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Posture Control and Stability of a *General* 1-Trailer System: A Lyapunov Approach

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1 Introduction

Posture control and stability has been an integral part of nonholonomic motion planning, and recently, has garnered monomaniacal support and attention. Basically, nonholonomic motion planning involves finding a feasible trajectory from some initial configuration to a desired one while satisfying the velocity constraints of the system. Posture control adds another dimension to the problem wherein one is also required to harvest exact orientations at the target positions. Inclusion of obstacles makes the overall task increasingly complicated, as many more points in the workspace are no longer reachable, hence a patently preponderant task to many researchers.

A typical example of the nonholonomic system is the tractor-trailer mobile robot. By and large, these articulated robots play a pivotal role in the road freight transportation, nowadays. A wide range of trailer systems have been utilized to support research on control and motion planning of nonholonomic systems [2]. In this ever-growing repertoire, researchers are continuously churning out new and more efficient algorithms for motion planning and control of these articulated or multi-body vehicles that are capable of performing wide-ranging tasks in various different environments, which may be hazardous or even inaccessible to humans [7].

This paper embarks upon improving, in general, trajectory planning and posture control of *general* 1-trailer robots. Specifically, we control its *point-to-point* motion via a new collision-avoidance scheme. Integral to this motion planning , inter alia, is the critical issue of *posture control and stability*, the main emphasis of this paper. In [6] "near-perfect" orientations were obtained by employing a novel technique of fixing obstacles at regular intervals on the boundary lines of a parking bay. However, this method easily becomes cumbersome and the computations tedious if the number of fixed obstacles along the target is increased. Instead, in this paper, we erect *ghost walls* along the non-entry routes of the target. Now to avoid these ghost walls, we draw inspiration from Khatib's collision avoidance scheme in [5] to propose a new technique to effectively avoid these ghost walls and *per se* orchestrate the desired orientations, of every solid body of the articulated robot, at its corresponding target position. This variant technique also contributes in reducing the complexity of the motion planning algorithm designed in [6].

2 Vehicle Model

In this paper we shall consider a *general* 1-trailer system which consists of a rear wheel driven tractor and an off-axle hitched two-wheeled passive trailer. Essentially, a kingpin joins the two solid bodies with c and L_2 as positive lengths, from the midpoint point of the rear axle of the car and the trailer, respectively (see Fig. 1). The tractor utilised in this research basically performs motions similar to that of a car-like robot (Reeds and Shepp's model), with front-wheel steering and decrees the path of the attached trailer.

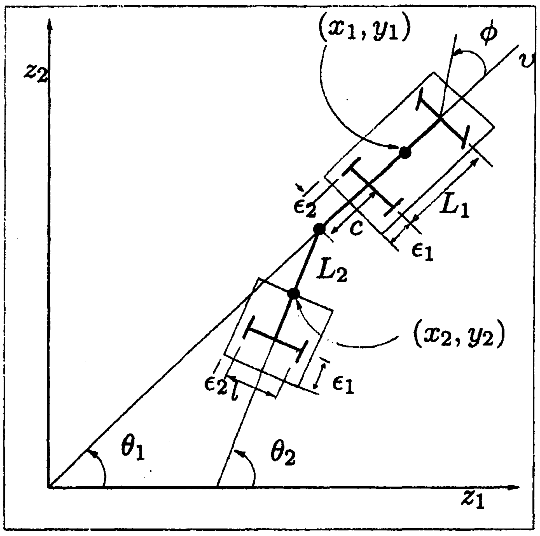

Figure 1: Kinematic model of a *general* 1-trailer robot

With reference to Fig. 1, (x_i, y_i) represents the Cartesian coordinates and gives the reference point of each solid body of the articulated robot, while θ_i gives its orientation with respect to the z_1-axis. The connections between the two bodies give rise to the following holonomic constraints (defining $c(.) = cos(.)$, $s(.) = sin(.)$ and $t(.) = tan(.)$):

$$\begin{aligned} x_2 &= x_1 - \left(\frac{L_1 + 2c}{2}\right) c(\theta_1) - \left(\frac{L_2 + a}{2}\right) c(\theta_2) \,, \\ y_2 &= y_1 - \left(\frac{L_1 + 2c}{2}\right) s(\theta_1) - \left(\frac{L_2 + a}{2}\right) s(\theta_2) \,. \end{aligned}$$

These constraints reduce the dimension of the configuration space since (x_2, y_2) could be expressed completely in terms of $(x_1, y_1, \theta_1, \theta_2)$.

If we let m be the mass of the robot, F the force along the axis of the tractor, Γ the torque about a vertical axis at (x_1, y_1) and I the moment of inertia of the tractor, then the dynamic model of a *general* 1-trailer system, with respect to the reference point of the tractor, is given by

$$\left.\begin{aligned} \dot{x}_1 &= c(\theta_1)v - \frac{L_1}{2}s(\theta_1)\omega \,, & \dot{\theta}_1 &= \frac{v}{L_1}t(\phi) := \omega \,, & \dot{v} &= \sigma_1 := F/m \,, \\ \dot{y}_1 &= s(\theta_1)v + \frac{L_1}{2}c(\theta_1)\omega \,, & \dot{\theta}_2 &= \frac{1}{L_2}\left(s((\theta_1 - \theta_2))v - c((\theta_1 - \theta_2))c\,\omega\right) \,, & \dot{\omega} &= \sigma_2 := \Gamma/I \,. \end{aligned}\right\} \tag{1}$$

Here v and ω are the translational and rotational velocities and, σ_1 and σ_2, are the instantaneous translational and rotational accelerations, respectively, of the tractor. For simplicity, we have let $\phi = \theta_1$. A state of the 1-trailer system is then described by $\mathbf{z} = (x_1, y_1, \theta_1, \theta_2, v, \omega) \in X \triangleq \mathbb{R}^6$. By and large, the motion is controlled via the instantaneous accelerations of the tractor.

To ensure that the entire vehicle safely steers pass an obstacle, the planar vehicle can be represented as a simpler fixed-shaped object, such as a circle, a polygon or a convex hull [8]. In [7], the authors represented a *standard* 1-trailer system by the smallest circle possible, given some clearance parameters. The obvious problem of the representation was the creation of unwarranted obstacle space, which further curtailed the set of reachable points in the configuration space. In this research, given the *clearance parameters* ϵ_1 and ϵ_2, we shall enclose the articulated vehicle within two separate protective circular regions, i.e. a protective region for each solid body, which basically palliates the unnecessary growth of the C-space in [7], and subsequently, presents a greater set of options. Hence, circular region C_1 is centered at (x_1, y_1) with a radius of $r_{V1} := \sqrt{(L_1 + 2\epsilon_1)^2 + (l + 2\epsilon_2)^2 / 4} := \frac{L_1}{2} + c$, while C_2 centered at (x_2, y_2) having a radius of $r_{V2} := \sqrt{(L_2 + \epsilon_1)^2 + (l + 2\epsilon_2)^2 / 4} := \frac{L_2 + a}{2}$. For simplicity we treat $L_2 := L_1 + a$ and $c := \epsilon_1 + a$, where a is a small offset as seen in Fig. 2.

3 Formulation of the Problem

This section formulates collision free trajectories the robot system under kinodynamic constraints in a fixed and bounded workspace. It is assumed that there is *a priori* knowledge of the whole workspace. Utilizing the Direct Method of Lyapunov, we want to design the acceleration controllers, σ_1 and σ_2, such that the robot will navigate safely in the workspace, reach a neighborhood of its target and be aligned to a pre-determined final posture. To obtain a feasible solution of this posture control problem, we utilize the method of artificial potentials, a prominent method in motion planning of nonholonomic systems. We begin by describing precisely, the target, the workspace, all obstacles, and discuss the new concept of *ghost walls* which facilitates the desired orientations.

3.1 Posture

We shall consider position and the orientation separately to highlight and elucidate the importance of our new technique.

3.1.1 Position

First, we affix a target for the robot to reach after some time t. For the ith body of the tractor-trailer system, we define a target $T_i = \{(z_1, z_2) \in \mathbb{R}^2 : (z_1 - p_{i1})^2 + (z_2 - p_{i2})^2 \leq rt_i^2\}$ with center (p_{i1}, p_{i2}) and radius rt_i. For attraction to the targets, we consider a potential function:

$$V(\mathbf{z}) = \frac{1}{2}\left(\sum_{i=1}^{2} \{(x_i - p_{i1})^2 + (y_i - p_{i2})^2\} + v^2 + \omega^2\right), \tag{2}$$

Note that if we define $\mathbf{z}_e := (p_{11}, p_{12}, p_{13}, p_{23}, 0, 0) \in \mathbb{R}^6$, then we see that $V(\mathbf{z}_e) = 0$. As a consequence, the role of V in the Lyapunov function is to ensure that system trajectories start and remain close to $\mathbf{z}_e$, forcing $\mathbf{z}_e$, via our controllers, to be an equilibrium point of system (1).

3.1.2 Orientation

One difficulty that exists with continuous time-invariant controllers is that although the final position is reachable, it is virtually impossible to harvest exact orientations at the equilibrium point of this special class of dynamical systems, a direct result of Brockett's Theorem [1]. In [6] we provided, inter alia, a *practical solution* to this problem by fixing a number of obstacles at regular intervals on the boundary lines of a parking bay. However, the method easily becomes cumbersome if the number of parking bays is increased, hence making the controllers computationally intensive.

In this paper, we introduce a new algorithm to obtain final orientations. The first part of the algorithm encompasses the construction of *ghost walls* along the sides of a target. As seen in figure 2, two ghost walls are constructed along the side of the target parallel to the final orientation of the robot, while a third ghost wall is erected in-front of the target to curtail the unnecessary and time-consuming backward-forward iterations commonly found in literature. The

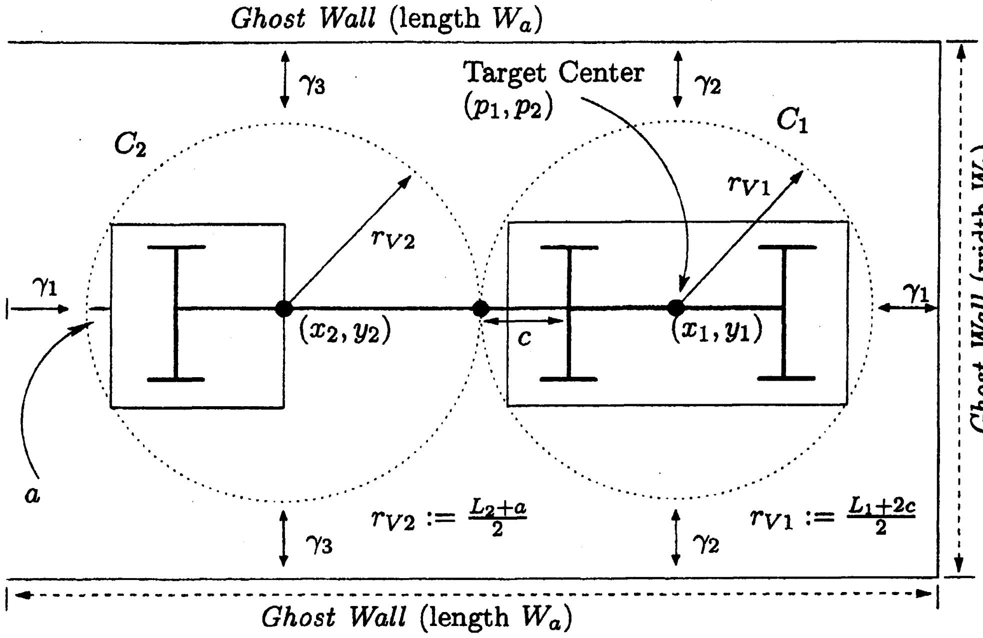

Figure 2: Schematic diagram of a *general* 1-trailer system in a parking bay showing the mandatory safety margins γ_1, γ_2 and γ_3.

second part of the algorithm is avoidance of these ghost walls in order to *force* the occurrence of desired orientations. Here we utilize an idea inspired by the work carried out by Khatib in [5]. We design a variant optimization technique where we calculate the minimum distance from the robot to a ghost wall and avoid the resultant point on that ghost wall. Avoiding the closest point on a ghost wall is basically affirming that the mobile robot avoids the whole wall. This algorithm helps greatly to retain the simplicity of the navigation laws.

Now let us consider the kth *ghost wall* in the z_1z_2-plane, from the point (a_{k1}, b_{k1}) to the point (a_{k2}, b_{k2}). We assume that the point (x_i, y_i) is closest to it at the tangent line which passes through the point. From geometry, it is known that if (Lx_{ik}, Ly_{ik}) is the point of intersection of the tangent, then $Lx_{ik} = a_{k1} + \lambda_{ik}(a_{k2} - a_{k1})$, $Ly_{ik} = b_{k1} + \lambda_{ik}(b_{k2} - b_{k1})$,

where $\lambda_{ik} = (x_i - a_{k1})d_k + (y_i - b_{k1})r_k$ and $d_k = \frac{a_{k2}-a_{k1}}{(a_{k2}-a_{k1})^2+(b_{k2}-b_{k1})^2}$, $r_k = \frac{b_{k2}-b_{k1}}{(a_{k2}-a_{k1})^2+(b_{k2}-b_{k1})^2}$.

If $\lambda_{ik} \geq 1$, then we let $\lambda_{ik} = 1$, if $\lambda_{ik} \leq 0$, then we let $\lambda_{ik} = 0$, otherwise we accept the value of λ_{ik} between 0 and 1, in which case there is a perpendicular line to the point (Lx_{ik}, Ly_{ik}) on the ghost wall from the center (x_i, y_i) of ith body of the vehicle at every time $t \geq 0$.

For the ith body of the robot to avoid the closest point of each of the kth ghost wall, we consider a repulsive function

$$LS_{ik}(\mathbf{z}) = \frac{1}{2}\left\{(x_i - Lx_{ik})^2 + (y_i - Ly_{ik})^2 - r_{Vi}^2\right\} , \tag{3}$$

for $k = 1, \ldots, m$ and $i = 1, 2$. The main idea here is to attach necessary and sufficient repulsive potentials to these *ghost walls* so that final orientations could be *forced* to eventuate.

3.2 Kinematic constraints

The kinematic constraints are the nonholonomy of the vehicle and any obstacle in the workspace. The nonholonomy of the vehicle is reflected in the dynamic model of system (1). The obstacles are (a) the four boundaries of a rectangular workspace, (b) stationary solids in the workspace, (c) the boundaries of the parking bay, and (d) the *artificial obstacles* due to the mechanical singularities of the systems. These constraints and the corresponding potential functions, in the interest of brevity, are discussed below.

3.2.1 Workspace limitations

We desire to setup a framework for the workspace of our robot. It is a fixed, closed and bounded rectangular region, defined as $WS = \{(z_1, z_2) \in \mathbb{R}^2 : 0 \le z_1 \le \eta_1, 0 \le z_2 \le \eta_2\}$. We require the robot to stay within the rectangular region at all time $t \ge 0$. Therefore, we impose the following boundary conditions; left boundary $(z_1, z_2) : z_1 = 0$; upper boundary $(z_1, z_2) : z_2 = \eta_2 > 0$; right boundary $(z_1, z_2) : z_1 = \eta_1 > 0$; and lower boundary $(z_1, z_2) : z_2 = 0$. In our Lyapunov-based scheme, these boundaries are considered as *fixed obstacles*. For the robot to avoid these, we define the following potential functions for the left, upper, right and lower boundaries, respectively:

$$W_{i1}(\mathbf{z}) = x_i - r_{Vi} , \qquad W_{i2}(\mathbf{z}) = \eta_2 - (y_i + r_{Vi}) , \tag{4a-b}$$

$$W_{i3}(\mathbf{z}) = \eta_1 - (x_i + r_{Vi}) , \qquad W_{i4}(\mathbf{z}) = y_i - r_{Vi} , \tag{4c-d}$$

each of which is positive over its domain, for $i = 1, 2$. Embedding these functions into the control laws will contain the motions of the tractor-trailer robot to within the specified boundaries of the workspace.

3.2.2 Fixed obstacles in the workspace

Let us fix w solid obstacles within the boundaries of the workspace. We assume that the qth obstacle is circular with center (o_{q1}, o_{q2}) and radius ro_q. For its avoidance, we adopt

$$FO_{iq}(\mathbf{z}) = \frac{1}{2}\left\{(x_i - o_{q1})^2 + (y_i - o_{q2})^2 - (ro_q + r_{Vi})^2\right\} , \tag{5}$$

for $q = 1, ..., w$ and $i = 1, 2$. This function has the same effect as the other repulsive functions defined above. A point of caution here is that there needs to be sufficient free space between any two fixed obstacles so that the entire mobile robot can steer through if warranted.

3.3 Dynamics constraints

Modulus bounds on the velocities and accelerations are treated as dynamics constraints. In practice, the steering and bending angles of an articulated robot is limited due to mechanical singularities, while the translational speed is restricted due to safety reasons. Subsequently, we incorporate the following constraints; (i) $|v| \leq v_{max}$, where v_{max} is the *maximal speed* of the tractor; (ii) $|\phi| \leq \phi_{max} < \frac{\pi}{2}$, where ϕ_{max} is the *maximal steering angle* of the tractor; and (iii) $|\theta_2 - \theta_1| \leq \theta_{max} < \frac{\pi}{2}$, where θ_{max} is the *maximum bending angle* of the trailer with respect to the orientation of the tractor. The trailer can freely rotate within $]-\frac{\pi}{2}, \frac{\pi}{2}[$ about their linking point with the tractor [4]. We consider these mechanical constraints as *artificial obstacles*, and for the avoidance, we choose positive functions

$$
\begin{aligned}
DC_1(\mathbf{z}) &= \frac{1}{2}(v_{max} - v)(v_{max} + v) , &(6)\\
DC_2(\mathbf{z}) &= \frac{1}{2}\left(\frac{v_{max}}{|\rho_{min}|} - \omega\right)\left(\frac{v_{max}}{|\rho_{min}|} + \omega\right) , &(7)\\
DC_3(\mathbf{z}) &= \frac{1}{2}\left(\theta_{max} - (\theta_2 - \theta_1)\right)\left(\theta_{max} + (\theta_2 - \theta_1)\right) , &(8)
\end{aligned}
$$

which would guarantee the adherence to the restrictions placed upon translational velocity v, steering angle ϕ, and the rotation θ_2 of the trailer, respectively.

3.4 Auxiliary Function

To guarantee the convergence of the tractor-trailer mobile robot to its target. This inclusion also takes care of other associated problems, in particular, goals nonreachable with obstacles nearby (GNRON) [3]. Thus we introduce

$$
G(\mathbf{z}) = \frac{1}{2}\sum_{i=1}^{2}\left\{(x_i - p_{i1})^2 + (y_i - p_{i2})^2 + (\theta_i - p_{i3})^2\right\} \geq 0 . \qquad (9)
$$

4 Design of Control Laws

Combining all the potential functions, (2−9), and introducing *control parameters*, $\alpha_{ik} > 0$, $\beta_{ij} > 0, \gamma_{iq} > 0$, $\zeta_s > 0$, for $i, j, k, q, s \in \mathbb{N}$, we define a candidate Lyapunov function for system (1) as

$$
L(\mathbf{z}) = V(\mathbf{z}) + G(\mathbf{z})\sum_{i=1}^{2}\left(\sum_{k=1}^{2}\frac{\alpha_{ik}}{LS_{ik}(\mathbf{z})} + \sum_{j=1}^{4}\frac{\beta_{ij}}{W_{ij}(\mathbf{z})} + \sum_{q=1}^{w}\frac{\gamma_{iq}}{FO_{iq}(\mathbf{z})}\right) + G(\mathbf{z})\sum_{s=1}^{3}\frac{\zeta_s}{DC_s(\mathbf{z})} . \qquad (10)
$$

Clearly, L is locally positive and continuous on the domain $D(L)$. Moreover, we see that $\mathbf{z}_e \in D(L)$ and $L(\mathbf{z}_e) = 0$. To extract the control laws for the 1-trailer system, we differentiate the various components of $L(\mathbf{z})$ separately, carry out the necessary substitutions from (1) to obtain

$$\begin{aligned}
\dot{L}(\mathbf{z}) &= \frac{1}{L_2}\left(\frac{L_2+a}{2}\left(f_2(\mathbf{z})\sin\theta_2 - g_2(\mathbf{z})\cos\theta_2\right) + h_2(\mathbf{z})\right)\sin\left(\theta_1-\theta_2\right)v \\
&+ \left(\left(f_1(\mathbf{z})+f_2(\mathbf{z})\right)\cos\theta_1 + \left(g_1(\mathbf{z})+g_2(\mathbf{z})\right)\sin\theta_1 + k_1(\mathbf{z})\sigma_1\right)v \\
&+ \frac{c}{L_2}\left(\frac{L_2+a}{2}\left(g_2(\mathbf{z})\cos\theta_2 - f_2(\mathbf{z})\sin\theta_2\right) - h_2(\mathbf{z})\right)\cos\left(\theta_1-\theta_2\right)\omega \\
&+ \left(c\left(f_2(\mathbf{z})\sin\theta_1 - g_2(\mathbf{z})\cos\theta_1\right) + \frac{L_1}{2}\left(g_1(\mathbf{z})\cos\theta_1 - f_1(\mathbf{z})\sin\theta_1\right) + h_1(\mathbf{z}) + k_2(\mathbf{z})\sigma_2\right)\omega\,,
\end{aligned}$$

where functions $f_i(\mathbf{z}), g_i(\mathbf{z}), h_i(\mathbf{z})$ and $k_i(\mathbf{z})$, for $i = 1, 2$ are defined as (on suppressing $\mathbf{z}$),

$$\begin{aligned}
f_i &= \left(1+\sum_{k=1}^{2}\frac{\alpha_{ik}}{LS_{ik}(\mathbf{z})} + \sum_{j=1}^{4}\frac{\beta_{ij}}{W_{ij}(\mathbf{z})} + \sum_{q=1}^{w}\frac{\gamma_{iq}}{FO_{iq}(\mathbf{z})} + \sum_{s=1}^{3}\frac{\zeta_s}{DC_s(\mathbf{z})}\right)(x_i - p_{i1}) \\
&-G\left\{\frac{\beta_{i1}}{W_{i1}^2} - \frac{\beta_{i3}}{W_{i3}^2} + \sum_{q=1}^{w}\frac{\gamma_{iq}}{FO_{iq}^2}(x_i - o_{q1})\right\} \\
&-G\sum_{k=1}^{2}\frac{\alpha_{ik}}{LS_{ik}^2}\left(\left[1-(a_{k2}-a_{k1})d_k\right](x_i - Lx_{ik}) - (b_{k2}-b_{k1})d_k\,(y_i - Ly_{ik})\right), \\
h_i &= \left(\sum_{k=1}^{2}\frac{\alpha_{ik}}{LS_{ik}(\mathbf{z})} + \sum_{j=1}^{4}\frac{\beta_{ij}}{W_{ij}(\mathbf{z})} + \sum_{q=1}^{w}\frac{\gamma_{iq}}{FO_{iq}(\mathbf{z})} + \sum_{s=1}^{3}\frac{\zeta_s}{DC_s(\mathbf{z})}\right)(\theta_i - p_{i3}) \\
&+(-1)^i G\frac{\zeta_3}{DC_3^2}(\theta_2-\theta_1)\,, \\
g_i &= \left(1+\sum_{k=1}^{2}\frac{\alpha_{ik}}{LS_{ik}(\mathbf{z})} + \sum_{j=1}^{4}\frac{\beta_{ij}}{W_{ij}(\mathbf{z})} + \sum_{q=1}^{w}\frac{\gamma_{iq}}{FO_{iq}(\mathbf{z})} + \sum_{s=1}^{3}\frac{\zeta_s}{DC_s(\mathbf{z})}\right)(y_i - p_{i2}) \\
&-G\left\{\frac{\beta_{i4}}{W_{i4}^2} - \frac{\beta_{i2}}{W_{i2}^2} + \sum_{q=1}^{w}\frac{\gamma_{iq}}{FO_{iq}^2}(y_i - o_{q2})\right\} \\
&-G\sum_{k=1}^{2}\frac{\alpha_{ik}}{LS_{ik}^2}\left(\left[1-(b_{k2}-b_{k1})r_k\right](y_i - Ly_{ik}) - (a_{k2}-a_{k1})r_k\,(x_i - Lx_{ik})\right), \\
k_i &= 1 + G\frac{\zeta_i}{DC_i^2}\,.
\end{aligned}$$

Next, given the *convergence parameters* $\delta_1, \delta_2 > 0$, $\dot{L}(\mathbf{z})$ can be made non-positive by letting the translational and rotational speeds for the 1-trailer system have the following form:

$$\begin{aligned}
-\delta_1 \times v &= \frac{1}{L_2}\left(\frac{L_2+a}{2}\left(f_2(\mathbf{z})\sin\theta_2 - g_2(\mathbf{z})\cos\theta_2\right) + h_2(\mathbf{z})\right)\sin\left(\theta_1-\theta_2\right) \\
&+ \left(f_1(\mathbf{z})+f_2(\mathbf{z})\right)\cos\theta_1 + \left(g_1(\mathbf{z})+g_2(\mathbf{z})\right)\sin\theta_1 + k_1(\mathbf{z})\sigma_1\,, \\
-\delta_2 \times \omega &= \frac{c}{L_2}\left(\frac{L_2+a}{2}\left(g_2(\mathbf{z})\cos\theta_2 - f_2(\mathbf{z})\sin\theta_2\right) - h_2(\mathbf{z})\right)\cos\left(\theta_1-\theta_2\right) \\
&+ c\left(f_2(\mathbf{z})\sin\theta_1 - g_2(\mathbf{z})\cos\theta_1\right) + \frac{L_1}{2}\left(g_1(\mathbf{z})\cos\theta_1 - f_1(\mathbf{z})\sin\theta_1\right) + h_1(\mathbf{z}) + k_2(\mathbf{z})\sigma_2\,.
\end{aligned}$$

The aforementioned conditions lead us to a negative semi-definite expression of the proposed candidate Lyapunov function provided our state feedback nonlinear controllers governing the robot are of the form:

$$\sigma_1 = -\frac{1}{L_2}\left(\frac{L_2+a}{2}\left(f_2(\mathbf{z})\sin\theta_2 - g_2(\mathbf{z})\cos\theta_2\right) + h_2(\mathbf{z})\right)\sin\left(\theta_1-\theta_2\right)/k_1(\mathbf{z}) \tag{11a}$$
$$- \left(\delta_1 v + \left(\left(f_1(\mathbf{z})+f_2(\mathbf{z})\right)\cos\theta_1 + \left(g_1(\mathbf{z})+g_2(\mathbf{z})\right)\sin\theta_1\right)\right)/k_1(\mathbf{z})\,,$$

$$\sigma_2 = -\frac{c}{L_2}\left(\frac{L_2+a}{2}\left(g_2(\mathbf{z})\cos\theta_2 - f_2(\mathbf{z})\sin\theta_2\right) - h_2(\mathbf{z})\right)\cos\left(\theta_1-\theta_2\right)/k_2(\mathbf{z}) \tag{11b}$$
$$- \left(\delta_2\omega + c\left(f_2(\mathbf{z})\sin\theta_1 - g_2(\mathbf{z})\cos\theta_1\right) + \frac{L_1}{2}\left(g_1(\mathbf{z})\cos\theta_1 - f_1(\mathbf{z})\sin\theta_1\right) + h_1\right)/k_2(\mathbf{z})\,.$$

We note that $\dot{L}(\mathbf{z}) \leq 0$ for all $\mathbf{z} \in D(L)$, and $\dot{L}(\mathbf{z}_e) = 0$. Interestingly, having $c = 0$ gives the controllers for the corresponding *standard* 1-trailer system. A careful scrutiny of the properties of our candidate function reveals that $\mathbf{z}_e$ is an equilibrium point of system (1) and L is a legitimate Lyapunov function that, *per se*, guarantees its stability. The following theorem ends our discussions thus far:

Theorem 1 *The equilibrium point* $\mathbf{z}_e$ *of system (1) is stable provided* σ_1 *and* σ_2 *are defined as in (11a) and (11b), respectively.*

5 Implementation of the Control Laws

To illustrate the effectiveness of the proposed controllers we fabricate a scenario where the tractor-trailer robot has to maneuver from an initial to a final state, in the workspace cluttered with fixed obstacles, by and by, attain a pre-determined posture at the target. We will verify numerically the stability results obtained from the Lyapunov function. The corresponding initial and final states and other details for the simulation are listed below (assuming that appropriate units have been taken into account).

1. **Robot Parameters:** $L_1 = 2$; $L_2 = 2.24$; $l = 1$; $c = 0.34$.
2. **Initial Configuration:** $(x_1, y_1) = (5, 8)\,; (\theta_1, \theta_2) = (\pi/4,\ 0\ )\,; (v, \omega) = (0.9, 0.5)$.
3. **Final Configuration:** $(p_{11}, p_{12}, p_{13}, p_{23}) = (22, 16.5, 0\ , 0\ )$; $rt_1 = 0.1$.
4. **Fixed Obstacle:** Center: $(o_{11}, o_{12}) = (12, 10)$, radius: $ro_1 = 2$.
5. **Physical Limitations:** $v_{\max} = 0.95$ and $\phi_{\max} = 60^o$.
6. **Control Parameters:**

Parameters related to	Tractor	Trailer
Ghost Walls	$\alpha_{11} = \alpha_{12} = 3$	$\alpha_{21} = \alpha_{22} = 2$
Workspace Restriction	$\beta_{11} = \beta_{12} = \beta_{13} = \beta_{14} = 1$	$\beta_{21} = \beta_{22} = \beta_{23} = \beta_{24} = 1$
Fixed Obstacle	$\gamma_{11} = 10$	$\gamma_{21} = 7$
Dynamic Constraints	$\zeta_1 = 0.009$, $\zeta_2 = 0.9$	$\zeta_3 = 0.9$

7. Parameters: Clearance: $\epsilon_1 = \epsilon_2 = 0.1$; Convergence: $\delta_1 = 180$ and $\delta_2 = 200$.

9. Boundaries and Ghost Walls: as shown in fig. 3

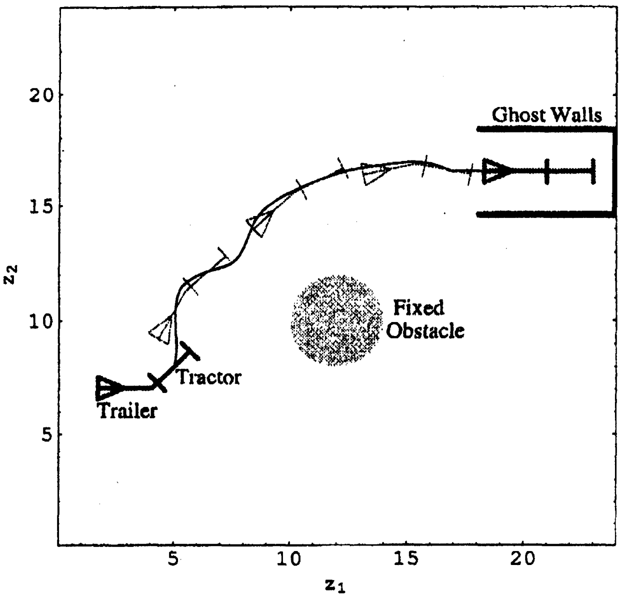

Figure 3: The resulting stable trajectory of the tractor-trailer robot in a specific traffic scenario.

The controllers were implemented to generate a feasible robot trajectory from an initial to a final state. Fine tuning of the control and convergence parameters were carried out to accomplish our research goal. Figure 3 shows how the tractor-trailer mobile robot converges to the desired state. With the inclusion of *ghost walls* and the new optimization technique, we generated the maneuvers that culminated to a pre-defined orientation at the target position (see Fig. 4), achieving the final predefined posture. Figure 5 shows explicitly the time evolution of the acceleration controllers along the trajectory of a two-body articulated vehicle. One can clearly notice the convergence of the controllers at the final state implying the effectiveness of the new controllers.

6 Discussion

This paper presents a new set of continuous time-invariant acceleration control laws that improves upon, in general, the posture control, with theoretically guaranteed point and posture stabilities, convergence and collision avoidance properties of a *general* tractor-trailer robot in *a priori* known environment. We basically utilize *ghost walls* and the new optimization technique to strengthen posture stability, in the sense of Lyapunov, of our dynamical model. The ghost walls are erected as required in the workspace and then we utilize the optimization technique to garner the predetermined final postures. This technique emerges as a convenient mechanism for obtaining feasible orientations of each and every body of the articulated robot system.

In a nutshell, we have a centralized trajectory planning, which to certain extent, demonstrates autonomy and multitasking capabilities of humans. The new algorithm provides us with a suitable and fitting platform to harvest collision-free trajectories from initial to desired states and achieving final postures within a dynamic environment, whilst satisfying the nonholonomic constraints of the system. The proposed controllers stabilize the configuration coordinates of the vehicle to an arbitrary small neighborhood of the target. We note here that convergence is only guaranteed from a number of initial states of the system. For now, we are satisfied with searching for these initial conditions numerically via the computer. This is, admittedly, still a long way from proving asymptotic stability, but we have now a starting point for using continuous control laws that guarantee, for some initial conditions, point and posture stabilities.

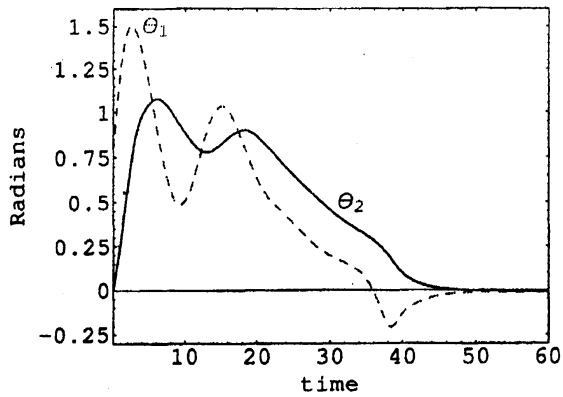

Figure 4: The resulting orientations of the tractor (dashed line) and the trailer.

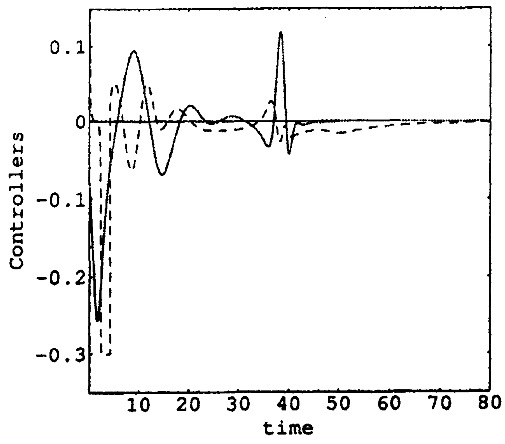

Figure 5: Evolution of the translational (dashed line) and rotational accelerations.

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