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Iteration dynamical systems of discrete Laplacians on the plane lattice (Its mathematical structure and computer simulations of designs)

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INTRODUCTION

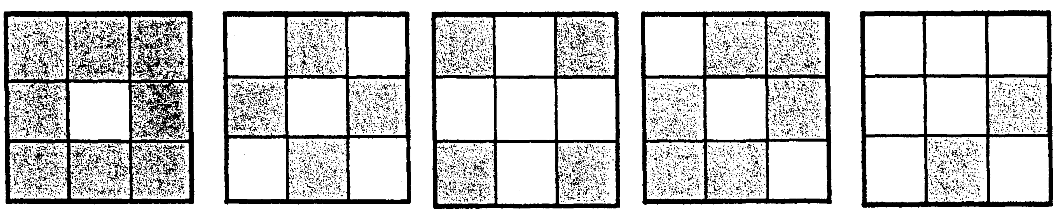
This is a continuation of papers on iteration dynamical sytems of discrete Lapalcians([2],[3]). In this paper we are concerned with (1) Mathematical structure of iteration dynamical system of discrete Laplacians on the plane lattice and (2)The design-patterns produced by the dynamical system .

At first we give a stability theorem for the dynamical system whose Laplacian is defined by even neighborhoods. Next we are concerned with computer simulations of designs. We can realize many kinds of designs and we can give a classification of designs by the choices of neighborhoods, sources and the steps of the iterations. Finally we analyze the variations of pattern and we can show that we supply the design-samplers using our software.

ITERATION DYNAMICAL SYSTEM OF DISCRETE LAPLACIAN

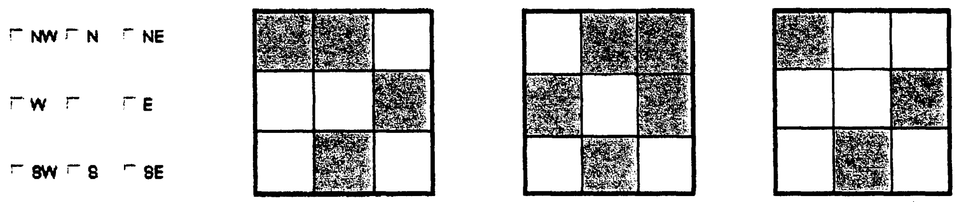
We recall the definition of the iteration dynamical system of discrete Laplacians([1]). We choose the plane lattice which is generated by two families of lines which are orthogonal each other. We identify a lattice point with a cell obtained by the lattice. We call a set of cells which are attached with the reference cells a neighborhood U_p. We call neighborhood even(or odd) if the number of the cells is even(respectively odd). We list several examples of neighborhoods.

(1) Even neighborhoods



Moor Neuman Diag Neuman Hexagonal Sierpinski

(2) Odd neighborhoods



NW N E S   N NE E S W   NW E S

We denote a neighborhood U_p by the directions N,NE,E,ES,S,SW,W,NW. For example we can denote the Neumann neighborhood by N,E,S,W. We take the space **F** of $\{0,1\}$ valued functions on the plane lattice and define the Lapacian operation by

$$\Delta_{U_p} f(p) = \Sigma_{q \in U_p} (f(q) - f(p))$$

Choosing an initial function $f_0 \in F$, we define the dynamical system defined by the iteration of the Laplacian:

$$\{f_n\}, f_n = \Delta_U f_{n-1} (n = 1,2,...)$$

We call point $p \in L$ a source of the dynamical system when $f_n(p) = 1$ for any $n \in N$. Then we can obtain the designs of distributions of 0 and 1 on the lattice plane and we can get various kind of designs by the choice of neighborhoods, sources.

SOME BASIC PROPERTIES ON THE DYNAMICAL SYSTEMS

Here we recall some basic notations on the dynamical systems and state assertions on mathematical structures([1],[2]). At first we notice that we consider dynamical systems under the periodic condition. Namely, choosing an integer M, which is called the size, we consider the following periodic functions:

$$F(M) = \{f \in F \mid F(x + mM, y + nM) = F(x)\ (m, n \in Z)\}$$

Choosing neighborhoods under the periodic condition, we can define the discrete Laplacian and we can consider the iteration dynamical system. We prepare several basic notations:

(1) A dynamical system is called stable if

$$\exists k \in N \ \text{s.t}\ f_n = f_k \ (\forall n \geq k)$$

(2) A dynamical system is called periodic, if

$$\exists n, \exists l \ \text{s.t}\ f_n = f_{n+kl} \ (\forall k \in N)$$

(3) A point $p \in F$ is called a source of a dynamical system, if $f_n(p) = 1$ for any $n \in N$.

We can state some basic properties on the dynamical systems:

(1) In the case $M = 2^p$ and a single source, we may expect the following results:

If the neighborhood is even, we see that the dynamical system is stable with the stability speed 2^p for Moor neigh., Hexagonal neigh., Neuman neigh., and Sierpinski neigh.
If the neighborhood is odd, we see that the dynamical system is periodic, period is different depending the neighborhoods.

(2) In the case where M is odd, we may expect the dynamical system is periodic in the case of a single source. We give the table of periods for smaller M:

M	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31
周期	1	5	6	13	30	62	29	30	511	126	2046	2045	1021	16384	61
再帰点	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

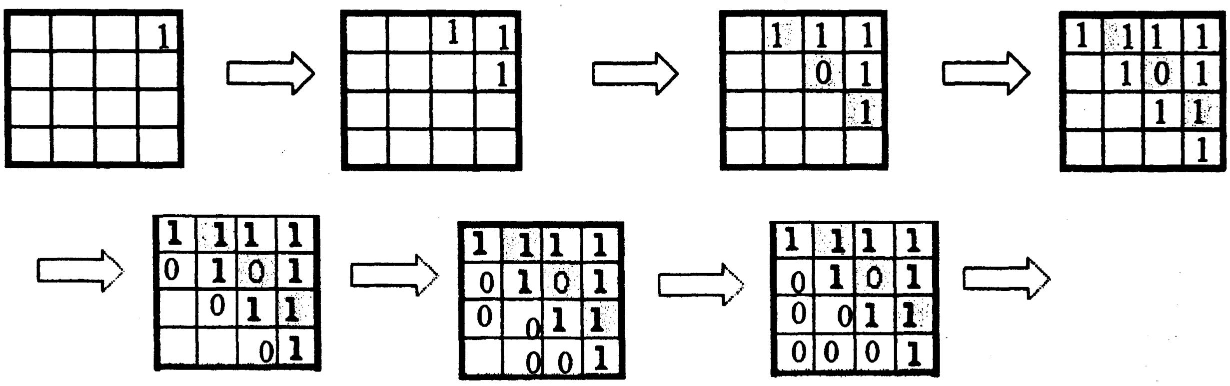
We can prove the following assertion:

PROPOSITION

In the case where $M = 2^p$, neighborhood is Sierpinski type, and it has one point source, the dynamical system is stable with the stability speed 2^p.

PROOF

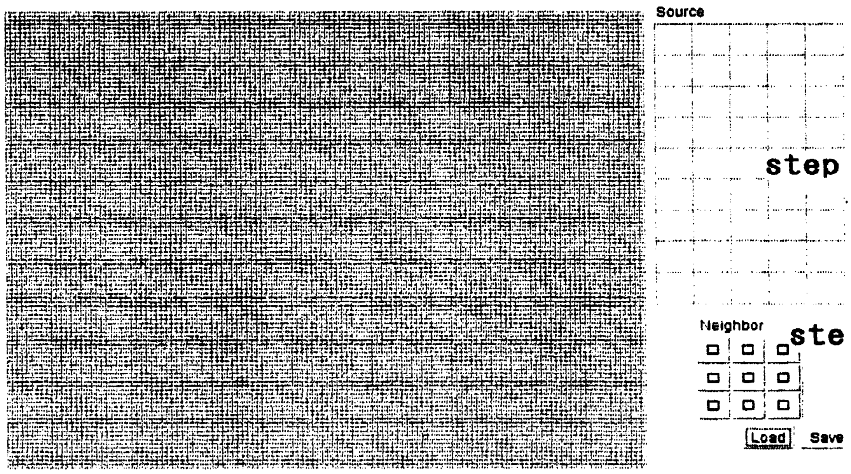
We give an outline of the proof of Proposition in the case $p = 4$. Only by an observation in this simple case we may understand that our assertion holds. The detail will be given in another paper.



We notice the following facts:
(1) Putting the source at the upper corner in the right side, we see that the Pascal triangle mod 2 appears in the upper triangle part.
(2) At the 4 step, every element in the diagonal is 1.
(3) The lower triangle is filled by 0.
By these facts we see that the dynamical system is stable.

SOFTWARE"DESIGNER KENTAURUS 2005"

We make a brief comment on our soft ware. We have well developed software named "Designer Kentaurus 2005". The software is written by JAVA.



The software has three panels:(1)The main panel describes the behavior of the dynamical system, and (2)the second panel describes the table of neighborhoods and (3)the third one describes the sources.

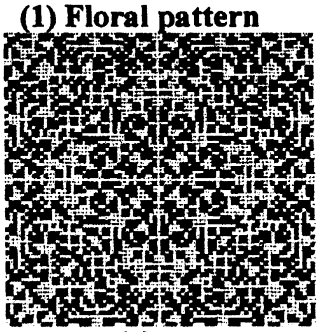
CLASSIFICATION OF DESIGN-PATTERNS

We can classify the design-patterns choosing neighborhoods, sources, steps. We notice that their characters depend on oddness and evenness of neighborhoods strongly as we have seen in the previous paper[3]. We treat dynamical systems with a single source.

Design-pattern I —even neighborhood—

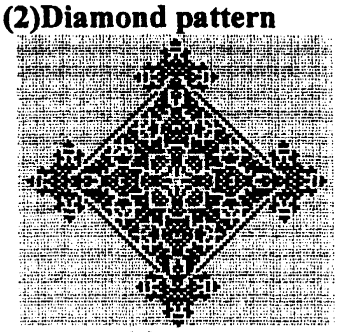
We can observe the following kinds of designs. The left side of the explanations under the pictures is type of neighborhood, for example moor, Neumann diag. Neumann etc and the middle [*] is the number of sources and the right side is the number of the step.

(1) Floral pattern



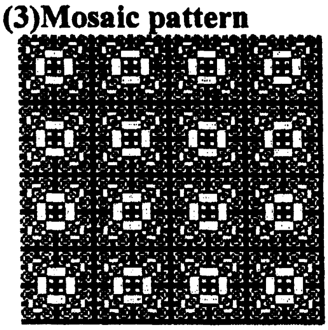
moor〔4〕 step=63

(2)Diamond pattern



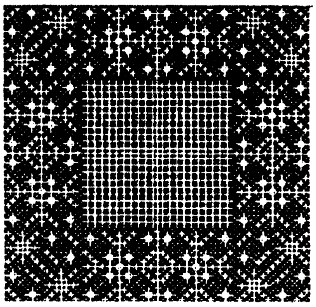
neuman〔4〕 step=42

(3)Mosaic pattern



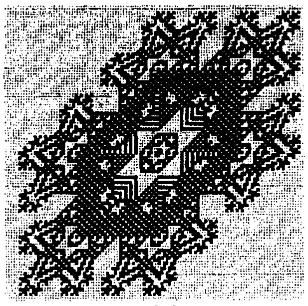
diag neuman〔4〕 step=114

(4)Bordered pattern



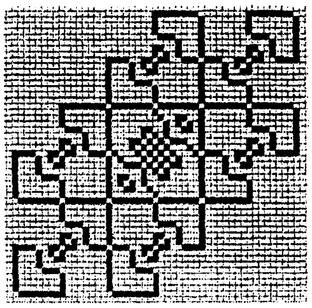
moor [4] step=94

(5)Crystal pattern



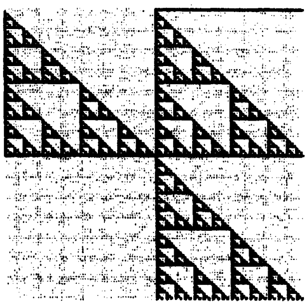
hexagonal [1] step=55

(6)Braid pattern



moor [8] step=48

(7) Gasket pattern



sierpinski [1] step=128

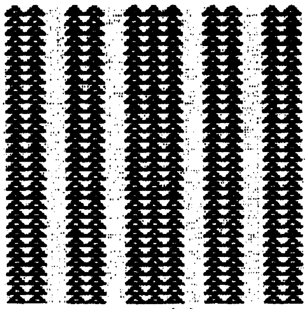
Table 1 examination of patterns I (even neighborhood)

property	design-pattern	Neighborhood
single	floral	Moor
		diag neuman
	diamond	Neuman
repetition	mosaic	Moor
		Neuman
		diag neuman
multiple	bordered	Moor
asymmetry-repetition	crystal	Hexagonal
	braid	
	gasket	Sierpinski
		N NE

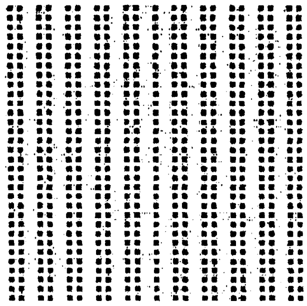
Design-pattern II —odd neighborhood with plural sources—

We can obtain design patterns with different taste :(1)stripe pattern, (2)mosaic pattern, (3)check pattern,(4)undulate pattern and(5)line pattern:

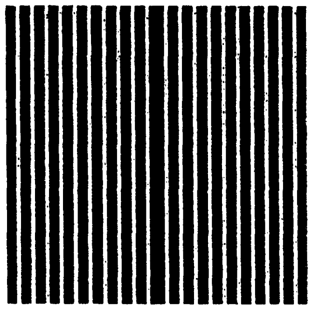
(1) Stripe pattern



N E SE SW W 〔4〕 step=124

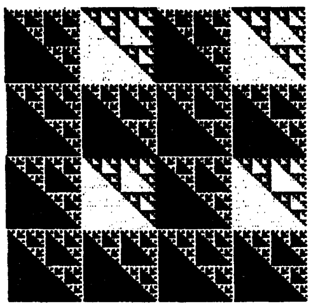


N E SE SW W 〔4〕 step=125

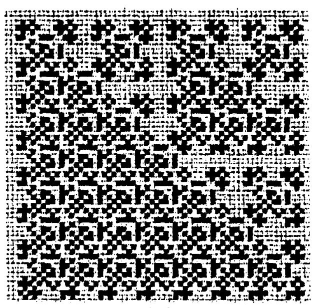


N E SE SW W 〔4〕 step=127

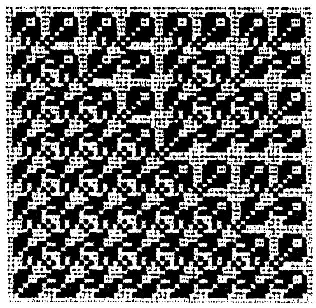
(2) Mosaic pattern



W NW NE SE S 〔1〕 step=96

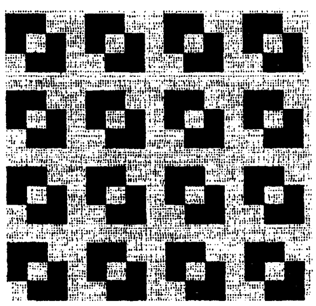


N NE E 〔4〕 step=59

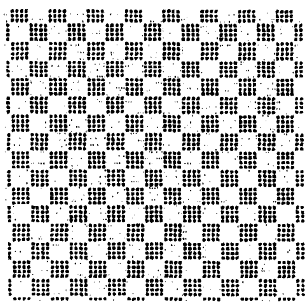


N NE E 〔4〕 step=60

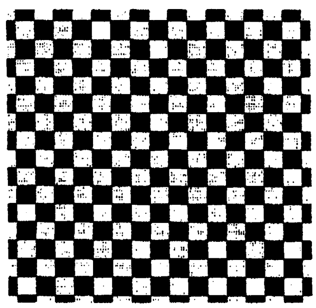
(3) Check pattern



N NE E 〔2〕 step=112

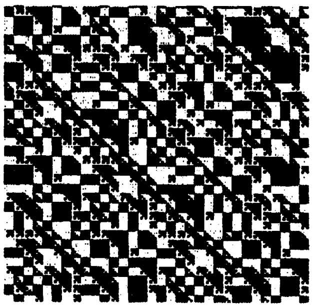


N NE E 〔2〕 step=119

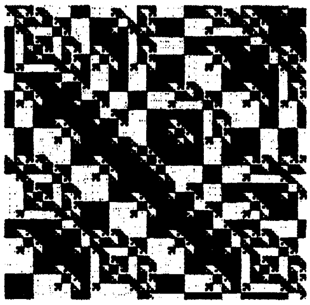


N NE E 〔2〕 step=120

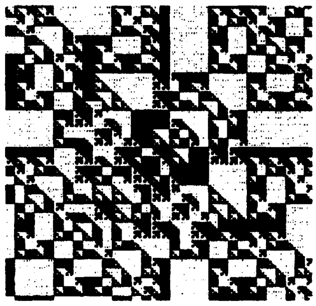
(4) Undulate pattern



NWNESESW 〔5〕 step=120

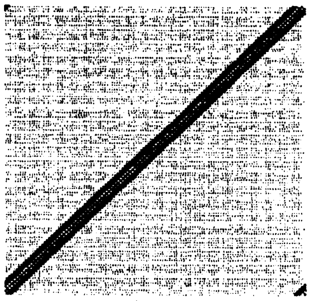


NWNESESW 〔5〕 step=128

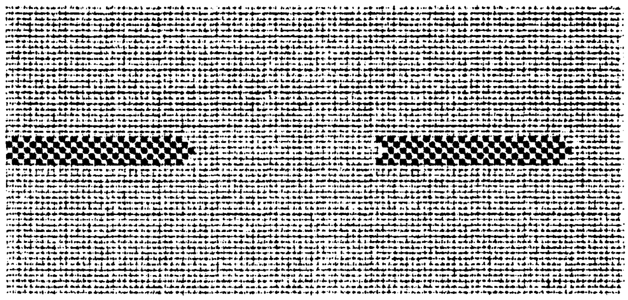


NWNESESW 〔5〕 step=152

(5) Line pattern

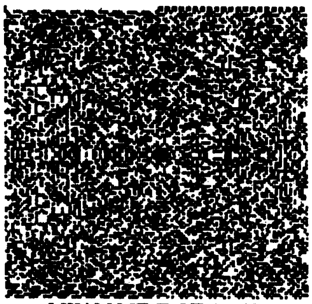


NE 〔9〕 step=128

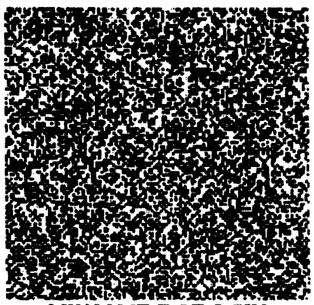


E 〔5〕 step=223

(6) Chaotic pattern



NW N NE E SE S SW
〔4〕 step=148



NW N NE E SE S SW
〔4〕 step=161

We can make chaotic patterns and designs of periodic characters choosing plural sources. When we put the sources without symmetries, we may obtain chaotic patterns easily.

Table 2 examination of patterns II (odd neighborhood)

<table>
<tr><th>property</th><th>design- pattern</th><th colspan="2">Neighborhood</th></tr>
<tr><td rowspan="2">flow</td><td rowspan="2">line</td><td>E</td><td rowspan="2">1</td></tr>
<tr><td>NE</td></tr>
<tr><td>equality</td><td>check</td><td>N NE E</td><td rowspan="2">3</td></tr>
<tr><td rowspan="2">lack</td><td rowspan="2">mosaic (asymmetry)</td><td>W NW SE</td></tr>
<tr><td>W NW NE SE S</td><td rowspan="4">5</td></tr>
<tr><td>multiple</td><td>undulate</td><td>NW N E SE SW</td></tr>
<tr><td>symmetry</td><td>stripe</td><td>N E SE SW W</td></tr>
<tr><td>squirm</td><td>chaotic</td><td>NW N SN S SW</td></tr>
</table>

CONSTRUCTION OF DESIGN-SAMPLERS

By these observations, we can obtain the following manual of constructing designs.

Table 3 structure of software

<table>
<tr><th colspan="3">neighborhood</th><th rowspan="2">design-samples</th></tr>
<tr><th>even/odd</th><th>layout</th><th>denomination</th></tr>
<tr><td rowspan="6">even neighborhood</td><td rowspan="11">symmetry</td><td rowspan="3">moor</td><td>Bordered</td></tr>
<tr><td>Mosaic</td></tr>
<tr><td rowspan="2">Floral</td></tr>
<tr><td>diag neuman</td></tr>
<tr><td>neuman</td><td>Diamond</td></tr>
<tr><td>hexagonal</td><td>Crystal</td></tr>
<tr><td rowspan="6">odd neighborhood</td><td>N E SE SW W</td><td>Stripe</td></tr>
<tr><td>W NW NE SE S</td><td rowspan="2">mosaic (asymmetry)</td></tr>
<tr><td rowspan="2">N NE E</td></tr>
<tr><td>Check</td></tr>
<tr><td>N E SE SW NW</td><td>Undulate</td></tr>
<tr><td>asymmetry</td><td>NW N SN S SW</td><td>Chaotic</td></tr>
</table>

We need some experiences for getting desired patterns. We will give a manual of producing designs systematically.

CONCLUSIONS AND DISCUSSIONS

(1) We have given a theorem on the mathematical structure of iteration dynamical system on our dynamical system in the simplest case. Namely we have proved a stability theorem for the Sierpinski's neighborhood with a source. We may expect to obtain analogous results for general even neighborhoods. This will be given in the forthcoming paper.
(2) We notice that our dynamical system in the case of line lattice is identical with the dynamical system #90 of Wolfram's table of cellular automata([7]). Hence we may expect to obtain analogous results for the plane lattice.
(3) We have produced many kinds of designs by use of our simulators and make analysis on them. Our simulations are defined by the iteration dynamical systems and we can reproduce them as one wants to obtain them. By this fact we have given the manual of producing designs following the consumer's needs.
(4) The evenness/oddness of neighborhoods give big difference not only in their impressions given by the designs but also in their mathematical structures. This can be observed in the psychological experiments on the visual impressions([3]). We notice that this character plays an important role in the discussions on the difference of Japanese and European designs. Also we have seen that we have made the simulations of the time change of numbers of families of extinct animals by use of the even neighborhoods([4]). Here we want to express our stress on the fact that our simulations may expect to describe the evolution of the universe. This topics will be discussed in the forthcoming paper.

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