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On Entropies of Quantum Communication Processes

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Abstract

The mutual entropy (information) denotes an amount of information transmitted correctly from the input system to the output system through a channel. The (semi-classical) mutual entropies for classical input and quantum output were defined by several researchers. The fully quantum mutual entropy, which is called Ohya mutual entropy, for quantum input and output by using the relative entropy was defined by Ohya in 1983. In this paper, we compare with mutual entropy-type measures and show some resuls for quantum capacity

1 Introduction

The development of communication theory is closely connected with study of entropy theory. The signal of the input system is carried through a physical device, which is called a channel. The mathematical representation of the channel is a mapping from the input state space to the output state space. In classical communication theory, the mutual entropy was formulated by using the joint probability distribution between the input system and the output system. The (semi-classical) mutual entropies for classical input and quantum output were defined by several researchers [9, 10]. In fully quantum system, there does not exist the joint probability distribution in general. Instead of the joint probability distribution, Ohya [14] invented the quantum (Ohya) compound state, and he introduced the fully quantum mutual entropy (information), which is called Ohya mutual entropy, for quantum input and output systems, describes the amount of information correctly sent from the quantum input system to the quantum output system through the quantum channel.

In this paper, we compare with mutual entropy-type measures and show some resuls for quantum capacity for the attenuation channel. $\mathcal{H}$

2 Quantum Channels

The concept of channel has been carried out an important role in the progress of the quantum communication theory. In particular, an attenuation channel introduced in [14] is one of the most inportaint model for discussing the information transmission in quantum optical communication. Here we review the definition of the quantum channels.

Let $\mathcal{H}_1, \mathcal{H}_2$ be the complex separable Hilbert spaces of an input and an output systems, respectively, and let $\mathbf{B}(\mathcal{H}_k)$ be the set of all bounded linear operators on $\mathcal{H}_k$. We denote the set of all density operators on $\mathcal{H}_k \quad (k = 1, 2)$ by

$$\mathfrak{S}(\mathcal{H}_k) \equiv \{\rho \in \mathbf{B}(\mathcal{H}_k); \rho \geq 0, tr\rho = 1\}. \tag{1}$$

A map Λ^* from the quantum input system to the quantum output system is called a **(fully) quantum channel**.

1. Λ^* is called a **linear channel** if it satisfies the affine property, i.e.,

$$\begin{aligned} \sum_k \lambda_k &= 1\,(\forall \lambda_k \geq 0) \\ \Rightarrow \quad \Lambda^*\left(\sum_k \lambda_k \rho_k\right) &= \sum_k \lambda_k \Lambda^*(\rho_k), \forall \rho_k \in \mathfrak{S}(\mathcal{H}_1). \end{aligned}$$

2. $\Lambda^* : \mathfrak{S}(\mathcal{H}_1) \to \mathfrak{S}(\mathcal{H}_2)$ is called a **completely positive (CP) channel** if its dual map Λ satisfies

$$\sum_{j,k=1}^{n} B_j^* \Lambda(A_j^* A_k) B_k \geq 0 \tag{2}$$

for any $n \in \mathbb{N}$, any $B_j \in \mathbf{B}(\mathcal{H}_1)$ and any $A_k \in \mathbf{B}(\mathcal{H}_2)$, where the dual map $\Lambda : \mathbf{B}(\mathcal{H}_2) \to \mathbf{B}(\mathcal{H}_1)$ of

$\Lambda^* : \mathfrak{S}(\mathcal{H}_1) \to \mathfrak{S}(\mathcal{H}_2)$ satisfies $tr\rho\Lambda(A) = tr\Lambda^*(\rho)A$ for any $\rho \in \mathfrak{S}(\mathcal{H}_1)$ and any $A \in \mathbf{B}(\mathcal{H}_2)$.

2.1 Quantum Communication Process

Let $\mathcal{K}_1$ and $\mathcal{K}_2$ be two Hilbert spaces expressing noise and loss systems, respectively. Quantum communication process including the influence of noise and loss is described by the quantum channel [14]:

$$\Lambda^*(\rho) \equiv tr_{\mathcal{K}_2} \pi^*(\rho \otimes \xi)$$

for any input state ρ in $\mathfrak{S}(\mathcal{H}_1)$ and a noise state ξ in $\mathfrak{S}(\mathcal{K}_1)$, where the map π^* is a CP channel from $\mathfrak{S}(\mathcal{H}_1 \otimes \mathcal{K}_1)$ to $\mathfrak{S}(\mathcal{H}_2 \otimes \mathcal{K}_2)$ determined by physical properties of the communication device.

3 Ohya Mutual Entropy and Capacity

The quantum entropy was introduced by von Neumann around 1932 [13], which is defined by

$$S(\rho) \equiv -tr\rho \log \rho$$

for any density operators ρ in $S(\mathcal{H}_1)$. It denotes the amount of information of the quantum state ρ.

In order to define such a quantum mutual entropy, we need the quantum relative entropy and the joint state, which is called a compound state, describing the correlation between an input state ρ and the output state $\Lambda^*\rho$ through a channel Λ^*. For a state $\rho \in \mathfrak{S}(\mathcal{H}_1)$,

$$\rho = \Sigma_k \lambda_k E_k, \tag{3}$$

is called a Schatten decomposition [24] of ρ, where E_k is the one-dimensional orthogonal projection associated with λ_k. The Schatten decomposition is not unique usually depending on a degeneracy of the eigenvalue of ρ.For $\rho \in \mathfrak{S}(\mathcal{H}_1)$ and $\Lambda^* : \mathfrak{S}(\mathcal{H}_1) \to \mathfrak{S}(\mathcal{H}_2)$, the compound states are define by

$$\sigma_E = \sum_n \lambda_n E_n \otimes \Lambda^* E_n, \quad \sigma_0 = \varphi \otimes \Lambda^* \varphi. \tag{4}$$

The first compound state is called a Ohya compund state associating with the Schatten decomposition $\rho = \Sigma_k \lambda_k E_k$, which generalizes the joint probability in classical dynamical system and it shows a certain correlation between the initial state ρ and the final state $\Lambda^*\rho$.

Ohya mutual entropy with respect to ρ and Λ^* is defined by

$$I(\rho;\Lambda^*) \equiv \sup \{S(\sigma_E, \sigma_0); E = \{E_n\}\}, \tag{5}$$

where $S(\sigma_E, \sigma_0)$ is Umegaki's relative entropy [25]. $I(\rho;\Lambda^*)$ satisfies the Shannon's type inequality :

$$0 \leq I(\rho, \Lambda^*) \leq \min \{S(\rho), S(\Lambda^*\rho)\}.$$

3.1 Quantum Capacity

The capacity means the ability of the information transmission of the channel, which is used as a measure for construction of channels. The quantum capacity is formulated by taking the supremum of the Ohya mutual entropy with respect to a certain subset of the initial state space. The capacity of quantum channel was studied in [17, 18, 19, 20].

Let $\mathcal{S}$ be the set of all input states satisfying some physical conditions. Let us consider the ability of information transmission for the quantum channel Λ^*. The answer of this question is the capacity of quantum channel Λ^* for a certain set $\mathcal{S} \subset S(\mathcal{H}_1)$ defined by

$$C_q^{\mathcal{S}}(\Lambda^*) \equiv \sup \{I(\rho;\Lambda^*); \rho \in \mathcal{S}\}. \tag{6}$$

When $\mathcal{S} = S(\mathcal{H}_1)$, the capacity of quantum channel Λ^* is denoted by $C_q(\Lambda^*)$.

4 Quantum Entropy for Quantum Communication Channels

4.1 Attenuation channel

One of the example of the quantum communication channels is the attenuation channel Λ_0^* introduced by Ohya [14], which is defined by

$$\Lambda_0^*(\rho) \equiv tr_{\mathcal{K}_2}\pi_0^*(\rho \otimes \xi_0), \quad \xi_0 \equiv |0\rangle\langle 0| \text{ and } \pi_0^*(\cdot) \equiv V_0(\cdot)V_0^*, \tag{7}$$

where $|0\rangle\langle 0|$ is vacuum state in $\mathcal{H}_1$ and V_0 is a linear mapping from $\mathcal{H}_1 \otimes \mathcal{K}_1$ to $\mathcal{H}_2 \otimes \mathcal{K}_2$ given by

$$V_0(|n\rangle \otimes |0\rangle) \equiv \sum_{j=0}^{n} \sqrt{\frac{n!}{j!(n-j)!}} \alpha^j \bar{\beta}^{n-j} |j\rangle \otimes |n-j\rangle \tag{8}$$

for any $|n\rangle$ in $\mathcal{H}_1$ and α,β are complex numbers satisfying $|\alpha|^2 + |\beta|^2 = 1$. $\eta = |\alpha|^2$, which is the transmission rate of the channel. π_0^* is called a beam splittings, which means that one beam comes and two beams appear after passing through π_0^*. The attenuation channel is generalized by the noisy optical channel [20, 21], which is also reformulated by Accardi and Ohya [1] using the liftings. The noisy optical channel consists of the generaliezed beam splittings π^*, which was extended on generalized Fock space by Fichtner, Freudenberg and Libsher [6] by means of the concept of compound Hida-Malliavin derivative [8] and so on.

For the attenuation channel Λ_0^*, one can obtain the following theorem proved in [22].

Theorem 1 *For a subset $S_n \equiv \{\rho \in S(\mathcal{H}_1); \dim s(\rho) = n\}$, the capacity of the attenuation channel Λ_0^* satisfies*

$$C_q^{S_n}(\Lambda_0^*) = \log n,$$

where $s(\rho)$ is the support projection of ρ.

When the mean energy of the input state vectors $\{|\tau\theta_k\rangle\}$ can be taken infinite, i.e., $\lim_{\tau\to\infty} |\tau\theta_k|^2 = \infty$, the above theorem tells that the quantum capacity for the attenuation channel Λ_0^* with respect to S_n becomes $\log n$. It is a natural result, however it is impossible to take the mean energy of input state vector infinite.

4.2 Quantum Teleportation Channel

In usual quantum communication processes, quantum states are transmitted from input system to an output system through a channel. Since the channel representing a physical apparatus is affected by outside system, the final state sent through the channel is different from the initial state. On the contrary, in teleportation scheme, a particle is not transmitted from Alice's system to Bob's system, but one can reconstruct the initial state by means of the entangled state located at Bob's system. Bennett et al. [2, 3] proposed a state change, so-called a quantum teleportation, in terms of EPR entangled state denoted by Bell's base. It is difficult to realize such teleportation scheme because the EPR entangled state dissipates easily. In order to avoid this demerit, Ohya and Fichtner [4, 5] introduced a new teleportation scheme on boson-Fock space by means of the entangled coherent states and the general beam splitting.

In perfect quantum teleportation scheme, the total process of the quantum teleportation is consist of three systems denoted by the complex Hilbert spaces $\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3$. Alice controls $\mathcal{H}_1$,$\mathcal{H}_2$ and Bob treats $\mathcal{H}_3$. Alice has an unknown initial state $\rho^{(1)} \in \mathfrak{S}(\mathcal{H}_1)$ and she teleports it to Bob by using an entangled state $\sigma^{(23)} \in \mathfrak{S}(\mathcal{H}_2 \otimes \mathcal{H}_3)$ belonging to Alice and Bob. At first, Alice measures for the part of $\mathcal{H}_1 \otimes \mathcal{H}_2$ in the state $\rho^{(1)} \otimes \sigma^{(23)}$ by means of an observable $F^{(12)} \equiv \sum_{l,m} z_{lm} F_{lm}^{(12)} \in \mathbb{B}(\mathcal{H}_1 \otimes \mathcal{H}_2)$, where $\left\{F_{lm}^{(12)}\right\}$ is a set of orthogonal projections $F_{lm}^{(12)} \in \mathbb{B}(\mathcal{H}_1 \otimes \mathcal{H}_2)$. If Alice obtains a value z_{lm} after measurement, then the state $\rho^{(1)} \otimes \sigma^{(23)}$ is changed to

$$\rho_{lm}^{(123)} = \frac{\left(F_{lm}^{(12)} \otimes I_3\right)\left(\rho^{(1)} \otimes \sigma^{(23)}\right)\left(F_{lm}^{(12)} \otimes I_3\right)}{tr_{123}\left(F_{lm}^{(12)} \otimes I_3\right)\left(\rho^{(1)} \otimes \sigma^{(23)}\right)\left(F_{lm}^{(12)} \otimes I_3\right)}. \tag{9}$$

Alice send the result z_{lm} of the measurement to Bob through a classical communication channel. Bob reconstructs the unknown initial state $\rho^{(1)}$ from $\Lambda_{lm}^{*}\left(\rho^{(1)}\right) \equiv tr_{12}\rho_{lm}^{(123)} \in \mathfrak{S}(\mathcal{H}_3)$ by applying a unitary key U_{lm} created by the value z_{lm} received from Alice. The total process of the perfect teleportation scheme is described by

$$\rho^{(1)} \equiv U_{lm}\left(\Lambda_{lm}^{*}\left(\rho^{(1)}\right)\right)U_{lm}^{*}. \tag{10}$$

In the perfect teleportation scheme, Bob can reconstruct the unknown initial state $\rho^{(1)}$ by applying the unitary key once. On the contrary, in the non-perfect teleportation scheme, Bob can obtain the unknown initial state $\rho^{(1)}$ by applying the unitary operator V_{nm} more than twice. Applying the unitary operator V_{nm} once to the state $\Lambda_{nm}^{*}\left(\rho^{(1)}\right)$, the state change of the non-perfect teleportation scheme is denoted by

$$\Xi_{nm}^{*}\left(\rho^{(1)}\right) \equiv V_{nm}\Lambda_{nm}^{*}\left(\rho^{(1)}\right)V_{nm}^{*}. \tag{11}$$

For the perfect teeportation, we have the following theorem [11]:

Theorem 2 *If the perfect teleportation channel is lnear and the input state $\rho^{(1)}$ is finite rank operator, then one can obtain*

$$I\left(\rho^{(1)}, \Lambda_T^{*}\right) = S\left(\rho^{(1)}\right)$$

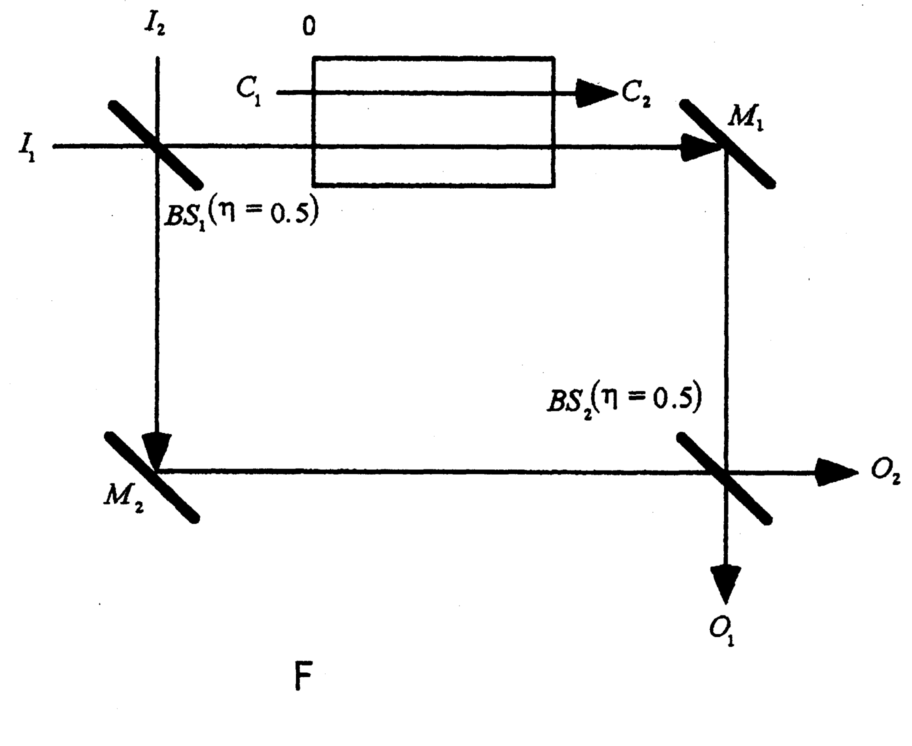
for

4.3 Quantum channel for Fredkin-Toffoli-Milburn gate

In usual computer, we could not determine two inputs for the logical gates AND and OR after we know the output for these gates. This property is called an irreversibility of logical gate. This property leads to the loss of information and the heat generation. Thus there exists an upper bound of computational speed.

Fredkin and Toffoli proposed a conservative gate, by which any logical gate is realized and it is shown to be a reversible gate in the sense that there is no loss of information. This gate was developed by Milburn as a quantum gate with quantum input and output. We call this gate Fredkin-Toffoli-Milburn (FTM) gate. Recently, we reformulate a quantum channel for the FTM gate and we rigorously study the conservation of information for FTM gate [23].

The FTM gate is composed of two input gates I_1, I_2 and one control gate C. Two inputs come to the first beam splitter and one spliting input passes through the control gate made from an optical Kerr device, then two spliting inputs come in the second beam splitter and appear as two outputs (Fig.2.1). Two beam splitters and the optical Kerr medium are needed to describe the gate.



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(1) Beam splitters: (a) Based on [19], let V_1 be a mapping from $\mathcal{H}_1 \otimes \mathcal{H}_2$ to $\mathcal{H}_1 \otimes \mathcal{H}_2$ with transmission rate η_1 given by

$$V_1 \left(|n_1\rangle \otimes |n_2\rangle \right) \equiv \sum_{j=0}^{n_1+n_2} C_j^{n_1,n_2} |j\rangle \otimes |n_1 + n_2 - j\rangle \tag{12}$$

for any photon number state vectors $|n_1\rangle \otimes |n_2\rangle \in \mathcal{H}_1 \otimes \mathcal{H}_2$. The quantum channel Π^*_{BS1} expressing the first beam splitter (beam splitter 1) is defined by

$$\Pi^*_{BS1}(\rho_1 \otimes \rho_2) \equiv V_1(\rho_1 \otimes \rho_2) V_1^* \tag{13}$$

for any states $\rho_1 \otimes \rho_2 \in \mathfrak{S}(\mathcal{H}_1 \otimes \mathcal{H}_2)$. In particular, for an input state in two gates I_1 and I_2 given by the tensor product of two coherent states $\rho_1 \otimes \rho_2 = |\theta_1\rangle\langle\theta_1| \otimes |\theta_2\rangle\langle\theta_2|$, $\Pi^*_{BS1}(\rho_1 \otimes \rho_2)$ is written as

$$\begin{aligned} & \Pi^*_{BS1}(\rho_1 \otimes \rho_2) \\ = \; & \left|\sqrt{\eta_1}\theta_1 + \sqrt{1-\eta_1}\theta_2\right\rangle \left\langle \sqrt{\eta_1}\theta_1 + \sqrt{1-\eta_1}\theta_2\right| \\ & \otimes \left|-\sqrt{1-\eta_1}\theta_1 + \sqrt{\eta_1}\theta_2\right\rangle \left\langle -\sqrt{1-\eta_1}\theta_1 + \sqrt{\eta_1}\theta_2\right|. \end{aligned} \tag{14}$$

(b) Let V_2 be a mapping from $\mathcal{H}_1 \otimes \mathcal{H}_2$ to $\mathcal{H}_1 \otimes \mathcal{H}_2$ with transmission rate η_2 given by

$$V_2(|n_1\rangle \otimes |n_2\rangle) \equiv \sum_{j=0}^{n_1+n_2} C_j^{n_2,n_1} |n_1+n_2-j\rangle \otimes |j\rangle \tag{15}$$

for any photon number state vectors $|n_1\rangle \otimes |n_2\rangle \in \mathcal{H}_1 \otimes \mathcal{H}_2$. The quantum channel Π^*_{BS2} expressing the second beam splitter (beam splitter 2) is defined by

$$\Pi^*_{BS2}(\rho_1 \otimes \rho_2) \equiv V_2(\rho_1 \otimes \rho_2) V_2^* \tag{16}$$

for any states $\rho_1 \otimes \rho_2 \in \mathfrak{S}(\mathcal{H}_1 \otimes \mathcal{H}_2)$. In particular, for coherent input states $\rho_1 \otimes \rho_2 = |\theta_1\rangle\langle\theta_1| \otimes |\theta_2\rangle\langle\theta_2|$, $\Pi^*_{BS2}(\rho_1 \otimes \rho_2)$ is written as

$$\begin{aligned} \Pi^*_{BS2}(\rho_1 \otimes \rho_2) = \; & \left|\sqrt{\eta_2}\theta_1 - \sqrt{1-\eta_2}\theta_2\right\rangle \left\langle \sqrt{\eta_2}\theta_1 - \sqrt{1-\eta_2}\theta_2\right| \\ & \otimes \left|\sqrt{1-\eta_2}\theta_1 + \sqrt{\eta_2}\theta_2\right\rangle \left\langle \sqrt{1-\eta_2}\theta_1 + \sqrt{\eta_2}\theta_2\right|. \end{aligned} \tag{17}$$

(2) Optical Kerr medium: The interaction Hamiltonian in the optical Kerr medium is given by the number operators N_1 and N_c for the input system 1 and the Kerr medium, respectively, such as

$$H_{int} = \hbar\chi(N_1 \otimes I_2 \otimes N_c), \tag{18}$$

where $\hbar$ is the Plank constant divided by 2π, χ is a constant proportional to the susceptibility of the Kerr medium and I_2 is the identity operator on $\mathcal{H}_2$. Let T be the passing time of a beam through the Kerr medium and put $\sqrt{F} = \hbar\chi T$, a parameter exhibiting the power of the Kerr effect. Then the unitary operator U_K describing the evolution for time T in the Kerr medium is given by

$$U_K = \exp\left(-i\sqrt{F}(N_1 \otimes I_2 \otimes N_c)\right). \tag{19}$$

We assume that an initial (input) state of the control gate is the n photon number state $\xi = |n\rangle\langle n|$, a quantum channel Λ_K^* representing the optical Kerr effect is given by

$$\Lambda_K^*(\rho_1 \otimes \rho_2 \otimes \xi) \equiv U_K(\rho_1 \otimes \rho_2 \otimes \xi)U_K^* \tag{20}$$

for any state $\rho_1 \otimes \rho_2 \otimes \xi \in \mathfrak{S}(\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{K})$. In particular, for an initial state $\rho_1 \otimes \rho_2 \otimes \xi = |\theta_1\rangle\langle\theta_1| \otimes |\theta_2\rangle\langle\theta_2| \otimes |n\rangle\langle n|$, $\Lambda_K^*(\rho_1 \otimes \rho_2 \otimes \xi)$ is denoted by

$$\begin{aligned}
&\Lambda_K^*(\rho_1 \otimes \rho_2 \otimes \xi) \\
=\ &\left|\exp\left(-i\sqrt{F}n\right)\theta_1\right\rangle\left\langle\exp\left(-i\sqrt{F}n\right)\theta_1\right| \\
&\otimes |\theta_2\rangle\langle\theta_2| \otimes |n\rangle\langle n|,
\end{aligned} \tag{21}$$

Using the above channels, the quantum channel for the whole FTM gate is constructed as follows: Let both one input and output gates be described by $\mathcal{H}_1$, another input and output gates be described by $\mathcal{H}_2$ and the control gate be done by $\mathcal{K}$, all of which are Fock spaces. For a total state $\rho_1 \otimes \rho_2 \otimes \xi$ of two input states and a control state, the quantum channels $\Lambda_{BS1}^*, \Lambda_{BS2}^*$ from $\mathfrak{S}(\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{K})$ to $\mathfrak{S}(\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{K})$ are written by

$$\Lambda_{BSk}^*(\rho_1 \otimes \rho_2 \otimes \xi) = \Pi_{BSk}^*(\rho_1 \otimes \rho_2) \otimes \xi \quad (k = 1, 2) \tag{22}$$

Therefore, the whole quantum channel Λ_{FTM}^* of the FTM gate is defined by

$$\Lambda_{\mathrm{FTM}}^* \equiv \Lambda_{BS2}^* \circ \Lambda_K^* \circ \Lambda_{BS1}^*. \tag{23}$$

In particular, for an initial state $\rho_1 \otimes \rho_2 \otimes \xi = |\theta_1\rangle\langle\theta_1| \otimes |\theta_2\rangle\langle\theta_2| \otimes |n\rangle\langle n|$, $\Lambda_{\mathrm{FTM}}^*(\rho_1 \otimes \rho_2 \otimes \xi)$ is obtained by

$$\begin{aligned}
&\Lambda_{\mathrm{FTM}}^*(\rho_1 \otimes \rho_2 \otimes \xi) \\
=\ &|\mu_n\theta_1 + \nu_n\theta_2\rangle\langle\mu_n\theta_1 + \nu_n\theta_2| \\
&\otimes |\nu_n\theta_1 + \mu_n\theta_2\rangle\langle\nu_n\theta_1 + \mu_n\theta_2| \otimes |n\rangle\langle n|
\end{aligned} \tag{24}$$

where

$$\mu_n = \frac{1}{2}\left\{\exp\left(-i\sqrt{F}n\right) + 1\right\}, \tag{25}$$

$$\nu_n = \frac{1}{2}\left\{\exp\left(-i\sqrt{F}n\right) - 1\right\}, \quad (k = 0, 1, 2, \cdots). \tag{26}$$

Then we obtain the following theorem:

Theorem 3 *If $\sqrt{F}$ satisfies the conditions $\sqrt{F}n = 0$ or $\sqrt{F}n = (2m+1)\pi$ $(m = 0, 1, 2, \cdots)$, then one can obtain*

$$I\left(\rho_1 \otimes \rho_2, \widetilde{\Lambda}_{FTM}^*\right) = S(\rho_1) + S(\rho_2)$$

for input states $\rho_1 \otimes \rho_2$ *given by*

$$\rho_i = \lambda_i |0\rangle \langle 0| + (1-\lambda_i) |x_i\rangle \langle x_i| \in \mathfrak{S}(\mathcal{H}_i), \quad \lambda_i \in [0,1], \quad (i=1,2),$$

where $|x_i\rangle$ *is defined by*

$$|x_i\rangle = \frac{|\theta_i\rangle - |-\theta_i\rangle}{\sqrt{2\left(1+\exp\left(-\frac{1}{2}|\theta_i|^2\right)\right)}}.$$

References

[1] Accardi, L., and Ohya, M., Compond channnels, transition expectation and liftings, Appl. Math, Optim., 39, 33-59 (1999).

[2] C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K.Wootters, *Teleporting an unknown quantum state via dual classical and EPR channels*, Physical Review Letters, **70**, 1895 (1993).

[3] C.H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J.A. Smolin,W.K. Wootters, *Purification of noisy entanglement and faithful teleportation via noisy channels*, Physical Review Letters, **76**, 722 (1996).

[4] K-H. Fichtner and M. Ohya, *Quantum teleportation with entangled states given by beam splittings*, Communications in Mathematical Physics, **222**, 229 (2001).

[5] K-H. Fichtner and M. Ohya, *Quantum teleportation and beam splitting*, Communications in Mathematical Physics, **225**, 67 (2002).

[6] Fichtner, K.H., Freudenberg, W., and Liebscher, V., Beam splittings and time evolutions of Boson systems, Fakultat fur Mathematik und Informatik, Math/ Inf/96/ 39, Jena, 105 (1996).

[7] Fredkin, E., and Toffoli T., Conservative logic, International Journal of Theoretical Physics , **21** , 219-253, (1982).

[8] Hida, T., Analysis of Brownian Functionals, Carleton Mathematical Lecture Notes, 13, 1975.

[9] Holevo, A.S., Some estimates for the amount of information transmittable by a quantum communication channel (in Russian)}, Problemy Peredachi Informacii, 9, 3-11 (1973).

[10] Ingarden, R.S., Kossakowski, A., and Ohya, M., Information Dynamics and Open Systems, Kluwer, 1997.

[11] K. Inoue, H. Suyari and M. Ohya, Characterization of quantum teleportation by nonlinear quantum channel and quantum mutual entropy, Physical D. ,**120**, 117 (1998).

[12] Milburn G.J., Quantum optical Fredkin gate, Physical Review Letters , **62**, 2124-2127, (1989).

[13] von Neumann, J., Die Mathematischen Grundlagen der Quantenmechanik, Springer-Berlin, 1932.

[14] Ohya, M., On compound state and mutual information in quantum information theory, IEEE Trans. Information Theory, 29, 770-774 (1983).

[15] Ohya, M., Some aspects of quantum information theory and their applications to irreversible processes, Rep. Math. Phys., 27, 19-47 (1989).

[16] Ohya, M., and Petz, D., Quantum Entropy and its Use, Springer, Berlin, 1993.

[17] Ohya, M., Petz, D., and Watanabe, N., On capacity of quantum channels, Probability and Mathematical Statistics, 17, 179-196 (1997).

[18] Ohya, M., Petz, D., and Watanabe, N., Numerical computation of quantum capacity, International Journal of Theoretical Physics, 37, No.1, 507-510 (1998).

[19] Ohya, M., and Watanabe, N., Quantum capacity of noisy quantum channel, Quantum Communication and Measurement, 3, 213-220 (1997).

[20] Ohya, M., and Watanabe, N., Foundatin of Quantum Communication Theory (in Japanese), Makino Pub. Co., 1998.

[21] Ohya, M., and Watanabe, N., Construction and analysis of a mathematical model in quantum communication processes, Electronics and Communications in Japan, Part 1, 68, No.2, 29-34 (1985).

[22] Ohya, M., and Watanabe, N., Comparison of mutual entropy - type measures, TUS preprint.

[23] Ohya M., and Watanabe N., On mathematical treatment of optical Fredkin - Toffoli - Milburn gate, Physica D, **120**, 206-213, (1998).

[24] Schatten, R., Norm Ideals of Completely Continuous Operators, Springer-Verlag, 1970.

[25] Umegaki, H., Conditional expectations in an operator algebra IV (entropy and information), Kodai Math. Sem. Rep., 14, 59-85 (1962).