Energy Dependences of Cross Sections for (p, 2p) Quasifree Scatterings on $^2$H, $^3$He and $^4$He

Shigeru Kakigi*, Tadahiko Yoshimura** and Akira Okihana**

Received February 9, 1994

Energy dependences of (p, 2p) quasifree scatterings on $^2$H, $^3$He and $^4$He were compared with calculations in the plane wave impulse approximation and with those corrected by multiple scattering effects. Large cross sections observed in the $^2$H (p, 2p) n reaction around 500 MeV are found to be owing mainly to the bump of cross section for pp elastic scattering.

KEY WORDS: (p, 2p) Reaction / Quasifree Scattering / Few-Nucleon System

1. INTRODUCTION

In the last two decades, (p, 2p) reactions on target nuclei $^2$H, $^3$He and $^4$He have been studied in the quasifree scattering (QFS) region over a wide range of incident energy from about 10 to 1000 MeV. Figure 1 shows experimental differential cross sections measured in symmetric kinematics. That is, two outgoing protons were detected in coincidence at the symmetric angles with respect to the beam direction in the reaction plane and at the symmetric energies with the undetected particle of zero recoil momentum in the target rest system (the laboratory system). The data for the $^2$H (p, 2p) n reaction were taken from refs. 1-10), those for the $^3$He (p, 2p) $^2$H reaction from refs. 11-17) and those for the $^4$He (p, 2p) $^3$H reaction from refs. 18-22).

For the $^2$H (p, 2p) n reaction, large cross sections around 500 MeV are noticeable. It is worthwhile to study incident energy dependence of (p, 2p) QFS in few-nucleon systems over a wide energy range to clarify reaction mechanisms. In the report, the data are compared with calculations in the plane wave impulse approximation (PWIA), which can be evaluated over a wide energy range although is one of the most simple theories. The multiple scattering (MS) and the Δ-excitation effects are discussed.

2. KINEMATICS

Let us consider a three-body reaction $a + A \rightarrow 1 + 2 + 3$ in the QFS region, where the particle $a$ is the projectile and the particle $A$ the target nucleus assumed to consist of particles $b$ and $c$ bounded, $A = (bc)$. For the particle $i$, the rest mass, the kinetic energy, the total energy and the momentum are denoted as $m_i$, $T_i$, $E_i$ and $p_i$, respectively.

* 椋木 茂: Institute for Chemical Research, Kyoto University, Uji, Kyoto 611
** 吉村忠彦, 沢村 彰: Kyoto University of Education, Kyoto 612
In the measurements mentioned above, the target nucleus A is at rest in the laboratory system, thus $E_A = m_A$ and $p_A = 0$. The particles 1 and 2 are identical and are detected in coincidence at angles $(\theta_1, \phi_1)$ and $(\theta_2, \phi_2)$, which are symmetric with respect to the beam direction, thus $m_1 = m_2$ and $\theta_1 = \theta_2$ and $\phi_2 - \phi_1 = 180^\circ$. At the point of symmetric energies $T_1 = T_2$, the energy $T_3$ has the minimum (or maximum) value and $p_3$ lies along the beam direction. Under these kinematic condition, the energy and momentum conservations are written as

$$E_A + m_A = 2E_1 + E_3, \quad (1)$$

$$p_a = 2p_1 \cos \theta_1 + p_3. \quad (2)$$

We consider a simple quasifree process that can be treated in the impulse approximation (IA). The projectile a interacts with the particle b bounded and assumed to have a momentum $p_b$ in the target A, producing the particles 1 and 2 in the final state and, on the other hand, the particle c remains as a spectator retaining the momentum $p_c = -p_b$ throughout the reaction process. Thus the relations $m_c = m_3$ and $p_c = p_3$ hold. For the s-state bound, the momentum distribution has a maximum at $p_b = 0$. For this maximum point ($T_3 = 0$), the equations (1) and (2) are solved simply as

$$E_1 = (E_a + m_A - m_3) / 2. \quad (3)$$
Energy Dependence of \((p, 2p)\) QFS Cross Sections on \(^2\text{H}, ^3\text{He}\) and \(^4\text{He}\)

The corresponding angle \(\theta_1\) is given by

\[
\cos \theta_1 = \frac{p_a}{(2p_1)}.
\] (4)

For \((p, 2p)\) QFS in the symmetric kinematics, \(m_a = m_b = m_1 = m_2\), \(\theta_1 = \theta_2\), \(T_1 = T_2\) and \(T_3 = 0\). Let us call the angle calculated by eq. (4) QFS angle \((\theta_{\text{QFS}})\). The relative kinetic energy between the particles \(i\) and \(j\) \((T_{i-j})\) is calculated from the invariant mass of the \(i + j\) system \((M_{ij})\) as

\[
T_{i-j} = M_{ij} - (m_i + m_j).
\] (5)

Invariant masses are given as

\[
M_{ab}^2 = 2m_a (2m_a + T_a),
\] (6)

\[
M_{12}^2 = (m_a + m_A - m_3)^2 + 2(m_A - m_3) T_a
\] (7)

and

\[
M_{23}^2 = (m_2 + m_3)^2 + 2m_3 T_1, \quad (M_{23} = M_{31}).
\] (8)

The binding energy \(B\) of the particle \(b\) in the target nucleus \(A\) is defined by

\[
m_A = m_b + m_c - B.
\] (9)

Thus

\[
m_A - m_3 = m_b - B.
\] (10)

The proton binding energies are 2.22, 5.49 and 19.8 MeV for \(^2\text{H}, ^3\text{He}\) and \(^4\text{He}\), respectively.

The QFS angles were calculated using eq. (4) for three targets and are shown in Fig. 2 as functions of the incident energy together with that for free pp scattering. The QFS angles have maximums at intermediate energies, for instance, 43.6° at about 100 MeV for the \(^2\text{H}~(p, 2p)\ n\) reaction, approaching that for free pp scattering with increase of the energy and rapidly decreasing to 0° at lower energies owing to the binding effect. The points \(\theta_{\text{QFS}} = 0°\) correspond to the thresholds for \((p, 2p)\) QFS in the symmetric kinematics. The threshold energies are determined by \(T_{1-2} = 0\) corresponding to \(\theta_{\text{QFS}} = 0°\) and given as

\[
T_a = 2B \left[1 - B/(4m_a)\right] / (1 - B/m_a),
\] (11)

which are \(T_a = 4.5, 11.0\) and 40.3 MeV for \(^2\text{H}, ^3\text{He}\) and \(^4\text{He}\) respectively. For the case \(B < m_a, T_a \sim 2B\).

Relative kinetic energies \(T_{1-2}\) for \((p, 2p)\) QFS on \(^2\text{H}, ^3\text{He}\) and \(^4\text{He}\) are shown in Fig. 3 as functions of the incident energy, together with that for free pp collision. The \(\pi\) -production
Fig. 2 (p, 2p) QFS angles for $^2$H (2), $^3$He (3) and $^4$He (4) together with that for pp scattering (1).

threshold is at $T_{1-2} = 138$ MeV and corresponds to the incident energy $T_a = 296$ MeV. The $\Delta$-excitation is expected to occur around $T_{1-2} = 300$ MeV corresponding to $T_a = 654$ MeV.

3. PWIA CALCULATIONS

The differential cross section calculated in the plane wave impulse approximation (PWIA) is given as

$$\left(\frac{d\sigma}{d\Omega_{1}d\Omega_{2}dE_2}\right)_{\text{PWIA}} = N \times K \times \left(\frac{d\sigma}{d\Omega}\right)_{12} \times |\Phi(\eta_3)|^2,$$

where $\left(\frac{d\sigma}{d\Omega}\right)_{12}$ is the half-off energy shell differential cross section for the $1 + 2$
scattering in the (p, 2p) c.m. system, \(|\Phi(p)|^2\) is the momentum density, \(N\) is the spectroscopic factor and \(K\) is the kinetic factor given by

\[
K = \frac{M_{12}^2 p_1 p_2^2 E_3}{(p_2 E_b)} / \left[ p_2 (E_2 + E_3) - E_2 (p_2 + p_3) p_2 / p_2 \right].
\] (13)

where

\[
E_b = (p_3^2 + m_b^2)^{1/2}.
\] (14)

For (p, 2p) QFS in the symmetric kinematics,

\[
K = M_{12}^2 p_1^2 / (p_2 E_b)
\] (15)

with \(E_b = m_b\).

The kinetic factors \(K\) are shown in Fig. 4 for (p, 2p) QFS on the targets \(^2\)H, \(^3\)He and \(^4\)He as functions of the incident energy. The half-off-energy-shell pp cross sections were replaced by the on-shell values at \(T_{1,2}\) and \(\theta_{1,2} = 90^\circ\), shown in Fig. 5. The momentum function \(\Phi(p)\) is given by

\[
\Phi(p) = g(p) / (\alpha^2 + p^2)
\] (16)

with

\[
g(p) = [\alpha \beta (\alpha + \beta)^3]^{1/2} / [\pi (\beta^2 + p^2)].
\] (17)

Thus

\[
|\Phi(0)|^2 = [(\alpha + \beta) / (\alpha \beta)]^3 / \pi^2.
\] (18)

Fig. 4 Kinetic factors for (p, 2p) QFS on \(^2\)H (2), \(^3\)He (3) and \(^4\)He (4).
The parameters \((a, \beta)\) in fm\(^{-1}\) are \((0.2316, 1.415)\), \((0.4201, 1.202)\) and \((0.8505, 0.7249)\) for \(^2\text{H}\) (pn)\(^{20}\), \(^3\text{He}\) (pd)\(^{20}\) and \(^4\text{He}\) (pt)\(^{20}\) respectively. Thus \(|\Phi(0)|^2 = 1.67, 0.437\) and \(0.220 \times 10^{-6}\) MeV\(^{-3}\), and the spectroscopic factors are \(N = 1, 3/2\) and 2 for the target \(^2\text{H}\), \(^3\text{He}\) and \(^4\text{He}\) respectively.

The PWIA cross sections are shown in Fig. 6 with solid lines (1), (2) and (3) for \(^2\text{H}\), \(^3\text{He}\) and \(^4\text{He}\) respectively.
Energy Dependence of (p, 2p) QFS Cross Sections on $^2$H, $^3$He and $^4$He and compared with the experimental data being the same as those in Fig. 1.

4. MULTIPLE SCATTERING EFFECTS

The PWIA calculations have given cross sections considerably larger than those experimentally measured. This fact can be ascribed to multiple scattering processes, where the QFS particles interact with the spectator. The multiple scattering effect can be measured by the multiple scattering (MS) factor $|\eta|^2$ defined in ref.26). The MS factors have been calculated for (p, 2p) QFS on $^2$H and $^3$He in the symmetric kinematics26,13). In the calculations, double-scattering terms were included in addition to that of single-scattering. The PWIA cross sections were multiplied by the MS factors,

$$\left(\frac{d\sigma}{d\Omega_1 d\Omega_2 dE_1}\right)_{MS} = \left(\frac{d\sigma}{d\Omega_1 d\Omega_2 dE_1}\right)_{PWIA} \times |\eta|^2,$$

shown in Fig. 6 with the lines (4) and (5) for $^2$H and $^3$He respectively and compared with the experimental data.

5. DISCUSSION

For the $^2$H (p, 2p) n reaction, the PWIA calculation roughly reproduces the measured cross sections above 500 MeV. Therefore the large cross sections are owing mainly to the bump of the cross section of pp elastic scattering. The $\Delta$-excitation effect in three-nucleon system seems to be small. The MS effect and the $\Delta$-excitation effect have been investigated in detail at 508 MeV7). Near $p_3 = 0$, $d\sigma_{exp} = 12.6$ mb/sr$^2$-MeV and $d\sigma_{IA} = 13.9$ mb/sr$^2$-MeV. The difference between both is 1.3 mb/sr$^2$-MeV (10%) and calculations show 10% of it are due to the MS effect. The rest remains not explained. The $\Delta$-excitation effect has been calculated separately to be about $1 \mu$b/sr$^2$-MeV. Interferences with other contributing mechanisms seem to be important but have not been calculated. At lower energies, the MS effect is large. Faddeev calculations below 150 MeV, shown in Fig. 6 with the line (6), well reproduce the experimental data.

For both the $^2$H (p, 2p) n and $^3$He (p, 2p) $^3$H reactions, no measurement in the symmetric kinematics exists in the energy region from 200 to 400 MeV. Thus it is worth-while to measure cross sections in this region.

For the $^4$He (p, 2p) $^3$He reaction, experimental cross sections are remarkably small compared with the PWIA calculations. For instance, the ratio $d\sigma_{exp}/d\sigma_{PWIA}$ at 100 MeV is 0.092 for $^4$He in contrast to those of 0.77 and 0.56 for $^2$H and $^3$He respectively29). DWIA calculations, shown in Fig. 6 with the line (7), were compared with experimental data at energies between 100 and 600 MeV21). At 250 MeV the DWIA calculation reproduced experimental data and, however, at lower energies experimental data are smaller than the DWIA values. The experimental cross section is 0.05 mb/sr$^2$-MeV at 70 MeV and grows up to 0.20 mb/sr$^2$-MeV at 100 MeV. These facts seem to show that the QFS in the $^4$He (p, 2p) $^3$H
reaction just rises around 100 MeV and proceeds sufficiently at 250 MeV.

Around 400 MeV, the following problems must be investigated. (1) The MS effects could not be neglected even in this energy region. Discrepancies between experimental data and theoretical calculations with the MS effects included remain not explained. (2) The $\Delta$-excitation effect seems to be week compared with pp QFS yet in this energy region. However more strong effects are expected from interferencies between the $\Delta$-excitation and other contributing mechanisms. (3) Relativistic effects have not been studied sufficiently for $(p, 2p)$ QFS on few-nucleon systems.

REFERENCES

Energy Dependence of \((p, 2p)\) QFS Cross Sections on \({}^2\text{H}, {}^3\text{He}\) and \({}^4\text{He}\)