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Undetermined Coefficient Method for Higher Order Scheme

Xiaofeng ZHANG*, Tamotsu TAKAHASHI, Hajime NAKAGAWA
and Yoshifumi SATOFUKA

*Department of River Engineering, Wuhan University, China

Synopsis

Undetermined coefficient method is proposed to derive higher order schemes. With this method, some famous schemes can be derived although their original derivations are different from each other. A new scheme named as HAUC1 is also presented using this method. Stability and accuracy of the new scheme are analyzed by comparing the computed results with exact solutions of 1-D pure convection equation and Burgers' equation. Effectiveness of the HAUC1 is also examined by comparing with the results using other schemes. Finally, it is applied to simulate 1-D dam break flow with different ratios of initial upstream water depth to downstream one. It shows that the scheme has the ability to simulate both undular bore and moving hydraulic jump well.

Keywords: higher order scheme; undetermined coefficient method; dam break flow; undular bore; moving hydraulic jump

1. Introduction

With the development of modern numerical methods and practical needs for more efficient simulation, higher order scheme is becoming more and more important in solving practical problems. Because one of the most difficult problems in the numerical simulation of flow is the treatment of convection terms in the momentum equations, one dimensional convection equation is usually used as a model equation for the assessment of accuracy. Among numbers of the higher order schemes, Holly-Preissmann (Holly & Preissmann, 1977) established a third order scheme, that uses an explicit three point interpolation polynomial with Courant number as a parameter. In the interpolation polynomial, terms of first order derivatives are introduced. Yang-Cunge (Yang & Cunge, 1989) developed a fifth order scheme, introducing both first and second order derivatives into the interpolation polynomial. Komatsu and others developed several higher order schemes (Komatsu *et al* , 1992). In their latest research (Asai *et al*, 1998), extra terms of higher order derivatives are introduced in the original equation to offset the second to fourth order numerical diffusion terms resulted from Crank-Nicolson scheme, and they obtained an implicit scheme which has fourth order accuracy. There are many other methods on this subject. Most of them have their own special technique in the derivation. But, a universal derivation method which will simplify the establishment of higher order accuracy scheme is not available yet. In this paper, the undetermined coefficient method which can be used to establish various order schemes is presented, and accuracy of a new scheme derived using this method is checked and compared with some of the previous methods.

The accuracy and efficiency are also verified through application to the rapidly varied flow in the experimental flume.

2. Undetermined coefficient method

A partial differential equation in general consists of several terms of the partial derivatives. Common procedure to establish a numerical scheme first selects or derives an appropriate scheme for each term in the equation to reflect the physical meaning of that term, then it obtains a discrete finite difference equation by substituting the selected scheme for each partial differential term into the original equation. But, regardless of forms of partial differential equations and scheme used in each term, in most conditions, final results describing unsteady processes can be written in a general form as follows

$$\sum a_i \phi_i^{n+1} = \sum b_i \phi_i^n \qquad (1)$$

where ϕ_i^n and ϕ_i^{n+1} are the values of variable ϕ at points (x_i, t_n) and (x_i, t_{n+1}), respectively. When the number of terms of the left hand side of equation (1) is more than 1, the scheme is called implicit, otherwise it is called explicit. The a_i and b_i are undetermined coefficients. Values of them change in different schemes.

Accuracy of equation (1) is mainly determined by: (i) concrete form of the partial differential equation, (ii) number of terms and values of a_i and b_i. When the concrete form of the partial differential equation and the grid points (node number) in the scheme are given, accuracy of the difference equation (1) is determined only by a_i and b_i. The undetermined coefficient method, which is presented in this paper and hereafter called 'HAUC' method, aims to obtain the possible highest accuracy of the difference equation through appropriate selection of a_i and b_i.

For an example, let's consider the one dimensional convection equation;

$$\frac{\partial\phi}{\partial t} + c\frac{\partial\phi}{\partial x} = 0 \qquad (2)$$

where, ϕ is a physical quantity transported by the flow, such as the solids concentration, c is the velocity of flow to x direction which is assumed to be a constant in the following discussion, t is time. Let assume the given difference equation is a 5 points explicit scheme as

$$\phi_i^{n+1} = b_1\phi_{i-2}^n + b_2\phi_{i-1}^n + b_3\phi_i^n + b_4\phi_{i+1}^n + b_5\phi_{xi-1}^n + b_6\phi_{xi}^n \qquad (3)$$

in which b_i are undetermined coefficients, and $\phi_{xi}^n = \left.\frac{\partial\phi}{\partial x}\right|_i^n$. Using Taylor expansions at (i, n), equation (3) gives

$$\sum_{i=0}^{5}\frac{\partial^i\phi}{\partial t^i} + O(\Delta t^6) =$$

$$\sum_{i=0}^{5}\left[[(-1)^i\left(2^i b_1 + b_2\right) + b_4]\frac{\partial^i\phi}{\partial x^i}\frac{(\Delta x)^i}{i!}\right] + O\left(\Delta x^6\right)$$

$$+ b_3\phi + b_6\frac{\partial\phi}{\partial x} + \sum_{i=1}^{5}(-1)^{i-1} b_5 \frac{\partial^i\phi}{\partial x^i}\frac{(\Delta x)^{i-1}}{(i-1)^{i-1}} + O\left(\Delta x^5\right) \qquad (4)$$

Comparing equation (4) with equation (2), following 6 independent algebraic equations are obtained

$$\begin{aligned}
2b_1 + b_2 - b_4 - \frac{b_5}{\Delta x} - \frac{b_6}{\Delta x} &= C_r \\
b_1 + b_2 + b_3 + b_4 &= 1 \\
-C_r^2 + 4b_1 + b_2 + b_4 - 2\frac{b_5}{\Delta x} &= 0 \\
C_r^3 - 8b_1 - b_2 + b_4 + 3\frac{b_5}{\Delta x} &= 0 \\
-C_r^4 + 16b_1 + b_2 + b_4 - 4\frac{b_5}{\Delta x} &= 0 \\
C_r^5 - 32b_1 - b_2 + b_4 + 5\frac{b_5}{\Delta x} &= 0
\end{aligned} \qquad (5)$$

This system of equations has unique solutions as

$$\begin{aligned}
b_1 &= \frac{1}{12}(C_r^5 - C_r^4 - C_r^3 + C_r^2) \\
b_2 &= \frac{1}{12}(9C_r^5 - 24C_r^4 - 3C_r^3 + 30C_r^2) \\
b_3 &= 1 - \frac{1}{12}(9C_r^5 - 21C_r^4 - 9C_r^3 + 33C_r^2) \\
b_4 &= \frac{1}{12}(-C_r^5 + 4C_r^4 - 5C_r^3 + 2C_r^2) \\
b_5 &= \frac{1}{12}(6C_r^5 - 12C_r^4 - 6C_r^3 + 12C_r^2)\Delta x \\
b_6 &= \frac{1}{12}(6C_r^5 - 18C_r^4 + 6C_r^3 + 18C_r^2 - 12C_r)\Delta x
\end{aligned} \qquad (6)$$

in which C_r is Courant number, $C_r = c\Delta t/\Delta x$. Equations (3) and (6) constitute a scheme which has fifth order accuracy to ϕ_{i-2}, ϕ_{i-1}, ϕ_i and ϕ_{i+1}, fourth order to ϕ_{xi-1} and ϕ_{xi}. It is hereafter named as HAUC1 scheme. This scheme is valid for the condition of c>0. When c<0, to keep the scheme upwind, equation (3) should be adjusted accordingly.

In order to keep higher order accuracy of first order derivative terms in equation (3), Holly-Preissmann method (Holly & Preissmann, 1977) can be used. Because c is a constant, differentiation of equation (2) by x gives

$$\frac{\partial \phi_x}{\partial t} + c\frac{\partial \phi_x}{\partial x} = 0 \tag{7}$$

which means that ϕ_x meets the convection equation, and its difference scheme can be written as

$$\phi_i^{n+1} = d_1\phi_{i-2}^n + d_2\phi_{i-1}^n + d_3\phi_i^n + d_4\phi_{i+1}^n + d_5\phi_{xi-1}^n + d_6\phi_{xi}^n \tag{8}$$

in which

$$d_i = \frac{db_i}{dC_r}\left(-\frac{1}{\Delta x}\right) \quad (i = 1,2,\cdots\cdots,6) \tag{9}$$

From equation (4), it can be found that if one want to derive more higher order schemes, more terms in equation (3) are needed. The available methods to obtain them are: (i) to increase number of nodes, or (ii) to introduce first or second order derivatives into the scheme as done in equation (3).

By changing terms in equation (3), different kinds of schemes can be derived using HAUC method. When terms in equation (3) are replaced by

$$\phi_i^{n+1} = b_1\phi_{i-1}^n + b_2\phi_i^n + b_3\phi_{xi-1}^n + b_4\phi_{xi}^n + b_5\phi_{xxi-1}^n + b_6\phi_{xxi}^n \tag{10}$$

values of undetermined coefficients derived are the same as the result of Yang-Cunge scheme (Yang & Cunge, 1989). If number of terms is decreased as

$$\phi_i^{n+1} = b_1\phi_{i-1}^n + b_2\phi_i^n + b_3\phi_{xi-1}^n + b_4\phi_{xi}^n \tag{11}$$

the coefficients become the same as those given by Holly-Preissmann scheme, which has third order accuracy to ϕ_i^n, and second to ϕ_{xi}^n. Similarly, if an implicit scheme is used as

$$a_1\phi_{i-1}^{n+1} + a_2\phi_i^{n+1} + a_3\phi_{i+1}^{n+1} = b_1\phi_{i-1}^n + b_2\phi_i^n + b_3\phi_{i+1}^n \tag{12}$$

the same coefficients are derived with Implicit HORNET scheme (Asai *et al*, 1998). If terms of $\phi_t = \frac{\partial \phi}{\partial t}$ are introduced as,

$$a_1\phi_{i-1}^{n+1} + a_2\phi_i^{n+1} + a_3\phi_{ti-1}^{n+1} = b_1\phi_{i-1}^n + b_2\phi_{ti-1}^n \tag{13}$$

the results are the same as Belleudy scheme (Belleudy & Sauvaget,1985).

3. Stability of HAUC1 scheme

Any Fourier component of numerical solution of equation (2) by HAUC1 scheme can be written as

$$\phi(x,t) = A\exp(-i\omega t)\exp(i\sigma x) \tag{14}$$

in which $i = \sqrt{-1}$, ω is angular velocity, σ wave number, A coefficient. Substitution of equation (14) into (3) gives

$$e^{-i\omega\Delta t} = B_1 + iB_2 \tag{15}$$

in which

$$\begin{aligned} B_1 &= b_1\cos 2\sigma\Delta x + b_2\cos\sigma\Delta x + b_3 \\ &\quad + b_4\cos\sigma\Delta x + b_5\sigma\sin\sigma\Delta x \\ B_2 &= -b_1\sin 2\sigma\Delta x - b_2\sin\sigma\Delta x \\ &\quad + b_4\sin\sigma\Delta x + b_5\sigma\cos\sigma\Delta x + b_6\sigma \end{aligned} \tag{16}$$

From equation (15), the corresponding amplification module $|G|$ and the relative phase error $\frac{\beta}{\beta_e}$, respectively, can be determined as following:

$$|G| = \left|e^{-i\omega\Delta t}\right| = \sqrt{B_1^2 + B_2^2} \tag{17}$$

$$\frac{\beta}{\beta_e} = \frac{\mathrm{Re}(\omega)}{c\sigma} = \frac{1}{2\pi}\frac{1}{C_r}\frac{L}{\Delta x}\tan^{-1}\left(\frac{B_2}{B_1}\right) \tag{18}$$

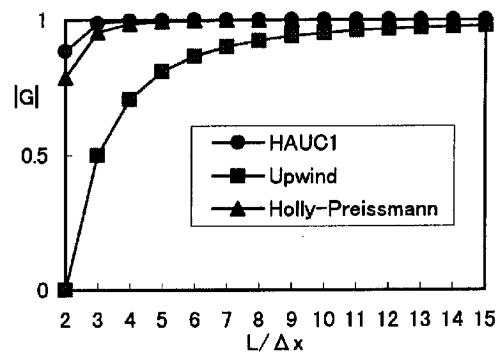


Fig. 1 Amplification modulus for the three schemes (C_r=0.5)

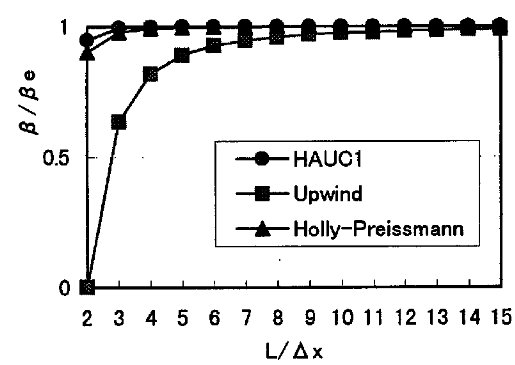


Fig. 2 Relative phase error for the three scheme (C_r=0.25)

in which L is the wave length, $L = 2\pi/\sigma$. Relations between $|G|$, β/β_e and the relative wave length $L/\Delta x$ are plotted in Fig. 1 and Fig. 2. Upwind and Holly-Preissmann schemes are also given in the figures for comparison. It can be seen that HAUC1 agree well with the exact solution at large $L/\Delta x$ and a little better than Holly-Preissmann scheme in the region of small $L/\Delta x$.

4. Numerical results and comparison with other schemes

4.1 Computation of convection equation (2)

Stability analysis shows that HAUC1 has the merits of small dissipation and dispersion error. But, there are derivative terms in the scheme. Sensitivity of initial error of these derivative terms to numerical result should be tested. To do it, Gauss distribution, in which the peak is 1(located at $x = 50\,\text{m}$) and the standard deviation is 1.5, is used as initial condition of the calculation. Initial conditions of ϕ_x are given in two cases. In case one, it is given according to the theoretical derivative function of Gauss distribution. In case two, all the initial values of ϕ_x in every nodes are assigned to be 0. The calculated results of these two cases with $\Delta t = 0.2\,\text{sec}$, $\Delta x = 1\,\text{m}$, $c = 0.5\,\text{m/sec}$ and time interval of calculation $T = 100\,\text{sec}$, and the exact solution are plotted in Fig. 3. Results of both two cases agree well with the exact solution. Error in initial condition of ϕ_x attenuates automatically in the calculation and has no obvious effects to the result.

In order to check adaptability of HAUC1 to discontinuous distribution and different shapes of distribution curve, and to compare with calculated results by many other schemes under the same condition(Asai, *et al*, 1998), a rectangular pulse and a half ellipse cases are added to the Gaussian distribution case. The height of the rectangular pulse is 1 with 10m in top width .Center of the ellipse is at $x = 125m$, the peak is 1 and radius along the flow direction is $10\,m$. Initial values of ϕ_x in the three cases are assigned with 0. Calculated results are shown in Fig. 4. Parameters in the calculation are $\Delta t = 0.2\,\text{sec}$, $\Delta x = 1\,\text{m}$, $c = 0.5\,\text{m/sec}$ and time interval of calculation $T = 100\,\text{sec}$. From Fig. 4, it can be found that numerical dissipation of HAUC1 is less than that of Holly-Preissmann scheme, and its numerical oscillation is less than that of Implicit HORNET (Asai, *et al*, 1998) and 6-point schemes(Komatsu, *et al*, 1985). HAUC1 can simulate both peak and discontinuity conditions well.

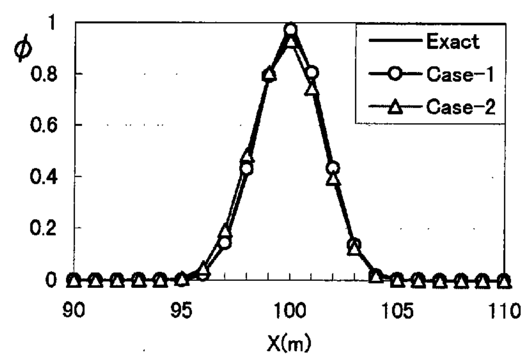


Fig. 3 Calculated results under different initial values of ϕ_x
(Δ x=1.0m, Δ t=0.2sec, c=0.5m/s, T=100sec)

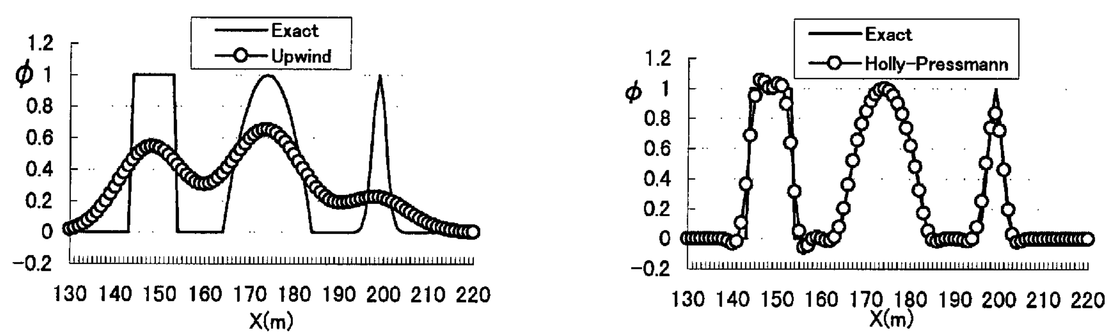


Fig. 4 Comparison of HAUC1 with other schemes

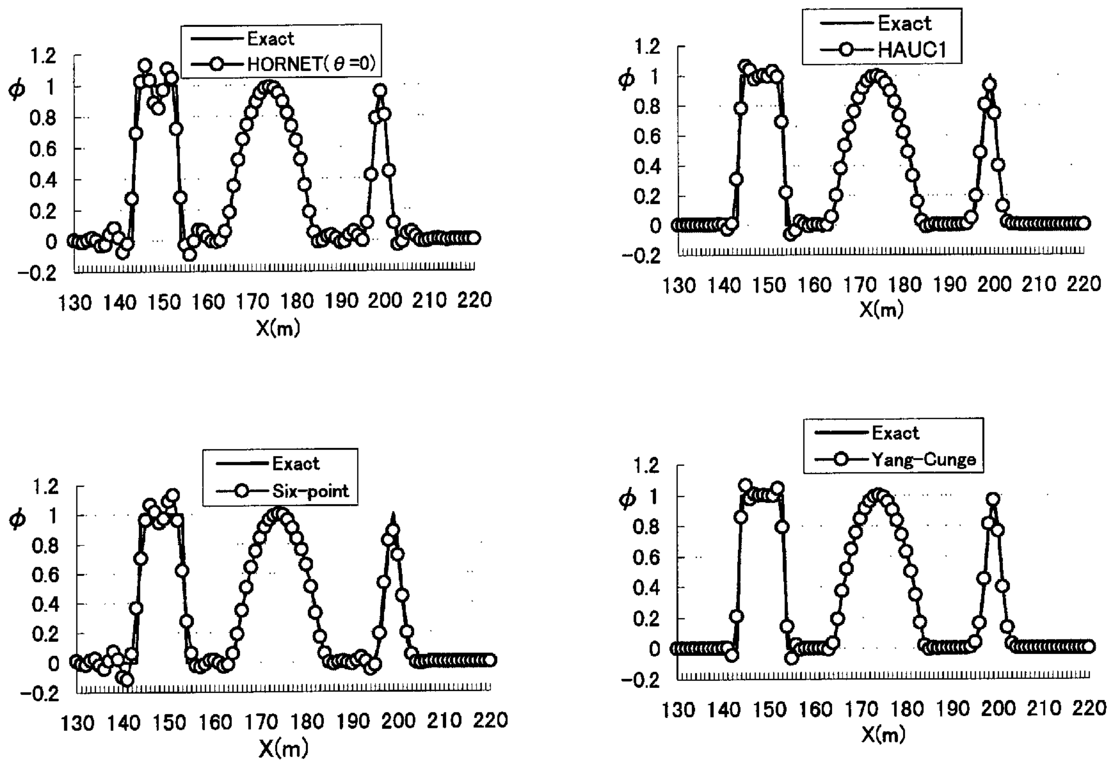


Fig. 4 Comparison of HAUC1 with other schemes (continued)

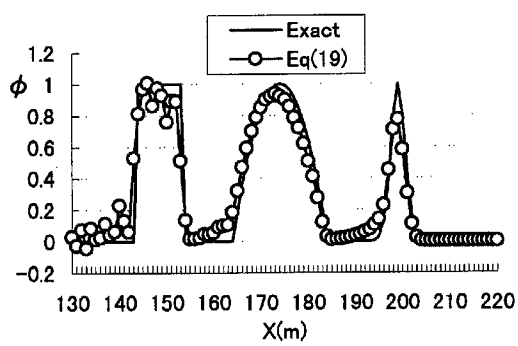


Fig. 5 Calculated result by scheme (19)

It should be noted, however, that not all high order schemes can work successfully. For example, the following scheme

$$\phi_i^{n+1} = b_1\phi_{i-1}^n + b_2\phi_i^n + b_3\phi_{i+1}^n + b_4\phi_{xi-1}^n + b_5\phi_{xi}^n + b_6\phi_{xi+1}^n \qquad (19)$$

$$b_1 = \frac{1}{4}\left(-3C_r^5 - 2C_r^4 + 5C_r^3 + 4C_r^2\right)$$

$$b_2 = C_r^4 - 2C_r^2 + 1$$

$$b_3 = \frac{1}{4}\left(3C_r^5 - 2C_r^4 - 5C_r^3 + 4C_r^2\right)$$

$$b_4 = \frac{1}{4}\left(-C_r^5 - C_r^4 + C_r^3 + C_r^2\right)\Delta x \qquad (20)$$

$$b_5 = -\left(C_r^5 - 2C_r^3 + C_r\right)\Delta x$$

$$b_6 = \frac{1}{4}\left(-C_r^5 + C_r^4 + C_r^3 - C_r^2\right)\Delta x$$

has a fifth order to ϕ_i^n, fourth to ϕ_{xi}^n. The results of simulation by this scheme are shown in Fig. 5. As is evident in the figure, this scheme can not well reproduce the exact solutions. Selection of terms and nodes in a scheme is very important to simultaneously keep both numerical dissipation and dispersion small.

4.2 Calculation of Burgers' equation

Burgers' equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0 \qquad (21)$$

is a nonlinear convection equation which may be viewed as a simple analog of the Euler equation for the flow of a non viscous fluid. One of the method to calculate this nonlinear equation is to replace the convection velocity u with local freezing constant c. Thus, equation (21) can be approximated by

$$\frac{\partial u}{\partial t}+c\frac{\partial u}{\partial x}=0 \tag{22}$$

Although theoretical relation between c and velocity u at nodes is difficult to obtain. If we assume

$$C_r=\frac{c_i^n \Delta t}{\Delta x} \tag{23}$$

in which

$$c_i^n=\theta u_{i-1}^n+(1-\theta)u_i^n, \quad 0\le\theta\le 1, \tag{24}$$

HAUC1 can be used to simulate equation (21).

Numerical result and the exact solution of equation (21) under the initial condition,

$$u(x,0)=\begin{cases}1, x\le 10\\ 0, x>10\end{cases} \tag{25}$$

is plotted in Fig. 6. The numerical computation agrees well with the exact solution. Thus, HAUC1 can be used to compute such a non linear equation.

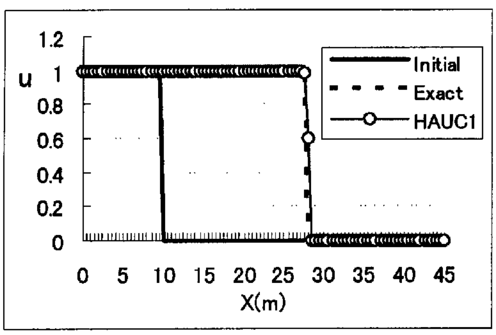


Fig.6 Comparison of the numerical results with the exact solution for Burgers' equation (θ =0.4)

5. Application to dam break flow

Equations of one dimensional open channel flow are

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=0 \tag{26}$$

$$\frac{\partial Q}{\partial t}+\frac{Q}{A}\frac{\partial Q}{\partial x}+Q\frac{\partial Q/A}{\partial x}+gA\left(\frac{\partial h}{\partial x}-S_0+S_f\right)=0 \tag{27}$$

where A is area of cross section, Q discharge, g acceleration of gravity, h water depth, S_0 bed slope, S_f friction slope. Using the HAUC1 scheme and staggered grid, computation procedure in each two time steps($2\Delta t$) can be divided into three phases.

The first phase is to calculate the first two terms of equation (27). The difference scheme is

$$Q_i^*=b_1Q_{i-2}^n+b_2Q_{i-1}^n+b_3Q_i^n+b_4Q_{i+1}^n+b_5Q_{xi-1}^n+b_6Q_{xi}^n \tag{28}$$

$$b_1=\frac{1}{12}(C_r^5-C_r^4-C_r^3+C_r^2)$$

$$b_2=\frac{1}{12}(9C_r^5-24C_r^4-3C_r^3+30C_r^2)$$

$$b_3=1-\frac{1}{12}(9C_r^5-21C_r^4-9C_r^3+33C_r^2)$$

$$b_4=\frac{1}{12}(-C_r^5+4C_r^4-5C_r^3+2C_r^2)$$

$$b_5=\frac{1}{12}(6C_r^5-12C_r^4-6C_r^3+12C_r^2)\Delta x$$

$$b_6=\frac{1}{12}(6C_r^5-18C_r^4+6C_r^3+18C_r^2-12C_r)\Delta x$$

in which $C_r=\dfrac{2\left[\theta(Q/A)_{i-1}^n+(1-\theta)(Q/A)_i^n\right]\Delta t}{\Delta x}$, $\theta=0.4$.

The second phase is, using the result Q_i^* calculated in the first phase, to calculate Q_i^{n+2} from equation (27) with the scheme

$$\frac{Q_i^{n+2}-Q_i^*}{2\Delta t}=-Q_i^n\frac{2u_{i+1}^n+3u_i^n-6u_{i-1}^n+u_{i-2}^n}{6\Delta x}$$
$$-g\frac{A_{i+1/2}^{n+1}-A_{i-1/2}^{n+1}}{2}\frac{(h+z_b)_{i+1/2}^{n+1}-(h+z_b)_{i-1/2}^{n+1}}{\Delta x}$$
$$-g\left(\frac{n_{i+1/2}+n_{i+1/2}}{2}\right)^2\frac{Q^{n+2}+Q^n}{R_{i+1/2}^{n+1}+R_{i-1/2}^{n+1}}\left|u_i^n\right|\left(\frac{R_{i+1/2}^{n+1}+R_{i-1/2}^{n+1}}{2}\right)^{-1/3} \tag{29}$$

in which, the convection term of equation (27) is not included. In equation (29), z_b is bed elevation, n is roughness, R is hydraulic radius, u is average velocity,

$$u_i^n=\frac{2Q_i^n}{h_{i+1/2}^{n+1}+h_{i-1/2}^{n+1}} \tag{30}$$

The third phase is to calculate A from continuity equation (26). The scheme is

$$A_{i+1/2}^{n+3}=A_{i+1/2}^{n+1}-\frac{2\Delta x}{\Delta x}\left(Q_{i+1}^{n+2}-Q_{i-1}^{n+2}\right) \tag{31}$$

At each phase, used schemes are explicit ones.

With the above schemes, a one-dimensional dam break flow is simulated. Computed result is compared with flume test data. This experiment was made by the third author in 1989. The flume is 5 m long with 0.5 m in width and 0.2 m in depth.. The gate is installed at 1.2 m from the exit. Water depth is 6.7 cm at the upstream with a dry tailwater. The bed is horizontal. Initially, the water is still and the gate is removed suddenly.

Fig. 7 is the comparison between computed (n=0.01, $\Delta t = 0.001\,\text{sec}$, $\Delta x = 0.05\,\text{m}$) and flume test results. Ritter's analytical result which is obtained from omitting friction term is also plotted. It shows that simulated result agrees well with both experimental and analytical results.

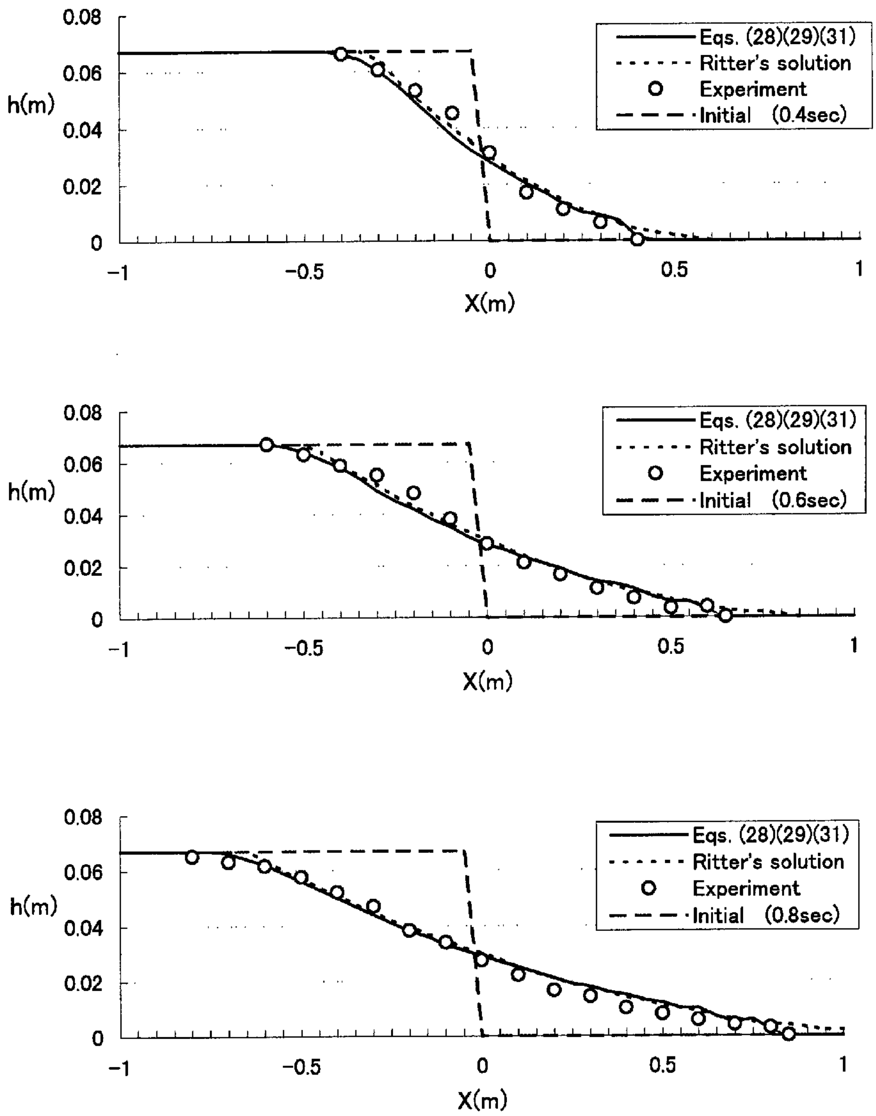


Fig. 7 Numerical result of one dimensional dam break flow (Initial water depth at the upper stream is 0.067m)

When the tailwater is not dry, according to Nakagawa's experimental results, surface profile of the dam break flow changes with the ratio of initial upstream water depth to downstream one and can be classified into: (1) $0 < h_1 / h_0 \leq 0.45$, moving hydraulic jump; (2) $0.45 < h_1 / h_0 \leq 0.77$, unstable state of undular bore; (3) $0.77 < h_1/h_0 < 1$, stable state of undular bore, where h_1, h_0 are initial water depth at downstream and upstream, respectively (Nakagawa, H. *et al.* 1969).

St. Venant equations (26), (27) are derived with the assumption that vertical pressure distribution is hydrostatic. While treating undular bore, vertical acceleration terms should be added to the right-hand side of equation (27) as follows (Iwasa, Y., 1955; Hosoda, T., *et al.* 1994):

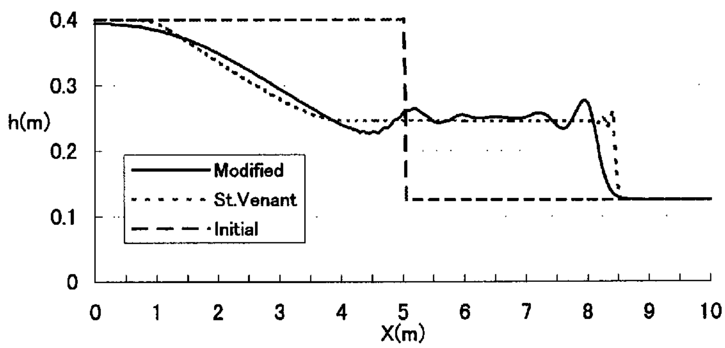
$$-\frac{\partial}{\partial x}\left(\frac{1}{3}h^2u^2\frac{\partial^2 A}{\partial x^2}+\frac{2}{3}h^2u\frac{\partial^2 A}{\partial x\partial t}+\frac{1}{3}h^2\frac{\partial^2 A}{\partial t^2}\right) \quad (32)$$

For the rectangular cross section with constant width, the above terms can be rewritten and discriminated to (in unit width)

$$\begin{aligned}
&\frac{1}{3}h^2\frac{\partial^3 M}{\partial t\partial x^2}+\frac{1}{3}\frac{\partial h^2}{\partial x}\frac{\partial^2 M}{\partial t\partial x}+\frac{2}{3}\frac{\partial Mh}{\partial x}\frac{\partial^2 M}{\partial x^2}\\
&+\frac{2}{3}Mh\frac{\partial^3 M}{\partial x^3}-\frac{1}{3}\frac{\partial M^2}{\partial x}\frac{\partial^2 h}{\partial x^2}-\frac{1}{3}M^2\frac{\partial^3 h}{\partial x^3}\\
&=\frac{1}{6\Delta t}\left(\frac{h_{i+1/2}^{n+2}+h_{i-1/2}^{n+2}}{2}\right)^2\\
&\quad\times\left(\frac{M_{i+1}^{n+3}-2M_i^{n+3}+M_{i-1}^{n+3}}{\Delta x^2}-\frac{M_{i+1}^{n+1}-2M_i^{n+1}+M_{i-1}^{n+1}}{\Delta x^2}\right)\\
&+\frac{1}{6\Delta t}\frac{\left(h_{i+1/2}^{n+2}\right)^2-\left(h_{i-1/2}^{n+2}\right)^2}{2\Delta x}\left(\frac{M_{i+1}^{n+3}-M_{i-1}^{n+3}}{2\Delta x}-\frac{M_{i+1}^{n+1}-M_{i-1}^{n+1}}{2\Delta x}\right)\\
&+\frac{2}{3}\frac{M_{i+1}^{n+1}\left(h_{i+3/2}^{n+2}+h_{i+1/2}^{n+2}\right)-M_{i-1}^{n+1}\left(h_{i-1/2}^{n+2}+h_{i-3/2}^{n+2}\right)}{4\Delta x}\\
&\quad\times\frac{M_{i+1}^{n+1}-2M_i^{n+1}+M_{i-1}^{n+1}}{\Delta x^2}\\
&+\frac{2}{3}\frac{h_{i+1/2}^{n+2}+h_{i-1/2}^{n+2}}{2}M_i^{n+1}\frac{M_{i+1}^{n+1}-3M_i^{n+1}+3M_{i-1}^{n+1}-M_{i-2}^{n+1}}{\Delta x^3}\\
&-\frac{1}{3}\frac{\left(M_{i+1}^{n+1}\right)^2-\left(M_{i-1}^{n+1}\right)^2}{2\Delta x}\frac{h_{i+3/2}^{n+2}-2h_i^{n+2}+h_{i-1/2}^{n+2}}{\Delta x^2}\\
&-\frac{1}{3}\left(M_i^{n+1}\right)^2\frac{h_{i+3/2}^{n+2}-2h_{i+1/2}^{n+2}+2h_{i-3/2}^{n+2}-h_{i-5/2}^{n+2}}{2\Delta x^3}
\end{aligned} \quad (33)$$

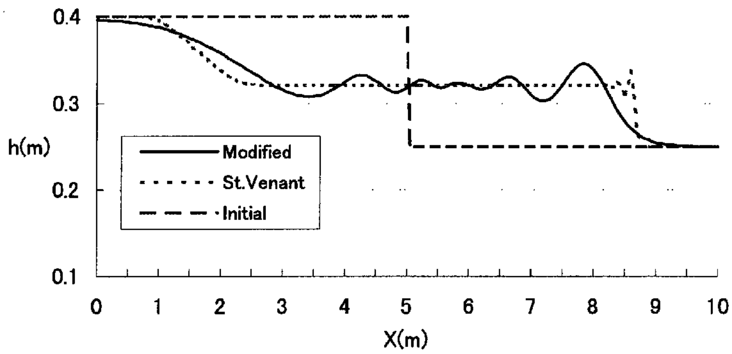
where $M=Q/B$, B is the width of the channel. Substitution of equation (33) multiplied by B into (29) gives an implicit scheme which can be computed using tridiagonal matrix algorithm (TDMA) method.

Computed results using thus modified Saint Venant equation with different initial ratios of water depth are shown in Fig. 8. The flow is simulated in the tank with 10 m long and 0.5 m wide. Manning roughness is 0.01. The bed is horizontal. There is no inflow and outflow at the ends. Initially, the gate is installed at the middle of the flume and removed suddenly. For the sake of comparison, results of Saint Venant equation with no modification are also plotted. It shows that the model using modified equations can simulate moving hydraulic jump and undular bore at least qualitatively. But the unmodified one can not simulate undular bore. In near future, experiments will be made to examine the numerical results of the modified equation.

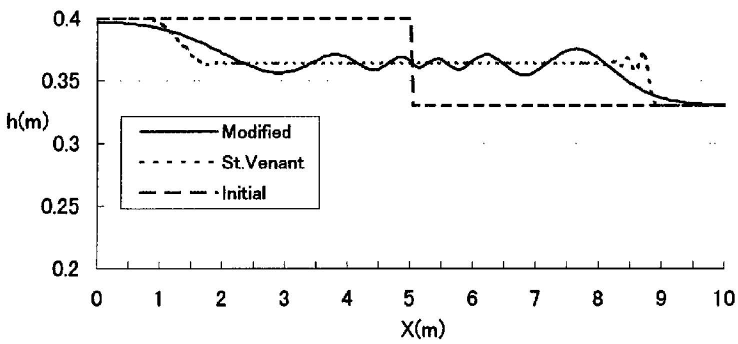


(h_1=0.125m, h_0=0.4m, h_1/h_0=0.313, 2 sec after removing of gate)

Fig.8 Simulations of moving hydraulic jump and undular bore



(h_l=0.25m, h_0=0.4m, h_l/h_0=0.625, 2 sec after removing of gate)



(h_l=0.33m, h_0=0.4m, h_l/h_0=0.825, 2 sec after removing of gate)

Fig.8 Simulations of moving hydraulic jump and undular bore (continued)

6. Conclusion

Effectiveness of the undetermined coefficient method, and accuracy of HAUC1 scheme can be concluded as follows:

(1) Undetermined coefficient method is a practical and applicable method to establish various finite difference schemes, especially, higher order schemes.

(2) Stability of HAUC1 scheme is analyzed and compared with other schemes. It has the merits of both less dissipation and less dispersion errors.

(3) Computational results using HAUC1 agree well with the exact solutions of one dimensional pure convection equation and Burgers' equation. HAUC1 can simulate discontinuous and fast-changing phenomena well.

(4) HAUC1 can be used to simulate one dimensional dam break flow and the result agrees well with the experimental data.

(5) Computation using a modified Saint Venant equation, that is added the vertical acceleration terms shows the scheme can simulate both undular bores and moving hydraulic jump. But, further experimental research is needed to verify the equation as well as the computation scheme.

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要　旨

未定係数法を用いた高精度数値計算スキームの誘導手法が提示されている。この手法を用いて、過去に別の手法で求められている高精度計算スキームを導出するとともに、HAUC1 と名付けた新たな高精度計算スキームを開発している。一次元移流方程式および Burgers 方程式の厳密解とこの計算スキームによる数値計算結果とを比較検討したところ、HAUC1 スキームは高い計算精度と安定な解を与えることが確認された。本計算スキームと他の高精度計算スキームの計算精度を比較したところ、本スキームの有用性が確認された。さらに、HAUC1 スキームを一次元 dam-break 流れに適用し、不連続波頭を持つ移動跳水と波状段波の再現が可能であることが分かった。

キーワード：高精度スキーム，未定係数方法，波状断波，移動跳水，ダムブレク流れ