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NUMERICAL REAL INVERSION FORMULAS OF THE LAPLACE TRANSFORM BY USING THE SINC FUNCTIONS

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Abstract

We shall give a very natural and numerical real inversion formula of the Laplace transform for general L_2 data following the ideas of best approximations, generalized inverses, the Tikhonov regularization and the theory of reproducing kernels. Furthermore, we shall additionally use the Sinc functions (the Sinc method) to our general theory, to solve the related integral equation. However, the new method in this paper for the real inversion formula will be reduced to the solution of linear simultaneous equations. This real inversion formula may be expected to be practical to calculate the inverses of the Laplace transform by computers when the real data contain noise or errors. We shall illustrate examples and justify our computational work.

1 Introduction

We shall give a very natural and numerical real inversion formula of the Laplace transform

$$(\mathcal{L}F)(p) = f(p) = \int_0^\infty e^{-pt}F(t)dt, \quad p > 0 \tag{1.1}$$

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for functions F of some natural function space. This integral transform is, of course, very fundamental in mathematical science. The inversion of the Laplace transform is, in general, given by a complex form, however, we are interested in and are requested to obtain its real inversion in many practical problems. However, the real inversion will be very involved and one might think that its real inversion will be essentially involved, because we must catch "analyticity" from the real or discrete data. Note that the image functions of the Laplace transform are analytic on some half complex plane. For complexity of the real inversion formula of the Laplace transform, we recall, for example, the following formulas:

$$\lim_{n\to\infty} \frac{(-1)^n}{n!} \left(\frac{n}{t}\right)^{n+1} f^{(n)}\left(\frac{n}{t}\right) = F(t)$$

(Post [15] and Widder [27,28]), and

$$\lim_{n\to\infty} \Pi_{k=1}^{n} \left(1 + \frac{t}{k}\frac{d}{dt}\right) \left[\frac{n}{t} f\left(\frac{n}{t}\right)\right] = F(t),$$

([27,28]).

Furthermore, see [1-8,16,17,21,25,26,28,29] and the recent related articles [10] and [11]. See also the great references [29,30]. The problem may be related to analytic extension problems, see [11] and [22]. In this paper, we shall give a new type and very natural real inversion formula from the viewpoints of best approximations, generalized inverses and the Tikhonov regularization by combining these fundamental ideas and methods by means of the theory of reproducing kernels. Furthermore, we shall use the sinc functions (the sinc method) as a new approarch to solve the crucial Fredholm integral equation of the second kind on the half space in our general theory. We shall also propose a new method for the real inversion of the Laplace transform based essentially on linear simultaneous equations. We may think that this real inversion formula is practical and natural. Error analysis will be also considered.

2 Background General Theorems

Let E be an arbitrary set, and let H_K be a reproducing kernel Hilbert space (RKHS) admitting the reproducing kernel $K(p, q)$ on E. For any Hilbert space $\mathcal{H}$ we consider a bounded linear operator L from H_K into $\mathcal{H}$. We are generally interested in the best approximation problem

$$\inf_{f \in H_K} \|Lf - \mathbf{d}\|_{\mathcal{H}} \tag{2.2}$$

for a vector $\mathbf{d}$ in $\mathcal{H}$. However, this extremal problem is quite involved in existence and representation. See [16,19,20] for the details.

Now, for the Tikhonov regularization, we set, for any fixed positive $\alpha > 0$

$$K_L(\cdot, p; \alpha) = (L^*L + \alpha I)^{-1} K(\cdot, p),$$

where L^* denotes the adjoint operator of L. Then, by introducing the inner product

$$(f, g)_{H_K(L;\alpha)} = \alpha (f, g)_{H_K} + (Lf, Lg)_{\mathcal{H}}, \tag{2.3}$$

we shall construct the Hilbert space $H_K(L; \alpha)$ comprising functions of H_K. This space, of course, admits a reproducing kernel. Furthermore, we dirctly obtain

Proposition 2.1 *([18-20]) The extremal function $f_{\mathbf{d},\alpha}(p)$ in the Tikhonov regularization*

$$\inf_{f \in H_K} \{\alpha \|f\|^2_{H_K} + \|\mathbf{d} - Lf\|^2_{\mathcal{H}}\} \tag{2.4}$$

exists uniquely and it is represented in terms of the kernel $K_L(p, q; \alpha)$ by:

$$f_{\mathbf{d},\alpha}(p) = (\mathbf{d}, LK_L(\cdot, p; \alpha))_{\mathcal{H}} \tag{2.5}$$

where the kernel $K_L(p, q; \alpha)$ is the reproducing kernel for the Hilbert space $H_K(L; \alpha)$ and it is determined as the unique solution $\tilde{K}(p, q; \alpha)$ of the equation:

$$\tilde{K}(p, q; \alpha) + \frac{1}{\alpha}(L\tilde{K}_q, LK_p)_{\mathcal{H}} = \frac{1}{\alpha} K(p, q) \tag{2.6}$$

with

$$\tilde{K}_q = \tilde{K}(\cdot, q; \alpha) \in H_K \quad \textit{for} \quad q \in E, \tag{2.7}$$

and

$$K_p = K(\cdot, p) \in H_K \quad \textit{for} \quad p \in E.$$

In (2.5), when $\mathbf{d}$ contains errors or noise, we need its error estimate. For this, we can use the general result:

Proposition 2.2 *([14]). In* (2.5), *we have the estimate*

$$|f_{\mathbf{d},\alpha}(p)| \leq \frac{1}{\sqrt{\alpha}}\sqrt{K(p,p)}\|\mathbf{d}\|_{\mathcal{H}}.$$

For the convergence rate or the results for noisy data, see, ([9]).

3 A Natural Situation for Real Inversion Formulas

In order to apply the general theory in Section 2 to the real inversion formula of the Laplace transform, we shall recall the "natural situation" based on [17].

We shall introduce the simple reproducing kernel Hilbert space (RKHS) H_K comprised of absolutely continuous functions F on the positive real line $\mathbf{R}^+$ with finite norms

$$\left\{\int_0^\infty |F'(t)|^2 \frac{1}{t} e^t dt\right\}^{1/2}$$

and satisfying $F(0) = 0$. This Hilbert space admits the reproducing kernel $K(t,t')$

$$K(t,t') = \int_0^{\min(t,t')} \xi e^{-\xi} d\xi \tag{3.8}$$

$$= \left\{ \begin{array}{lll} -te^{-t} - e^{-t} + 1 & \text{for} & t \leq t' \\ -t'e^{-t'} - e^{-t'} + 1 & \text{for} & t \geq t' \end{array} \right\}$$

(see [16], pages 55-56). Then we see that

$$\int_0^\infty |(\mathcal{L}F)(p)p|^2 dp \leq \frac{1}{2}\|F\|_{H_K}^2; \tag{3.9}$$

that is, the linear operator

$$(\mathcal{L}F)(p)p$$

on H_K into $L_2(\mathbf{R}^+, dp) = L_2(\mathbf{R}^+)$ is bounded. For the reproducing kernel Hilbert spaces H_K satisfying (3.9), we can find some general spaces ([17]). Therefore, from the general theory in Section 2, we obtain

Proposition 3.1 *([13]). For any $g \in L_2(\mathbf{R}^+)$ and for any $\alpha > 0$, the best approximation $F^*_{\alpha,g}$ in the sense*

$$\inf_{F\in H_K}\left\{\alpha\int_0^\infty |F'(t)|^2\frac{1}{t}e^t dt + \|(\mathcal{L}F)(p)p - g\|^2_{L_2(\mathbf{R}^+)}\right\}$$

$$= \alpha\int_0^\infty |F^{*\prime}_{\alpha,g}(t)|^2\frac{1}{t}e^t dt + \|(\mathcal{L}F^*_{\alpha,g})(p)p - g\|^2_{L_2(\mathbf{R}^+)} \tag{3.10}$$

exists uniquely and we obtain the representation

$$F^*_{\alpha,g}(t) = \int_0^\infty g(\xi)\left(\mathcal{L}K_\alpha(\cdot,t)\right)(\xi)\xi d\xi. \tag{3.11}$$

Here, $K_\alpha(\cdot,t)$ is determined by the functional equation

$$K_\alpha(t,t') = \frac{1}{\alpha}K(t,t') - \frac{1}{\alpha}((\mathcal{L}K_{\alpha,t'})(p)p, (\mathcal{L}K_t)(p)p)_{L_2(\mathbf{R}^+)} \tag{3.12}$$

for

$$K_{\alpha,t'} = K_\alpha(\cdot,t')$$

and

$$K_t = K(\cdot,t).$$

4 Sampling Theory and Reproducing Kernels

In order to solve the integral equation (3.11), numerically, we shall employ the sinc method. At first we shall fix notations and basic results in the sampling theory following the book by F. Stenger[23] and at the same time we shall show the basic relation of the sampling theory and the theory of reproducing kernels.

We shall consider the integral transform, for a function g in

$$L_2(-\pi/h, +\pi/h), (h > 0)$$

$$f(z)=\frac{1}{2\pi}\int_{-\pi/h}^{\pi/h} g(t)e^{-izt}dt. \tag{4.13}$$

In order to identify the image space following the theory of reproducing kernels [16], we form the reproducing kernel

$$\begin{aligned} K_h(z,\overline{u}) &= \frac{1}{2\pi}\int_{-\pi/h}^{\pi/h} e^{-izt}\overline{e^{-iut}}dt \qquad (4.14)\\ &= \frac{1}{\pi(z-\overline{u})}\sin\frac{\pi}{h}(z-\overline{u})\\ &:= \frac{1}{h}Sinc\left(\frac{z-\overline{u}}{h}\right)\\ &:= \frac{1}{h}S(k,h)(z-\overline{u}+hk), \end{aligned}$$

by the notations in [23]. The image space of (4.13) is called the Paley-Wiener space $W\left(\frac{\pi}{h}\right)$ comprised of all analytic functions of exponential type satisfying, for some constant C and as $z\to\infty$

$$|f(z)|\leq C\exp\left(\frac{\pi|z|}{h}\right)$$

and

$$\int_{\mathbf{R}}|f(x)|^2dx<\infty.$$

From the identity

$$K_h(jh,j'h)=\frac{1}{h}\delta(j,j')$$

(the Kronecker's δ), since $\delta(j,j')$ is the reproducing kernel for the Hilbert space ℓ^2, from the general theory of integral transforms and the Parseval's identity we have the isometric identities in (4.13)

$$\begin{aligned} &\frac{1}{2\pi}\int_{-\pi/h}^{\pi/h}|g(t)|^2dt\\ &= h\sum_j|f(jh)|^2\\ &= \int_{\mathbf{R}}|f(x)|^2dx. \end{aligned}$$

That is, the reproducing kernel Hilbert space H_{K_h} with $K_h(z,\overline{u})$ is characterized as a space comprising the Paley-Wiener space $W\left(\frac{\pi}{h}\right)$ and with the norm squares above. Here we used the well-known result that $\{jh)\}_j$ is a unique set for the Paley-Wiener space $W\left(\frac{\pi}{h}\right)$; that is, $f(jh)=0$ for all j implies $f\equiv 0$. Then, the reproducing property of $K_h(z,\overline{u})$ states that

$$f(x)=\langle f(\cdot),K_h(\cdot,x)\rangle_{H_{K_h}}=h\sum_j f(jh)K_h(jh,x)$$

$$=\int_{\mathbf{R}} f(\xi)K_h(\xi,x)d\xi.$$

In particular, on the real line x, this representation is the sampling theorem which represents the whole data $f(x)$ in terms of the discrete data $\{f(jh)\}_j$. For a general theory for the sampling theory and error estimates for some finite points $\{hj\}_j$, see [16].

5 New Algorithm

By setting

$$(\mathcal{L}K_\alpha(\cdot,t))(\xi)\xi=H_\alpha(\xi,t),$$

which is needed in (3.11), we obtain the Fredholm integral equation of the second type

$$\alpha H_\alpha(\xi,t)+\int_0^\infty H_\alpha(p,t)\frac{1}{(p+\xi+1)^2}dp$$

$$=f(\xi,t)=-\frac{e^{-t\xi}e^{-t}}{\xi+1}\left(t+\frac{1}{\xi+1}\right)+\frac{1}{(\xi+1)^2}. \tag{5.15}$$

We shall use the double exponential transform following the idea [24]

$$\xi=\phi(x)=\exp(\frac{\pi}{2}\sinh x),$$

$$\phi'(x)=\frac{\pi}{2}\cosh x\exp(\frac{\pi}{2}\sinh x).$$

Note that this $\phi(x)$ is a monotonically increasing function and $\phi(-\infty)=0$ and $\phi(\infty)=\infty$. In addition, for examples

$$\phi(-4)=2.416\times 10^{-19}$$

and

$$\phi(16) = 5.860 \times 10^{3030999}.$$

So there is no need for setting so wide interval of integration from a practical point of view.
Then, we have

$$H_\alpha(\xi, t) = H_\alpha(\phi(x), t) = \widetilde{H}_\alpha(x, t)$$

and so,

$$\widetilde{H}_\alpha(x, t) = \sum_{j=-\infty}^{j=\infty} \widetilde{H}_\alpha(jh, t) Sinc(\frac{x}{h} - j)$$

and

$$\alpha \widetilde{H}_\alpha(x, t) + \int_{-\infty}^{\infty} \widetilde{H}_\alpha(z, t) \frac{1}{(\phi(z) + \phi(x) + 1)^2} \phi'(z) dz = f(\phi(x), t). \tag{5.16}$$

We shall approximate as follows:

$$\widetilde{H}_\alpha(x, t) \simeq \sum_{j=-N}^{j=N} \widetilde{H}_\alpha(jh, t) Sinc(\frac{x}{h} - j).$$

For error estimates for some finite points $\{hj\}_j$, see [16]. Then, we have

$$\alpha \sum_j \widetilde{H}_\alpha(jh, t) Sinc(\frac{x}{h} - j) + \int_{-\infty}^{\infty} \sum_k \widetilde{H}_\alpha(kh, t) Sinc(\frac{z}{h} - k) \frac{1}{(\phi(z) + \phi(x) + 1)^2} \phi'(z) dz = f(\phi(x), t). \tag{5.17}$$

From the identities

$$\int_{-\infty}^{\infty} Sinc(\frac{x}{h} - i) Sinc(\frac{x}{h} - j) dx = h\delta_{ij}$$

and

$$\int_{-\infty}^{\infty} \frac{1}{(\phi(z) + \phi(x) + 1)^2} Sinc(\frac{x}{h} - l) dx = \frac{1}{(\phi(z) + \phi(lh) + 1)^2} h,$$

by setting

$$A_{lk} \equiv \frac{1}{(\phi(kh) + \phi(lh) + 1)^2} \phi'(kh) h$$

we obtain the equation

$$\alpha\widetilde{H}_\alpha(lh,t) + \sum_k \widetilde{H}_\alpha(kh,t)A_{lk} = f(\phi(lh),t) \equiv \tilde{f}(lh,t) \tag{5.18}$$

and the representation

$$F^*(t) = \int_0^\infty g(\xi)H_\alpha(\xi,t)d\xi = \int_{-\infty}^\infty g(\phi(x))H_\alpha(\phi(x),t)\phi'(x)dx$$
$$\simeq h\sum_i g(\phi(ih))\widetilde{H}_\alpha(ih,t)\phi'(ih).$$

6 Inverses for More General Functions

As one of the main features of our method, we can easily generalize the approximation function space. By a suitable transform, our inversion formula in Section 5 is applicable for more general functions as follows:

We assume that F satisfies the properties (P):

$$F \in C^1[0,\infty),$$
$$F'(t) = o(e^{\alpha t}), \quad 0 < \alpha < k - \frac{1}{2},$$

and

$$F(t) = o(e^{\beta t}), \quad 0 < \beta < k - \frac{1}{2}.$$

Then, the function

$$G(t) = \{F(t) - F(0) - tF'(0)\}e^{-kt} \tag{6.19}$$

belongs to H_K. Then,

$$(\mathcal{L}G)(p) = f(p+k) - \frac{F(0)}{p+k} - \frac{F'(0)}{(p+k)^2}. \tag{6.20}$$

Therefore, if we know $F(0)$ and $F'(0)$, then from

$$g(p) = (\mathcal{L}G)(p)$$

by the method in Section 5, we obtain $G(t)$ and so, from the identity

$$F(t) = G(t)e^{kt} + F(0) + tF'(0) \tag{6.21}$$

we have the inverse $F(t)$ from the data $f(p), F(0)$ and $F'(0)$ through the above procedures.

7 Numerical Experiments

We used $h = 0.05$ and for t, we take the span with 0.01. For the simultaneous equations (5.18), we take from $\ell = -200$ to $\ell = 799$; that is, 1000 equations. We solved such equations for $[0, 4.99]$ with the span 0.01 for t.

We shall give a numerical experiment for the typical example

$$F_0(t) = K(t,1) = \begin{cases} -te^{-t} - e^{-t} + 1 & \text{for} \quad 0 \le t \le 1 \\ 1 - 2e^{-1} & \text{for} \quad 1 \le t, \end{cases}$$

whose Laplace transform is

$$(\mathcal{L}F_0)(p) = \frac{1}{p(p+1)^2}\left[1 - (p+2)e^{-(p+1)}\right]. \tag{7.22}$$

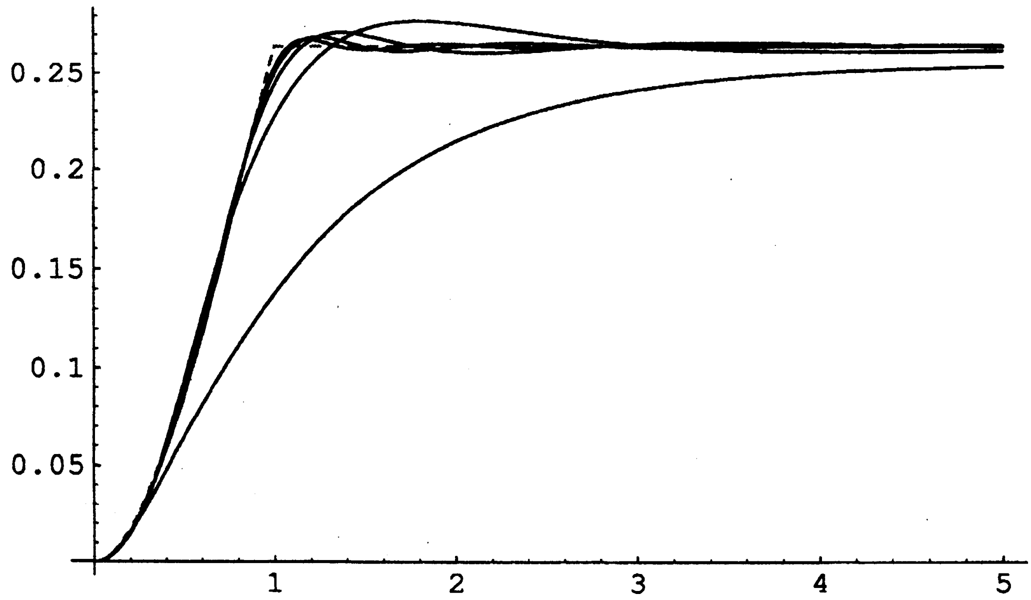

Figure 1: For $F_0(t) = K(t,1)$ and for $\alpha = 10^{-1}, 10^{-4}, 10^{-8}, 10^{-12}, 10^{-16}$.

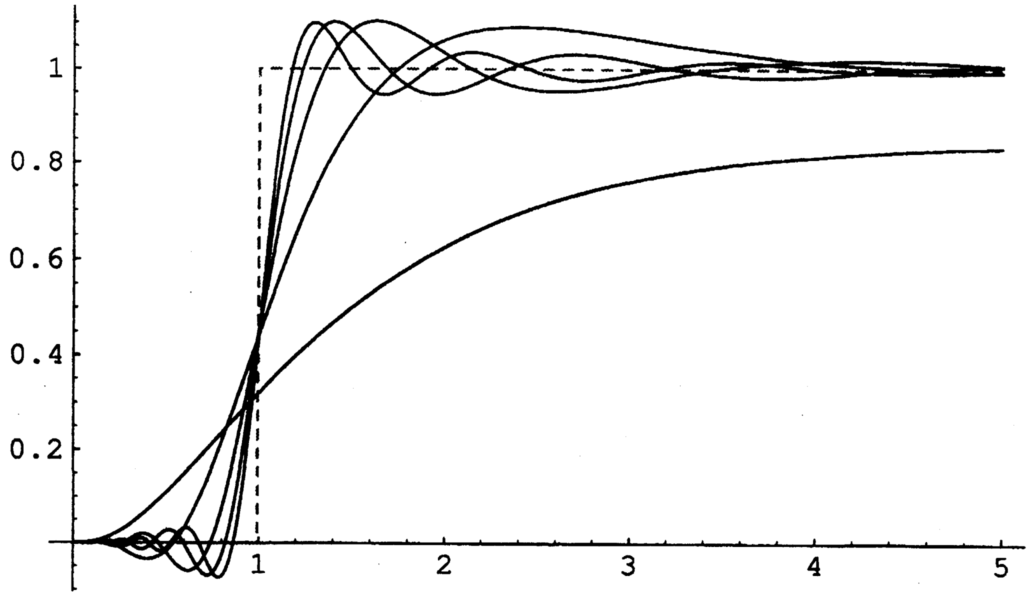

Figure 2: For the step function $F_0(t)$ and for $\alpha = 10^{-1}, 10^{-4}, 10^{-8}, 10^{-12}, 10^{-16}$.

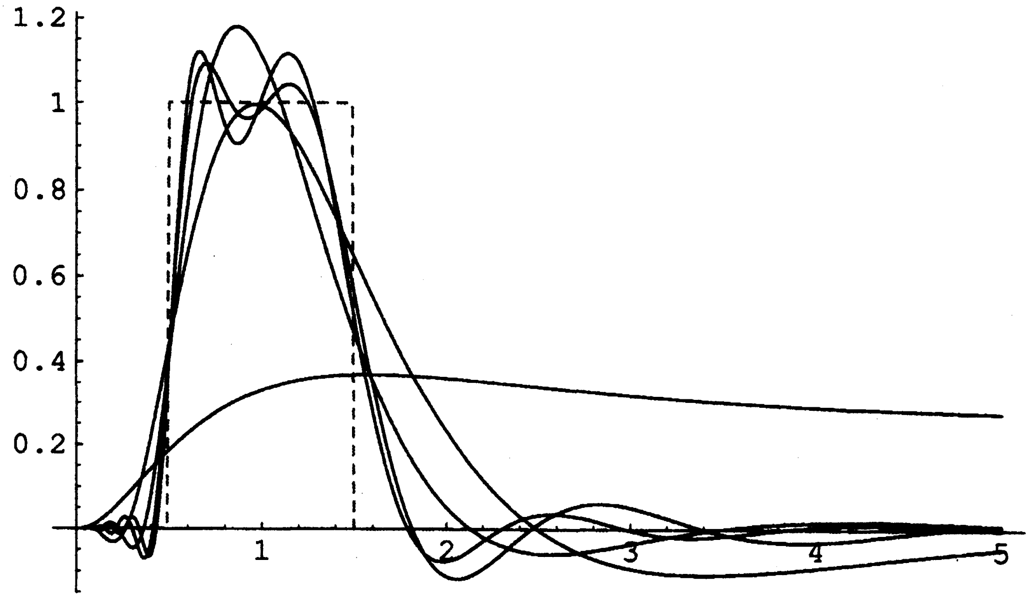

Figure 3: For the characteristic function $F_0(t)$ on $[1/2, 3/2]$ and for $\alpha = 10^{-1}, 10^{-4}, 10^{-8}, 10^{-12}, 10^{-16}$.

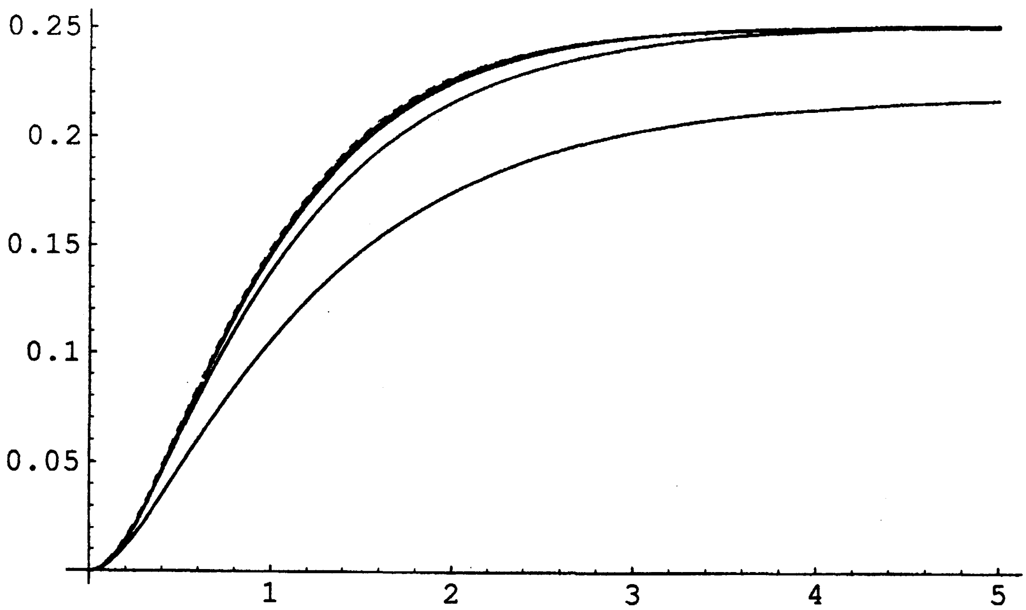

Figure 4: For the function $F_0(t) = 1/4 - 1/4e^{-2t}(1 + 2t)$ and for $\alpha = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}$.

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