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# Double-Scattering Corrections to the Quasifree Scattering in the $^3$He (p, pp)$^2$H Reaction

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A multiple-scattering effect is calculated in the quasifree scattering region for the breakup reaction on a few-nucleon system. For all pair interactions, spin states are taken into account explicitly and, on the other hand, zero orbital angular momentum is assumed.

KEY WORDS: Quasifree scattering/ Multiple scattering/ Few-nucleon system

## 1. INTRODUCTION

In recent years, breakup reactions for few-nucleon systems have been studied in the quasifree scattering (QFS) region over a wide range of incident energy. For the deuteron-breakup reaction induced by nucleons, the cross sections have been calculated by the Faddeev equation. However, its solutions are very involved and it is not easy for us to have an insight into incident-energy and correlation-angle dependences of the cross section.

Moreover, up to date, no exact calculations are available for the $^3$H-, $^3$He- and $^4$He-breakup reactions even in the case induced by nucleons because the systems must be considered as consisting of four or five nucleons. Thus model calculations have been done on several assumptions.

For the QFS leading to three particles in the final state, it can be assumed that either the projectile or the target nucleus consists of two particles bound and breaks up into its components through the interaction between the non-breakup nucleus and one of the initial-bound particles with the other particle as a spectator (spectator model). The plane-wave impulse approximation (PWIA) has been used in the calculations on the spectator model.

The PWIA calculation, however, gives the cross section considerably larger than that experimentally measured. The ratio of the measured cross section to the PWIA one is less than unity. This fact can be ascribed to multiple-scattering processes[1], where the QFS particles and spectator in the PWIA calculation interact with each other.

We calculated analytically the ratio taking into account a double-scattering for the $^2$H (p, pp) n and $^2$H (p, pn) $^1$H reactions in the QFS region[1]. The incident-energy and correlation-angle dependences of the ratio were explained successfully. Then we

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extended the calculation to the $^3$He (p, pp) $^2$H reaction assuming a three-body model[2]. The experimental results were well reproduced. In the study the global calculation results were compared with the experimental values. Then it is worth-while examining each contribution of the spin-state components separately because the spin-state dependence of the ratio was pointed out in ref. 1).

In the present report the calculation in ref. 1) is reformulated to take into account spins explicitly and contributions of spin-state components are examined for the $^3$He (p, pp) $^2$H reaction concerning the incident-energy and correlation-angle dependences.

## 2. CALCULATION FORMULAE

### 2.1 Notation

Let us consider a three-body reaction i+ (jk) → i+j+k in the QFS region. The particles j and k are bound in the initial state and the particles i and j in final state are detected in coincidence at the angles $(\theta_i, \phi_i)$ and $(\theta_j, \phi_j)$, respectively. These angles are chosen to satisfy the kinematical condition for the QFS. The pair k= (ij) is referred as the QFS pair. The trasition matrix for the breakup reaction is represented as

$$< X_i \mid U_i G_0 \mid X_{i0} > \tag{1}$$

using the AGS transition operator $U_i$ with $G_0=(H_0-z)^{-1}$. The initial and final states are denoted by

$$\mid X_{i0} > = \mid g_i \boldsymbol{q}_{i0} x_{i0} > \tag{2}$$

and

$$\mid X_i > = \mid \boldsymbol{p}_i \boldsymbol{q}_i x_i >, \tag{3}$$

respectively, where $(\mathsf{p}_i, \mathsf{q}_i)$ are the Jacobi momenta for the channel i and $g_i$ is the form factor of the initial bound pair i. The spin state $\mid x_i >$ is represented as

$$\mid x_i > = \mid S_i M_i m_i >, \tag{4}$$

where $m_i$ is the $z$-component of the spin $s_i$ for the particle i and $S_i$, $M_i$ are the spin and its $z$-component for the pair i. For the initial bound pair i, the relative orbital angular momentum is assumed to be zero and thus $S_{i0}$ corresponds to the spin of the target.

### 2.2 Definition of the Multiple-Scattering Factor

The AGS transition operator $U_i$ is expanded into a multiple-scattering series, that is,

$$U_i = T_k - T_i G_0 T_k - T_j G_0 T_k + \cdots + T_j - T_i G_0 T_j - T_k G_o T_j + \cdots, \tag{5}$$

where $T_i$ is the two-body transition operator for the i-pair interaction and the term $(H_0-z)$ is previously omitted because of the breakup reaction. We choose the pair k as the QFS pair and considering the kinematical condition for the QFS, we can take only the first three terms in eq. (5), a single-scattering term and two double-scattering terms,

$$U_i = T_k - T_i G_0 T_k - T_j G_0 T_k. \tag{6}$$

Taking the first term in eq. (6) corresponds to the impulse approximation (IA) and taking the double-scattering terms in addtion can be considered as a correction to the IA.

The multiple-scattering (MS) factor which measures the MS effect is difined in a factorized form by

$$< X_i \mid U_i G_0 \mid X_{i0} > = \eta_k < X_i \mid T_k G_0 \mid X_{i0} >. \tag{7}$$

For eq. (6), the MS factor is written as

$$\eta_k = 1 - \sum_{l=i,j} < X_i \mid T_l G_0 T_k G_0 \mid X_{i0} > / < X_i \mid T_k G_0 \mid X_{i0} > \tag{8}$$

We factorize the MS factor out of the matrix element as in eq. (7) without calculating the matrix elements of eq. (6) directly. The purposes of it are as follows. (a) The MS effect may appear magnified in the MS factor, (b) rather rough approximations can be taken in the calculation of the MS factor compared with that in the calculation of eq. (6) because two-body scattering data are available for the single-scattering matrix element in eq. (7) and (c) several experimental results exist in the form of the ratio of the breakup cross section to the PWIA one, appropriate to be compared with the MS factor.

### 2.3 Calculation of the MS Factor

We take an approximation in the calculation of eq. (8) that only the $L=0$ component contributes to all pair interactions, $L$ being the relative orbital angular momentum. As a result, the matrix elements of three terms in eq. (6) are written in the following form,

$$< X_i \mid T_k G_0 \mid X_{i0} > = \sum_{S'_k} I_1 (S'_k S_{i0})\ C_1 (x_i S'_k x_{i0}), \tag{9}$$

$$< X_i \mid T_i G_0 T_k G_0 \mid X_{i0} > = \sum_{S'_k} I_2 (S_i S'_k S_{i0})\ C_1 (x_i S'_k x_{i0}) \tag{10}$$

and

$$< X_i \mid T_j G_0 T_k G_0 \mid X_{i0} > = \sum_{S'_k} \sum_{S'_j} I_2 (S'_j S'_k S_{i0}) C_2 (x_i S'_j S'_k x_{i0}), \tag{11}$$

where

$$I_1\ (S'_k S_{i0}) = (4\pi)^{-3/2} \mid C^{ki}_{qp} \mid {}^{-3}T_k (p_k,\ \tilde{p}_k,\ p'^2_k;\ S'_k)\ (-1)\ \phi_i\ (\tilde{p}_i\ ;\ S_{i0}) \tag{12}$$

$$I_2\ (S'_l S'_k S_{i0}) = (4\pi)^{-5/2} \mid C^{ki}_{qp} \mid {}^{-3}$$
$$\times \int p'^2_l \mathrm{d}p'_l\ T_l\ (p_l,\ p'_l,\ p'^2_l;\ S'_l)\ (p'^2_l - p^2_l - i\varepsilon)^{-1}$$
$$\times \int \mathrm{d}\Omega'_l\ T_k\ (p'_k,\ \tilde{p}''_k,\ z - q'^2_k;\ S'_k)\ (-1)\ \phi_i\ (\tilde{p}''_i;\ S_{i0}),$$

$$(l=i,\ j;\ S'_l=S_i,\ S'_j) \tag{13}$$

and

$$C_1\ (x_iS'_kx_{i0})=[(2S_i+1)\ (2S_{i0}+1)]^{1/2}\ (2S'_k+1)$$
$$\times(\varepsilon_{ijk})^{\sigma}\ \sum_{S'}W(S's_is_ks_j\ ;\ S_iS'_k)$$
$$\times W(S's_ks_is_j\ ;\ S'_kS_{i0})\ F(x_iS'x_{i0}), \tag{14}$$

$$C_2\ (x_iS'_jS'_kx_{i0})=[(2S_i+1)\ (2S_{i0}+1)]^{1/2}\ (2S'_j+1)\ (2S'_k+1)$$
$$\times(\varepsilon_{ikj})^{\sigma}\ \sum_{S'}(-1)^{\tau}W(S's_is_js_k\ ;\ S_iS'_j)$$
$$\times W(S's_js_ks_i\ ;\ S'_j\ S'_k)\ W(S's_ks_is_j\ ;\ S'_k\ S_{i0})F(x_iS'x_{i0}) \tag{15}$$

with

$$F(x_iS'x_{i0})=\sum_{M'}\ (s_im_i,\ S_iM_i\mid S'M')\ (s_im_{i0},\ S_{i0}M_{i0}\mid S'M'),$$
$$\sigma=S_i-S_{i0},$$
$$\tau=s_i+s_j+s_k+S_i+S'_j+S'_k-3S'$$

and $\varepsilon_{ijk}$ is the Levi-Civita symbol.
In these equations, $T_k\ (p_k',\ p_k,\ z;\ S_k)$ is the matrix element of the two-body transition operator, $p_k$ and $p_k'$ are the moments in the initial and final states respectively, $z$ is the energy of the total system, $S_k$ is the spin of the pair k, $\phi_i\ (p_i;\ S_{i0})$ is the momentum distribuition function for the breakup nucleus. The momenta in eqs. (12) and (13) are the same as those in ref. 1) if $(\alpha,\ \gamma,\ \delta)$ replaced by (i, j, k). $W\ (S\ s_is_js_k;\ S_iS_j)$ are the Racah coefficients and $(s_im_i,\ S_iM_i\mid SM)$ are the Clebsch-Gordan coefficients.

### 2.4 A Special Case

If we take only the state of spin $S_k$ for the QFS pair, the MS factor $\eta_k$ reduces to a more simple form as

$$\eta_k(x_iS_kx_{i0})=1-J_{12}(S_iS_kS_{i0})-\sum_{S_j}J_{12}(S'_jS_kS_{i0})\ D_{12}(x_iS'_jS_kx_{i0}) \tag{16}$$

where

$$J_{12}(S_jS_kS_{i0})=I_2(S'_jS_kS_{i0})\ /\ I_1(S_kS_{i0}) \tag{17}$$

and

$$D_{12}(x_iS'_jS_kx_{i0})=C_2(x_iS'_jS_kx_{i0})\ /\ C_1(x_iS_kx_{i0}). \tag{18}$$

In addition, if we take only a value of the total spin $S$, the last ratio $D_{12}$ becames

$$D_{12}\ (x_iS'_jS_kSx_{i0})=(2S'_j+1)\ (-1)^{\lambda}W(S\ s_is_js_k\ ;\ S_iS'_j)$$
$$\times W(S\ s_js_ks_i\ ;\ S'_j\ S_k)/W(S\ s_is_ks_j\ ;\ S_i\ S_k) \tag{19}$$

with

$$\lambda=s_i+s_j+s_k+S_i+S'_j+S_k-3S.$$

Accordingly the MS factor is independent of all the $z$-components. As an example, let

us consider the (p, pp) reaction in the QFS region. We can take the singlet ($S_k=0$) component for the pp QFS pair. Thus the total spin $S$ is restricted to $S=s_k$ with $s_i=s_j$ by the Racah coefficient properties. Substituting these values into eq. (19), we obtain

$$D_{12}\ (x_i S'_j S_k=0\ S=s_k\ x_{i0})=(2S'_j+1)\ W(s_k s_i s_i s_k\ ;\ S_i\ S'_j). \tag{20}$$

## 3. CALCULATION RESULTS

We shall apply the calculation to the $^3$He (p, pp) $^2$H reaction in the QFS rigion. The $^3$He target nucleus is assumed to consist of a proton and a deuteron bound. The momentum distribution function and the separable form assumed for the s-wave T-matrix with the parameters $\varkappa$ and $\beta$ are given in ref. 1). For simplicity, in the calculation of eq. (13) the term $(p_1'^2-p_1^2-i\varepsilon)^{-1}$ is approximated as $i\pi\delta\ (p_1'^2-p_1^2)$ with the

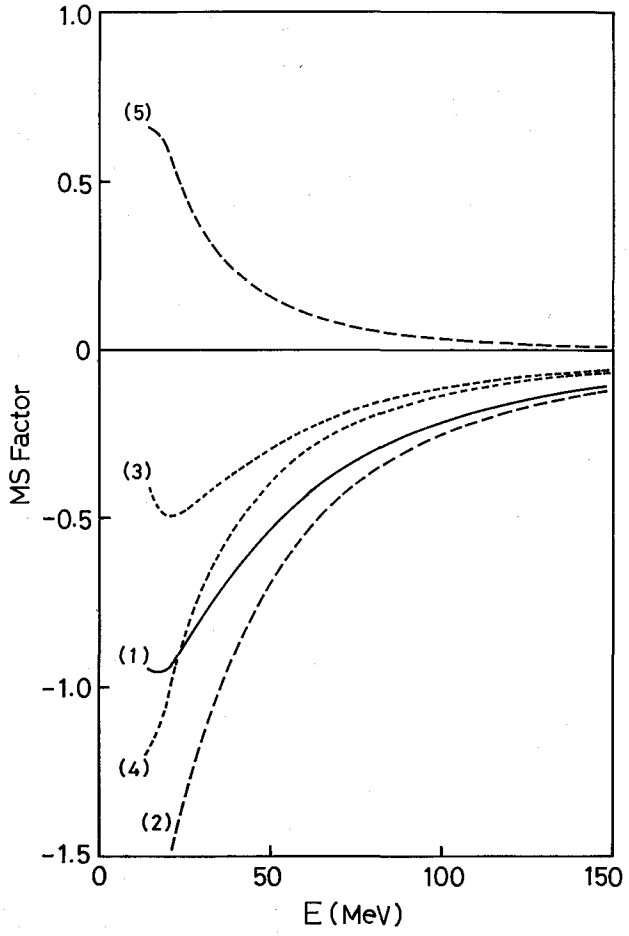


Fig. 1. Mutiple-scattering factor and its components as a function of the incident energy for the pp-QFS on $^3$He in the coplanar symmetric geometry. The curves show (1) $|\eta|^2-1$, (2) $2\mathrm{Re}\mu$, (3) $2\mathrm{Re}\mu$ (D), (4) $2\mathrm{Re}\mu$ (Q) and (5) $|\mu|^2$.

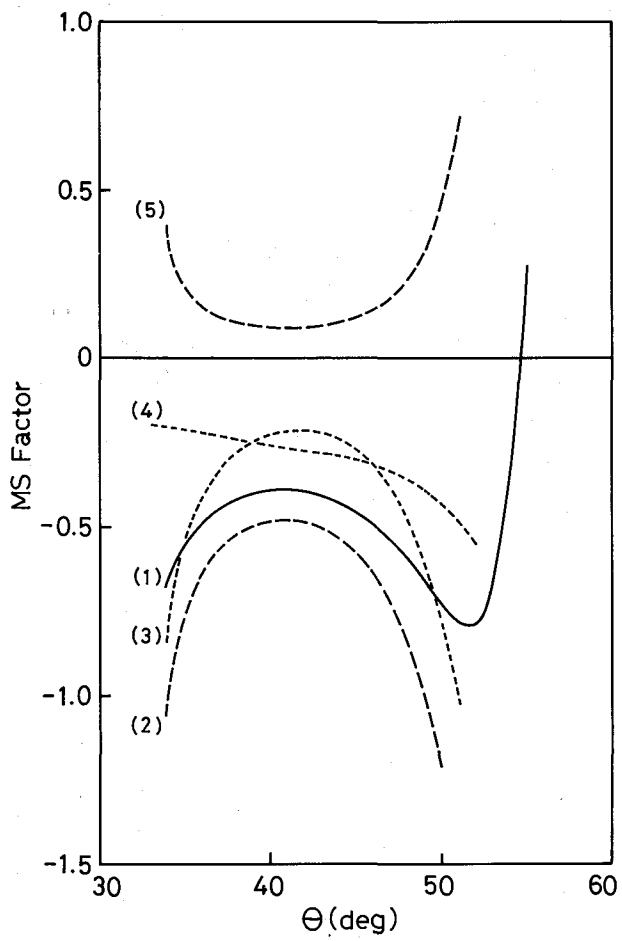


Fig. 2. Multiple-scattering factor and its components as a function of the angle for the pp-QFS on $^3$He in the coplanar symmetric geometry. The curves show (1) $|\eta|^2-1$, (2) $2\mathrm{Re}\mu$, (3) $2\mathrm{Re}\mu$ (D), (4) $2\mathrm{Re}\mu$ (Q) and (5) $|\mu|^2$.

principal value integral neglected. The integlation over solid angle is perfomed with an approximation $\beta \gg \kappa$ and the final formulae are represented in ref. 1). Let us write eq. (16) as

$$\eta = 1+\mu$$

and

$$\mu = \mu_i\ (\mathrm{D}) + \mu_j\ (\mathrm{D}) + \mu_j\ (\mathrm{Q}) = \mu\ (\mathrm{D}) + \mu\ (\mathrm{Q}),$$

where D and Q denote the singlet and quartet states for the second pd interaction respectively. Thus

$$|\eta|^2 = 1 + 2\mathrm{Re}\mu + |\mu|^2.$$

We evaluated the MS factor for $S_i = S_{i0} = 1/2$. The evaluated $|\eta|^2$ and its

components are shown in Fig. 1 as a function of the incident energy and in Fig. 2 as a function of correlation angle at 65 MeV. The kinematical condition for these two cases is the coplanar symmetric geometry. As seen from Fig. 1, the dependence of $|\eta|^2$ on the incident energy is given mainly by the term $2\mathrm{Re}\mu$, which corresponds to the interference between the single- and the double-scatterings. The doublet (D) and quartet (Q) spin states contribute almost equally to $|\eta|^2$ in particular at higher energies. The condition $|\mu|^2 \ll 1$ may be required for the double-scattering to be regarded as a correction to the single-scattering. The values of $|\mu|^2$ are less than 0.1 at energies higher than about 60 MeV. The dependence of $|\eta|^2$ on the symmetric angle is also given by the term $2\mathrm{Re}\mu$, however, by the doublet component alone. The quartet component varies slowly with the angle. The comparison of $|\eta|^2$ with the experimental results is shown in ref. 2). The incident-energy dependence is well reproduced.

Numerical calculations were carried out with the FACOM-M380Q computer at the Institute for Chemical Research of Kyoto University.

## REFERENCES

(1) K. Fukunaga, S. Kakigi, T. Ohsawa and A. Okihana, *J. Phys. Soc. Jpn.* **52** (1983) 69
(2) S. Kakigi, K. Fukunaga, T. Ohsawa, A. Okihana, T. Sekioka, H. Nakamura-Yokota, S. Tanaka and S. Kato, *Nucl. Phys.* **A473** (1987) 31