigma\, ma_{\mathrm{m}}$, where a_{m} is the population of each magnetic substate m, $\Sigma\, a_{\mathrm{m}}=1.$), P and E are the momentum and total energy of beta particle. A is the asymmetry parameter ($|A|\leqq 1$), the magnitude of which depends upon the properties of the particular β decay.

An experimental measure of the beta-ray asymmetry is the quantity ε described by

$$\varepsilon=\frac{W(0)-W(\pi)}{W(0)+W(\pi)}=A\rho_1\cdot\frac{P}{E}, \tag{2}$$

which is directly proportional to A and ρ_1. Thus it is very important to evaluate the asymmetry parameter for the estimation of the nuclear polarization.

The asymmetry parameter is independent of E in the allowed transitions. For example, in the transition from the parent state with its spin j to the daughter nuclear state $j-1$ without parity change (Gamow-Teller transition), A is equal to -1. On the other hand the asymmetry parameter in the forbidden transitions depends upon E because such transitions are due to the contributions of several nuclear matrix elements $\int\Omega$ (which means transition matrix elements from the initial to the final nuclear state through the operator Ω). Therefore we have to evaluate asymmetry parameters from the values of relevant nuclear matrix elements in the forbidden transitions.

Recently we achieved significant nuclear polarization of ^{170}Tm in cubic crystal SrF_2 by optical pumping method and detected beta-ray asymmetry for the measurement of the nuclear magnetic moment of ^{170}Tm (β-ray radiation detected optical pumping in solids: β-RADOPS)[3]. In order to determine the degree of nuclear polarization from the experimentally observed ε, we have evaluated the asymmetry parameter A from the relevant nuclear matrix elements obtained by other experimental data. The β-decay of ^{170}Tm ($^{170}Tm\rightarrow{}^{170}Yb$, half life is 128.6 days) is a typical example of the first forbidden beta decay. Spin-parity of the ground state of ^{170}Tm is 1^- and the spectrum of electrons emerging from this initial state has two components as shown in Figure 1: the 968 keV component corresponds to the $1^-\rightarrow 0^+$ ground state transition with a $\log ft$ value of 8.9 (branching ratio 76%) while the 884 keV component corresponds to the $1^-\rightarrow 2^+$ first exited state transition with a $\log ft$ value 9.3 (branching ratio 24%).

Following the method of Morita et al.[4], we evaluated the asymmetry parameter for the $1^-\rightarrow 0^+$ transition from the nuclear matrix elements to reproduce the experimental data on the longitudinal polarization of beta particles[5]. Also we evaluated the asymmetry parameter for the $1^-\rightarrow 2^+$ transition by using the nuclear matrix elements obtained by Runge to reproduce the experimental data on the beta-gamma directional correlation[6].

In addition to the importance in the nuclear polarization as described above, the asymmetry parameters and the nuclear matrix elements also play significant role in the understanding of nuclear structure relevant to the forbidden beta-decay process. Bogdan et al.[7] theoretically predicted the nuclear matrix elements in ^{170}Tm in the

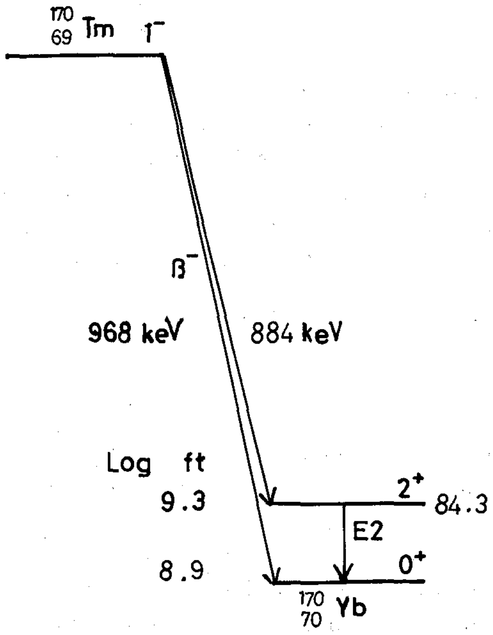

Fig. 1. The decay scheme of ^{170}Tm.

representation of Schülke and others[8] with the decoupling model for odd-odd nuclei. However no experimentally deduced asymmetry parameters and the ratio of nuclear matrix elements for the $1^- \rightarrow 0^+$ transition have been obtained to compare with such theoretical predictions.

We first discuss the ratio of nuclear matrix elements and evaluate beta-ray angular distribution for the $1^- \rightarrow 0^+$ transition in ^{170}Tm in § 2. The beta-ray angular distribution for the $1^- \rightarrow 2^+$ transition in ^{170}Tm is then evaluated in § 3. The effective asymmetry parameter taking into account the effect of beta-rays branching is discussed in § 4.

2. BETA-RAY ANGULAR DISTRIBUTION FOR THE $1^- \rightarrow 0^+$ TRANSITION IN ^{170}Tm

The spectral shape factor $C(E)$, the beta-ray angular distribution $W(\theta)$ and the longitudinal polarization of beta particle $P_L(E)$ in the case of the $1^- \rightarrow 0^+$ transition are respectively given by,

$$C(E) = -\left(\frac{1}{3}\right)^{1/2} b_{11}^{(0)}, \tag{3}$$

$$W(\theta) = -\left(\frac{1}{3}\right)^{1/2} b_{11}^{(0)} - \left(\frac{1}{2}\right)^{1/2} \rho_1 \, b_{11}^{(1)} \cos\theta, \tag{4}$$

$$P_L(E) = 2\left(\frac{3^{1/2}}{b_{11}^{(0)}}\right) \Big[\left|\int r\right|^2 C_V^2 [(2/3)\, q^2 \Lambda_1 - (2/3)\, q N_{11} + 4\Lambda_2 - 2m_1]$$

$$+ \left|\int \boldsymbol{\alpha}\right|^2 C_V^2 2\Lambda_1$$

$$+\left|\int\sigma\times r\right|^2 C_A^2[(1/3)\,q^2\Lambda_1+(2/3)\,qN_{11}+\Lambda_2-2m_1] \tag{5}$$

$$+\left\{\left(\int\boldsymbol{\alpha}\right)^*\left(\int\sigma\times r\right)\right\}2C_VC_A[(2/3)\,q\Lambda_1+N_{11}]$$

$$+\left\{i\left(\int r\right)^*\left(\int\boldsymbol{\alpha}\right)\right\}2C_V^2\,[(2/3)\,q\Lambda_1-N_{11}]$$

$$+\left\{i\left(\int r\right)^*\left(\int\sigma\times r\right)\right\}2C_VC_A\,[2\Lambda_2+2m_1]\Big]$$

The particle parameters, $b_{11}^{(n)}$ with $n=0$, 1 and 2 for the first forbidden transition are expressed in terms of three nuclear matrix elements, $i\int r$, $\int\boldsymbol{\alpha}$, and $\int\boldsymbol{\sigma}\times\boldsymbol{r}$. The quantities Λ_1, Λ_2, N_{11}, m_1, etc are various combinations of electron wave functions[9]. q is the neutrino energy, and the coupling constants V_V, C_A were assumed to be real. Throughout in this paper the electron wave function were obtained by solving the Dirac equation with the Coulomb field of finitevsize nucleus numerically[10].

We obtained an optimum set of relevant nuclear matrix elements to reproduce experimental data on the longitudinal polarization of the beta particle[5] by varying the parameters,

$$x=\frac{C_V i\int r}{C_A\int\boldsymbol{\sigma}\times\boldsymbol{r}}, \tag{6}$$

$$y=\frac{C_V\int\boldsymbol{\alpha}}{C_A\int\boldsymbol{\sigma}\times\boldsymbol{r}} \tag{7}$$

In this calculation, we assumed the relation

$$\int\boldsymbol{\alpha}=(E_i-E_f+E_c)\,i\int r \tag{8}$$

derived from the conserved vector current (CVC) theory[11]. Ei–Ef is the energy difference between the initial and final nuclear states. E_c is a Coulomb displacement energy, including the neutron-proton mass difference. Taking into account rather large experimental uncertainties, the parameters x and y were determined to be,

$$0.95\leq x\leq 1.05, \tag{9}$$

$$34.0\leq y\leq 37.8. \tag{10}$$

The asymmetry parameter A_{10} for the $1^-\rightarrow 0^+$ transition is expressed by,

$$A_{10}=\left(\frac{3}{2}\right)^{1/2}\cdot\frac{b_{11}^{(1)}}{b_{11}^{(0)}\cdot P/E}\cdot \tag{11}$$

Figure 2 shows the asymmetry parameter A_{10} determined from the set of x and y as a function of E.

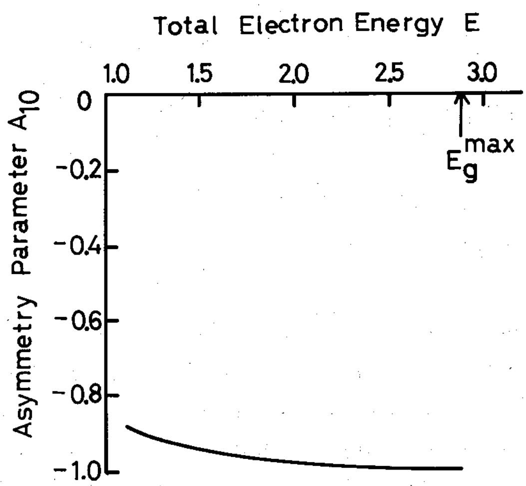

Fig. 2. The asymmetry parameter A_{10} for the $1^- \to 0^+$ transition. The unit of energy is in electron mass. Uncertanties of the asymmetry parameter are within the width of the line.

3. BETA-RAY ANGULAR DISTRIBUTION FOR THE $1^- \to 2^+$ TRANSITION IN ^{170}Tm

Unlike the $1^- \to 0^+$ transition, the nuclear matrix element $\int Bij$ also contributes to the spectral shape factor, and also the beta-ray angular distribution in the case of the $1^- \to 2^+$ transition. In this case the spectral shape factor $C(E)$ and the beta ray angular distribution $W(\theta)$ are respectively given by,

$$C(E) = -\left(\frac{1}{3}\right)^{1/2} b_{11}^{(0)} + \left(\frac{1}{5}\right)^{1/2} b_{22}^{(0)}, \tag{12}$$

$$W(\theta) = -\left(\frac{1}{3}\right)^{1/2} b_{11}^{(0)} + \left(\frac{1}{5}\right)^{1/2} b_{22}^{(0)} + 3\left(\frac{1}{2}\right)^{1/2} \left\{\frac{1}{6} b_{11}^{(1)} - \frac{1}{2\sqrt{5}} b_{12}^{(1)} + \frac{1}{6\sqrt{5}} b_{22}^{(1)}\right\} \rho_1 \cos\theta. \tag{13}$$

In these formulas, we neglected higher order particle parameters, $b_{LL'}^{(n)}$ (L or $L' \geqq 3$), L, L' are orbital angular momentum of the lepton system. In addition to x and y, the new parameter u is necessary for the calculation of $C(E)$, $W(\theta)$;

$$u = \frac{i \int Bij}{\int \sigma \times r}. \tag{14}$$

Runge determined a set of nuclear matrix elements in this case by comparison to the experimental data on the beta-gamma directional correlation[6]. In our notation his conclusion is respectively

$$x = 2.0^{+0.4}_{-0.4}, \tag{15}$$

$$y = 80.2^{+23.0}_{-19.0}, \tag{16}$$

$$u=1.34^{+0.05}_{-0.27}. \tag{17}$$

The asymmetry parameter A_{12} for the $1^{-}\to 2^{+}$ transition was thus evaluated by using this parameter set, where A_{12} is expressed by,

$$A_{12}=3\left(\frac{1}{2}\right)^{1/2}\frac{\left\{\frac{1}{6}b_{11}^{(1)}-\frac{1}{2\sqrt{5}}b_{12}^{(1)}+\frac{1}{6\sqrt{5}}b_{22}^{(1)}\right\}}{\left\{-\left(\frac{1}{3}\right)^{1/2}b_{11}^{(0)}+\left(\frac{1}{5}\right)b_{22}^{(0)}\right\}}\cdot\frac{P}{E} \tag{18}$$

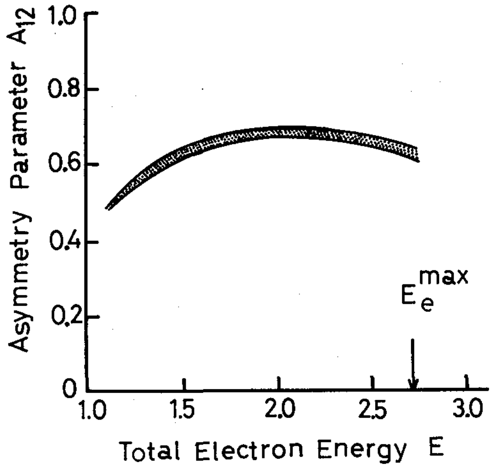

Fig. 3. The asymmetry parameter A_{12} for the $1^{-}\to 2^{+}$ transition. Shaded area represents uncertanties due to the errors of experimental data used in the present evaluation.

Figure 3 shows the asymmetry parameter A_{12} as a function of E.

4. DISCUSSION AND CONCLUSION

In the beta-decay in ^{170}Tm both experimentally observed $C(E)$ and $P_L(E)$ are similar to those expected in the allowed transition. Thus the asymmetry parameter is expected also to be allowed like. From the asymmetry parameter obtained in the present evaluation we knew this expectation holds rather well.

It is interesting to compare the determined parameters x and y with theoretical predictions based on the various nuclear models. The detailed discussion of this point will be presented elsewhere.

Beta-rays from the $1^{-}\to 0^{+}$ and the $1^{-}\to 2^{+}$ transitions are not separated in the usual measurement of beta-ray distribution. Thus "effective" asymmetry parameters, A_{eff}, taking into account both transitions are important from the experimental point of view. The asymmetry parameter, A_{eff}, was evaluated by averaging the values of asymmetry parameters for both transitions taking into account also the branching ratio and the shape factor. Figure 4 shows effective asymmetry parameter multiplied by p/E, as a function of E.

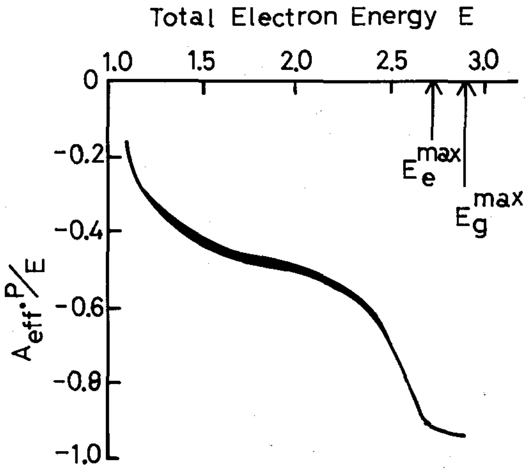

Fig. 4. The effective asymmetry parameter A_{eff} multiplied by p/E. See the text for the meaning of "effective".

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REFERENCES

1) Donald Conner, *Phys. Rev. Lett.* **3**, 429 (1959).
2) M. Morita, Beta decay and Muon Capture (Benjamin, Readig, Mass, 1973), Chap. 3.
3) K. Shimomura, S. Matsuki, S. Uemura, T. Kohmoto, Y. Fukuda and T. Hashi, Proc. Int. Conf. on Quantum Electronics (July 1988, Tokyo), p. 298.
4) M. Morita, H. Ohtsubo and K. Arita, *Prog. Theor. Phys.* Suppl. **No. 60**, 125 (1976).
5) A. I. Alikahanov, G. P. Eliseiev and V. A. Liubimov, *Nucl. Phys.* **7**, 655 (1958).
6) K. Runge, *Z. Physik* **183**, 184 (1965).
7) D. Bogdan, A. Faessler, K. W. Schmid and S. Holan, *J. Phys. G: Nucl. Phys.* **8**, 1231 (1982).
8) H. F. Schopper, Nuclear Beta Decay and Weak Interactions (Amsterdam, North-Holland).
9) Ref. 2, Chap. 6.
10) A. subroutine program by H. Ohtsubo was used for this calculation.
11) J. Fujita, *Phys. Rev.* **126**, 202 (1962).
J. Eichler, *Z. Phys.* **171**, 463 (1963).