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タイプの排除と順序型

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Abstract

可算言語 L が順序のための 2 変数述語 $<$ を含むとする. T を L で表現された理論とする. Lopez-Escobar の定理により, タイプ $p(x)$ を排除する T のモデルで, 順序型が ω_1 のものが存在すれば, 同じタイプ $p(x)$ を排除して, 整列でないモデルが存在することがわかる. この結果は証明方法を変えることなく, 可算個のタイプの集合の場合に拡張される.

本稿においては, 完全なタイプを扱う限りにおいては, タイプの数が連続濃度未満の場合でも同様な結果が成り立つことをを示す. これから Morley のタイプ排除定理のある種の変形が得られる.

Proposition 1 *Let L be a countable language with $<$. Let T be an L-theory and $p(x)$ a type. Let $M \models T$ be a model omitting $p(x)$ such that $otp(<^M) = \omega_1$. Then there is a model $N \models T$ with*

- *N omits $p(x)$, and*
- *N has an infinite descending sequence with respect to $<^N$.*

Proof: We assume L contains all the Skolem functions. We prepare a countable set $X = \{x_i : i \in \omega\}$ of variables. Let $\{t_i : i \in \omega\}$ be an enumeration of all the L-terms whose variables belong to X. We may assume that the variables of t_n is contained in $\bar{x}_n = x_0, ..., x_{n-1}$. So we may assume $t_n = t_n(\bar{x}_n)$.

By $otp(<^M) = \omega_1$, we assume $(M, <) = (\omega_1, <)$. By induction on $n \in \omega$, we choose sets $S_n \subset \omega_1$ and formulas $\varphi_n(x) \in p(x)$ with the following properties:

1. $S_n \subset \omega_1$ is a set of descending sequences of length n.

2. For each $i \in \omega_1$, $S_n|i = \{(a_0, ..., a_{n-1}) \in S_n : a_{n-1} > i\}$ has the cardinality ω_1.
3. If $(a_0, ..., a_{n-1}, a_n) \in S_n$ then $(a_0, ..., a_{n-1}) \in S_{n-1}$,
4. For all $\bar{a} \in S_n$, we have $M \models \neg\varphi_n(t_n(\bar{a}))$.

Now we assume that we have successfully chosen S_n and $\varphi_n(x)$. For each $i \in \omega$, using condition 2, choose $\bar{a}_i$ from $S_n|i$. Let $\bar{a}(i)$ denote the sequence $\bar{a}_i, i$. Notice that $\bar{a}(i)$ is a decreasing sequence of length $n+1$. Now consider elements $t_{n+1}(\bar{a}(i))$ $(i \in \omega)$. Since M omits $p(x)$, we can choose a formula $\varphi^i_{n+1}(x) \in p(x)$ with $M \models \neg\varphi^i_{n+1}(t_{n+1}(\bar{a}(i)))$. Since there are only countably many such formulas, we can choose an uncountable set $U \subset \omega_1$ such that for any $i \in U$ φ^i_{n+1} is the same formula. Let φ_{n+1} be the fixed formula. We put $S_{n+1} = \{\bar{a}(i) : i \in U\}$. Now it is easy to see that S_{n+1} and φ_{n+1} satisfy our requirements.

The following claim is easily proven, using S_n's.

Claim A $\Gamma(x_0, x_1, ...) = \{x_0 > x_1 > x_2 > ...\} \cup \{\neg\varphi_n(t_n(\bar{x}_n)) : n \in \omega\}$ *is a consistent set.*

Let $I = (b_i)_{i\in\omega}$ realize Γ. Let N be the Skolem hull of I. $N \supset I$ has an inifinite descending sequence. Since $\{t_n\}$ is an enumeration of all the Skolem terms, we have $N = \{t_n(\bar{b}_n) : n \in \omega\}$, where $\bar{b}_n = b_0, ..., b_{n-1}$. Since each $t_n(\bar{b}_n)$ satisies $\neg\varphi_n(x)$, we conclude that N omits $p(x)$.

Corollary 2 *Let T be a countable complete theory and R a set of complete types with $|R| < 2^\omega$. Suppose that for each $i < \omega_1$, there is a model $M_i \models T$ with the following properties:*

1. $|M_i| \geq \beth_i(\omega)$,

2. M_i omits each member of R.

Then for each κ there is a model M omitting R and with $|M| \geq \kappa$.

References

[1] Chen Chung ChangH. Jerome Keisler, Model Theory (Studies in Logic and the Foundations of Mathematics) , 1990.

[2] Saharon Shelah, Classification Theory and the Number of Non-Isomorphic Models (Studies in Logic and the Foundations of Mathematics), 1990.

[3] Newelski, Omitting types and the real line, Journal of Symbolic Logic, vol. 62 (1987), no. 4, pp. 1020-1026.