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京都大学
CM-TRIVIALITY AND GEOMETRIC ELIMINATION OF IMAGINARIES

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1. INTRODUCTION

To show CM-triviality (of generic relational structures), first of all, we showed weak elimination of imaginaries, and then, working in the real sort, we could show CM-triviality. In this note, we show that CM-triviality in the real sort, defined in the second section, implies geometric elimination of imaginaries and CM-triviality (in the real and imaginary sorts). To show this, we give a characterization of geometric elimination of imaginaries in simple theories.

Our notation is standard. Let $T$ be a complete $L$-theory, and let $\mathcal{M}$ be the big model of $T$. $\bar{a}, \bar{b}, \ldots \subset \omega \mathcal{M}$ denote finite sequences in $\mathcal{M}$. We work in $\mathcal{M}^{eq}$, which consists of $\bar{a}_E$, the $E$-class of $\bar{a}$, for any 0-definable equivalence relation $E$ and $\bar{a} \subset \omega \mathcal{M}$. $AB$ denotes $A \cup B$ for any $A, B \subset \mathcal{M}^{eq}$.

For $a \in \mathcal{M}^{eq}, A \subset \mathcal{M}^{eq}$, we write $a \in \text{dcl}^{eq}(A)$, if $a$ is fixed by any automorphism pointwise fixing $A$. And we write $a \in \text{acl}^{eq}(A)$, if the orbit of $a$ by automorphisms pointwise fixing $A$, is finite. We write $\bar{a} \equiv_A \bar{b}$ for $\text{tp}(\bar{a}/A) = \text{tp}(\bar{b}/A)$ in $T$.

We said that $T$ geometrically eliminates imaginaries ($T$ has GEI), if for any $e \in \mathcal{M}^{eq}$, there exists $\bar{b} \subset \omega \mathcal{M}$ such that $e \in \text{acl}^{eq}(\bar{b})$ and $\bar{b} \in \text{acl}^{eq}(e)$.

2. A CHARACTERIZATION OF GEI IN SIMPLE THEORIES

Let $T$ be a simple theory.

Definition 2.1. We say that $T$ has the independence over intersections ($T$ has $\text{IND}/I$), for any $\bar{a}, A, B \subset \mathcal{M}$ with $\bar{a} \downarrow_A B, \bar{a} \downarrow_B A$, we have $\bar{a} \downarrow_{\text{acl}(A) \cap \text{acl}(B)} AB$.

Proposition 2.2. $\text{IND}/I$ implies GEI.

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Proof. Fix \( e = \bar{a}_E \in \mathcal{M}^\text{eq} \). Take \( \bar{b}, \bar{c} \models \text{tp}(\bar{a}/e) \) such that \( \bar{b}, \bar{c}, \bar{a} \) are independent over \( e \). Let \( A = \text{acl}(\bar{b}) \cap \text{acl}(\bar{c}) \). Then \( \bar{a} \downarrow_A \bar{b} \bar{c} \) by IND/I. By \( e \in \text{dcl}^\text{eq}(\bar{a}) \cap \text{dcl}^\text{eq}(\bar{b} \bar{c}) \), \( e \in \text{ac1}^\text{eq}(A) \). On the other hand, \( A \subset \text{ac1}^\text{eq}(e) \) follows from \( \bar{b} \downarrow_e \bar{c} \).

\( \square \)

Lemma 2.3. Suppose that \( T \) has GEI. Then, for any \( \text{acl}(A) = A, \text{acl}(B) = B \subset \mathcal{M} \), we have

\[
\text{acl}^\text{eq}(A) \cap \text{acl}^\text{eq}(B) = \text{acl}^\text{eq}(A \cap B).
\]

Proof. Let \( e \in \text{acl}^\text{eq}(A) \cap \text{acl}^\text{eq}(B) \). By GEI, there exists \( \bar{a} \subset \omega \mathcal{M} \) such that \( e \in \text{acl}^\text{eq}(\bar{a}) \) and \( \bar{a} \in \text{acl}^\text{eq}(e) \). As \( \bar{a} \in \text{acl}^\text{eq}(A) \) and \( \bar{a} \in \text{acl}^\text{eq}(B) \), we see \( \bar{a} \subset A \cap B \). Thus, \( e \in \text{acl}^\text{eq}(A \cap B) \).

\( \square \)

From now on, we assume elimination of hyperimaginaries (EHI). Then the converse of Proposition 2.2 follows.

Proposition 2.4. GEI \( \iff \) IND/I

Proof. (\( \Leftarrow \)) by Proposition 2.2. (\( \Rightarrow \)): Suppose that \( \bar{a} \downarrow_A B, \bar{a} \downarrow_B A \) and \( \text{acl}(A) = A, \text{acl}(B) = B \). By the above lemma and EHI, we see \( \text{Cb}(\bar{a}/AB) \subseteq \text{acl}^\text{eq}(A) \cap \text{acl}^\text{eq}(B) = \text{acl}^\text{eq}(A \cap B) \).

\( \square \)

3. MAIN THEOREM

Definition 3.1. We say that \( T \) is CM-trivial in the real sort, if, for any \( \bar{a}, A = \text{acl}(A), B = \text{acl}(B) \subset \mathcal{M}, \bar{a} \downarrow_A B \) implies \( \bar{a} \downarrow_{A \cap \text{acl}(\bar{a}, B)} B \).

Remark 3.2. The original definition of CM-triviality is as follows: For any \( a, A = \text{acl}^\text{eq}(A), B = \text{acl}^\text{eq}(B) \subset \mathcal{M}^\text{eq}, a \downarrow_A B \) implies \( a \downarrow_{A \cap \text{acl}^\text{eq}(a, B)} B \). Clearly, under assuming GEI, CM-triviality is equivalent to CM-triviality in the real sort. In the next remark, we lay out an example which shows the difference of the definitions.

Theorem 3.3. If \( T \) is CM-trivial in the real sort, then \( T \) has GEI. So CM-triviality in the real sort implies (the original) CM-triviality.

Proof. By Proposition 2.2, we will show that \( T \) has IND/I, i.e. if \( \bar{a}, A = \text{acl}(A), B = \text{acl}(B) \subset \mathcal{M} \) and \( \bar{a} \downarrow_A B, \bar{a} \downarrow_B A \), then \( \bar{a} \downarrow_{A \cap B} AB \). By CM-triviality in the real sort, we have \( \bar{a} \downarrow_{\text{acl}(\bar{a}, B) \cap A} B \). By \( \bar{a} \downarrow_B A \), we see \( \text{acl}(\bar{a}, B) \cap AB = B \). As \( A \cap B \subseteq A \cap \text{acl}(\bar{a}, B) \subseteq AB \cap \text{acl}(\bar{a}, B) = B \), we see

\[
\text{acl}(\bar{a}, B) \cap A = A \cap B.
\]

By \( \bar{a} \downarrow_{\text{acl}(\bar{a}, B) \cap A} B \) and \( \bar{a} \downarrow_B A \), we see \( \bar{a} \downarrow_{A \cap B} AB \).

}\( \square \)
Remark 3.4. (1) Let $T$ be the theory of a simple relational structure with a closure operator $\text{cl}(\ast)$ such that
- $\text{cl}(\text{acl}(A)) = \text{acl}(A) \text{ and } \text{cl}(\text{cl}(A) \cap \text{cl}(B)) = \text{cl}(A) \cap \text{cl}(B)$,
- for any algebraically closed sets $A, B \subseteq \mathcal{M}$, $A \downarrow_{\Lambda\cap B} B \Leftrightarrow "AB = \text{cl}(AB)\text{ and } R^{AB} = R^{A} \cup R^{B}\text{ for any predicate } R"$.

Then $T$ is CM-trivial in the real sort. (Suppose that $\bar{a} \downarrow_{A} B$.
Let $C = \text{acl}(\bar{a}, A), D = \text{acl}(AB)$. As $C \downarrow_{A} B$ and $C \cap B = A$,
$\text{cl}(CB) = CB$ and $R^{CB} = R^{C} \cup R^{B}$ for any predicate $R$. Let $E = \text{acl}(\bar{a}, B)$. Then $\text{cl}(CB \cap E) = CB \cap E$ and $R^{CB\cap E} = R^{C\cap E} \cup R^{B\cap E}$ for any predicate $R$. So, we see $C \cap E \downarrow_{A\cap E} B \cap E$.
As $\bar{a} \subset C \cap E, B \subset B \cap E, \bar{a} \downarrow_{A\cap \text{acl}(\bar{a}, B)} B$ follows.) So, by Theorem 3.3, CM-triviality of $T$ follows.

(2) CM-triviality does not imply CM-triviality in the real sort: In [E], Evans gave an $\omega$-categorical CM-trivial structure $\mathfrak{C}$, defined below, of SU-rank one without WEI.
Here, we check that $\mathfrak{C}$ does not have GEI.
Firstly, he constructed an $\omega$-categorical generic structure $M$
(coutable binary graph $R(x, y)$ with a predimension $\delta(A) = 2|A| - |R^{A}|$) of SU-rank two such that
- no triangles, no squares in $M$, and points and adjacent pairs
  of points are closed in $M$
- $\text{cl}(\ast) = \text{acl}(\ast)$ in $M$ and $M$ is of diameter 3.
Fix $a \in M$. Let $C, D$ be the sets of vertices at distance 1,2
from $a$. Then we have the canonical structure $\mathfrak{C}$ on $C$ such that $\text{Aut}(\mathfrak{C})$ is homeomorphic to $\text{Aut}(M/a)$, so $\mathfrak{C}$ and $(M, a)$
are biinterpretable. (See pp.136, 139, 348 in [H].) Then $\mathfrak{C}$ is of SU-rank one.
We see that $\mathfrak{C}$ does not have GEI as follows:
Let $c, c' \in C$ and $d, d' \in D$ be such that $M \models R(a, c) \cap R(a, c') \cap R(c, d) \cap R(c', d')$. As no triangles and squares in $M$, we have $M \models \neg R(c, c') \cap \neg R(c, d') \cap \neg R(c', d)$. Note that $c \in \text{dcl}(a, d)$
and $acd < acd'c$ or $acd'c$. So, $c', d' \notin \text{cl}(a, d, c) = \text{acl}(a, d, c)$.
Therefore $\text{cl}(a, d) = \text{acl}(a, d) = \{a, c, d\}$ follows. On the other hand, $\text{cl}(a, c) = \{a, c\}$. So, if $\mathfrak{C}$ has GEI, then, as $d \in \mathfrak{C}^{eq}$,
there exist $\bar{c} \subset_{\omega} C$ such that $d \in \text{acl}(a, \bar{c})$ and $\bar{c} \in \text{acl}(a, d)$
in the sense of $M$. But such $\bar{c}$ must be a singleton $c \in C$
with $M \models R(a, c) \cap R(c, d)$. Since $\text{acl}(a, c) = \{a, c\}$ in $M$, so $d \notin \text{acl}(a, c)$ in $M$.

Problem 3.5. In stable theories, is CM-triviality equivalent to CM-triviality in the real sort?
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