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A Value for Fuzzy Games with n Players and r Alternatives

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1. Introduction

Various social phenomena, for example, the allocation problem of expenditures and the election problem have been analyzed in the framework of cooperative games defined by characteristic functions, and some value functions have been presented for expressing the influence power or the evaluation value for the game by each player. As representative values, the followings are well-known : the Shapley value [9], the Banzhaf value [2] and the Deegan-Packel value [5]. Here it must be attended that the characteristic function is defined for each coalition of players and means the max-min payoff of the coalition. In the case that only two alternatives "Yes" or "No" are considered, there is no problem, but in the case of more than two alternatives, the max-min payoff is seemed to be more pessimistic than the actual payoff depended on the situation of other coalitions. Then a multi-alternative game should be considered in other frameworks. Bolger [3] considers a multi-alternative game by a generalized characteristic function defined for an arrangement, namely, a set of coalitions, and presents a new power value for multi-alternative games, which is a generalization of the Shapley value. The multi-alternative Banzhaf value (the MBZ value) [8] is a generalization of the Banzhaf value. The M-N index presented by Masuya and Nakai [6] is a generalized Deegan-Packel index for multi-alternative voting games. Furthermore Masuya and Nakai [7] presents the generalized multi-alternative Deegan-Packel value (the GMDP value) for general multi-alternative games.

Most of traditional cooperative games with characteristic functions treat crisp coalitions only. However there are many actual situations where some players participate partially in a coalition. For considering such a phenomenon Aubin [1] started the theory of cooperative fuzzy games. In this recent literature, Tsurumi et al. [10] proposes a fuzzy Shapley function using Choquet integral.

Furthermore most of traditional values are considered under the assumption that all coalitions are formed by equal probabilities, that is, under the homogeneity among players.

But in many actual social phenomena, this assumption is not always satisfied. Then some values permitting the non-homogeneity among players have been presented.

However, a value permitting all of three generalizations, multi-alternatives, fuzziness and non-homogeneity have not been developed as far as we know.

In this paper, inspired by these works, we develop fuzzy games with n players and r alternatives called multi-alternative fuzzy games and propose a new value permitting all of three generalizations, multi-alternatives, fuzziness and non-homogeneity. Multi-alternative fuzzy games are first defined by Tsurumi et al. [11]. The value they proposed is defined on multi-alternative "crisp" games. In this paper, we will first develop a value which is defined on multi-alternative fuzzy games. Then we develop multi-alternative fuzzy games which differ from those by Tsurumi et al. [11] and a value function on the games.

In Section 2, we formulate multi-alternative fuzzy games which are based on multi-alternative games and cooperative fuzzy games. In Section 3, we propose a new value function for multi-alternative crisp games. In Section 4, we propose a new value function for a class of multi-alternative fuzzy games and prove that it is the unique one satisfying a certain axioms system. In Section 5, we give a numerical example called "Three Alternative Job Game" and compare the new function with other values for traditional multi-alternative games.

2. Development of multi-alternative fuzzy games

First, we provide a definition of characteristic function form games. An n-person cooperative game is a pair (N,v) where N is a set of n players and the function $v:2^N \to \mathbb{R}$ satisfies $v(\phi)=0$.

We consider cooperative fuzzy games with the set of players $N=\{1,2,\cdots,n\}$. A fuzzy coalition is a fuzzy subset of N, which is identified with a function from N to $[0,1]$. Then for a fuzzy coalition S and player i, $S(i)$ indicates the membership grade of i in S, i.e., the rate of i^{th} player's participation in S. For a fuzzy coalition S, the level set is denoted by $[S]_h=\{i\in N|S(i)\geq h\}\ for \forall h\in[0,1]$, and the support is denoted by

$Supp\,S=\{i\in N|S(i)>0\}$.

Next, we develop a multi-alternative fuzzy game. First, we develop a multi-alternative crisp game which is a special case of a multi-alternative fuzzy game.

There are n players and r alternatives. Let $N=\{1,2,\cdots,n\}$ be the set of players and $R=\{1,2,\cdots,r\}$ be the set of alternatives. Each player chooses one of the r alternatives or chooses none of them. Let Γ_j be the set of players who choose the alternative $j \in R$. The set $\Gamma = (\Gamma_1, \Gamma_2, \cdots, \Gamma_r)$ is called a crisp arrangement. That is, each crisp arrangement Γ satisfies $\Gamma_1 \cup \cdots \cup \Gamma_r \subseteq N \; and \; \Gamma_k \cap \Gamma_l = \phi \; (\forall k \neq l)$. For any $S \in \Gamma$, we call (S, Γ) an embedded coalition(ECL). Let $EC(N,R)$ be the set of ECLs on N and R. Let $CA(N,R)$ be the set of crisp arrangements on N and R. Then the function $v: EC(N,R) \to \mathbb{R}_+ = \{z \in \mathbb{R} \mid z \geq 0\}$ is called a multi-alternative crisp game on N with r alternatives provided $v(\phi, \Gamma) = 0$. Let $MG_0(N,R)$ be the set of multi-alternative crisp games on N and R. These games are essentially equivalent to extended multi-alternative games by Tsurumi et al. [11].

Let S_j be the fuzzy coalition which chooses the alternative $j \in R$. Then we will call $S = (S_1, S_2, \cdots, S_r)$ a fuzzy arrangement. A fuzzy arrangement is a generalization of a crisp arrangement which is presented above. In multi-alternative fuzzy games, we assume that each player can not belong to more than one coalition simultaneously. That is, each fuzzy arrangement S satisfies $Supp\, S_k \cap Supp\, S_l = \phi (\forall k \neq l)$. If $T \in S$, we call (T, S) an embedded fuzzy coalition(EFC). Let $FA(N,R)$ be the set of fuzzy arrangements on N and R. That is, $FA(N,R)$ is the set of fuzzy arrangements which satisfy that each element of a fuzzy arrangement is a fuzzy subset of N. Let $EF(N,R)$ be the set of EFCs on N and R. Then the function $v: EF(N,R) \to \mathbb{R}_+$ is called a multi-alternative fuzzy game on N with r alternatives provided $v(\phi, S) = 0$. Let $MG(N,R)$ be the set of multi-alternative fuzzy games on N and R. Clearly, $MG_0(N,R) \subseteq MG(N,R)$ holds. Traditional cooperative fuzzy games are multi-alternative fuzzy games in case of $r = 1$.

In the rest of this section, we give some concepts which are used for following sections.

Definition 1. A membership grade matrix (grade matrix) is defined as follows:

$$U = [u_{ij}] \quad (i = 1, \cdots, n; j = 1, \cdots, r)$$
$$where \; 0 \leq u_{ij} \leq 1 \;\; for \; \forall i, j.$$

u_{ij} means the rate of "potential" participation of player i to the coalition which chooses the j^{th} alternative. The j^{th} column of U is denoted by U_j. Let N_R be the grade matrix with

$u_{ij} = 1$ *for* $\forall i, \forall j$.

Definition 2. For a grade matrix U, the set of level arrangement $[[U]]_h$ and the support $Supp[U]$ are defined as follows:

$$[[U]]_h = \{\Gamma \in CA(N,R) \mid i \in \Gamma_j \Rightarrow u_{ij} \geq h\ \forall i \in N, \forall j \in R\}\ for \forall h \in [0,1]$$

$$Supp[U] = \{\Gamma \in CA(N,R) \mid i \in \Gamma_j \Rightarrow u_{ij} > 0\ \forall i \in N, \forall j \in R\}\ .$$

$[[U]]_h$ is the set of crisp arrangements in which the rate of potential participation of players is no less than h.

Definition 3. For a fuzzy arrangement S, the level arrangement $[S]_h$ is defined as follows:

$$[S]_h = ([S_1]_h, \cdots, [S_r]_h)\ for \forall h \in [0,1].$$

$[S]_h$ is a crisp arrangement in which the rate of participation of each player for the coalition which chooses each alternative is no less than h.

The class of all fuzzy subsets of a fuzzy set $U \subseteq N_R$ is denoted by $L(U)$. Particularly, $L(N,R)$ denotes the class of all fuzzy subsets of N_R. $P(W)$ denotes the class of all crisp subsets of a set of crisp arrangements W. Particularly, $P(N,R)$ denotes the family of sets of crisp arrangements on N and R.

3. The new value for multi-alternative crisp games and its axioms system

First, we develop the new function on $MG_0(N,R)$. We permit the non-homogeneity among players in the new function. This means that each crisp arrangement is not always formed uniformly. Then we introduce a probability distribution on the set of crisp arrangements $p: CA(N,R) \to [0,1]$

$(\sum_{\Gamma \in CA(N,R)} p(\Gamma) = 1)$. p is an arbitrary discrete probability distribution and means a probability of forming the crisp arrangement.

Definition 4. Given $W \in P(N,R)$, $W_{i,j}$ ($i \in N$, $j \in R$) is defined as follows:

$$W_{i,j} = \{\Gamma \in W \mid i \in \Gamma_j\}.$$

Then we define the new function $\tilde{f}^j : MG_0(N,R) \to (\mathbb{R}^n_+)^{P(N,R)}$ $(j = 1,\cdots,r)$ as follows:

$$\tilde{f}^j_i(v)(W) = \begin{cases} \sum_{\substack{\Gamma\in W \\ i\in\Gamma_j}} p(\Gamma)\cdot\frac{v(\Gamma_j,\Gamma)}{|\Gamma_j|} & \text{if } W_{i,j} \neq \phi \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

W is the set of crisp arrangements which can be formed. $\tilde{f}^j_i(v)(W)$ means the expectation of payoffs of player i for the alternative j in the game v on W.

Definition 5. Given $W \in P(N,R), v \in MG_0(N,R)$ and $j \in R$, we define $\bar{v}_j(W)$ and $\bar{v}^i_j(W)$ as follows:

$$\bar{v}_j(W) = \sum_{\Gamma\in W} p(\Gamma)v(\Gamma_j,\Gamma)$$

$$\bar{v}^i_j(W) = \sum_{\substack{\Gamma\in W \\ i\in\Gamma_j}} p(\Gamma)v(\Gamma_j,\Gamma)$$

$\bar{v}_j(W)$ is the expectation of the payoffs which the coalition choosing j^{th} alternative gets when W is the set of arrangements which can be formed.

Definition 6.*(j-zero player* on multi-alternative crisp games*)*

Let $v \in MG_0(N,R), W \in P(N,R)$ satisfying $W_{i,j} \neq \phi$ for an alternative $j \in R$, and let p be a probability distribution on $CA(N,R)$. If $\bar{v}^i_j(W) = 0$ holds, i is called a *j-zero player* on W.

Definition 7. Let $W \in P(N,R)$ and two players $i,k \in N$, and let p be a probability distribution on $CA(N,R)$. Interchanging i with k in any $\Gamma \in W$, we make the new crisp arrangement Γ'. Two players i and k are called *symmetric* in the set (W,v,j,p) if and only if

$$p(\Gamma')v(\Gamma'_j,\Gamma') = p(\Gamma)v(\Gamma_j,\Gamma) \quad (\forall\Gamma \in W). \tag{2}$$

Definition 8. Given $v,w\in MG_0(N,R)$ the *sum game* $v+w$ is defined as follows.

$$(v+w)(\Gamma_j,\Gamma)=v(\Gamma_j,\Gamma)+w(\Gamma_j,\Gamma)\quad \forall(\Gamma_j,\Gamma)\in EC(N,R)$$

In the following, we give a new axioms system which a value for multi-alternative crisp games should satisfy. Let $\tilde{\pi}^j$ be a function from $MG_0(N,R)$ into $(\mathbb{R}^n_+)^{P(N,R)}$ $(j=1,\cdots,r)$. Note that for any $v,w\in MG_0(N,R)$, $v+w\in MG_0(N,R)$ holds.

Axiom MC_1. Given $v\in MG_0(N,R), W\in P(N,R)$ and a probability distribution p on $CA(N,R)$, the following holds.

$$\sum_{i\in N}\tilde{\pi}^j_i(v)(W)=\bar{v}_j(W)$$
$$\tilde{\pi}^j_i(v)(W)=0 \quad \text{if } W_{i,j}=\phi$$

Axiom MC_1 means that the sum of the power of each player for an alternative coincides with the expectation of the payoffs which the coalition choosing the alternative gets. This axiom is different from that of the Bolger value or the MBZ value.

Axiom MC_2. Given $v\in MG_0(N,R), W\in P(N,R)$ such that $W_{i,j}\neq\phi$ and a probability distribution p on $CA(N,R)$, the following holds.

$$\tilde{\pi}^j_i(v)(W)=0 \Leftrightarrow i \text{ is a } \textit{j-zero player} \text{ on } W$$

Axiom MC_2 is also different from that of the Bolger value or the MBZ value as well as Axiom MC_1. The Bolger value and the MBZ value give the value 0 for *j-null players*.
A *j-null player* makes a contribution to a game more than a *j-zero player*.

Axiom MC_3. Let $v\in MG_0(N,R), W\in P(N,R)$ and $i,k\in N$, and let p be a probability distribution on $CA(N,R)$. If i and k are symmetric in the set (W,v,j,p), the following holds.

$$\tilde{\pi}^j_i(v)(W)=\tilde{\pi}^j_k(v)(W)$$

Axiom MC_3 is a generalized axiom as that of the Bolger value or the MBZ value which is called the symmetry axiom.

Axiom MC_4 (*linearity*). Given $v_1, v_2 \in MG_0(N,R)$ and $W \in P(N,R)$, the following holds.

$$\tilde{\pi}_i^j(v_1+v_2)(W) = \tilde{\pi}_i^j(v_1)(W) + \tilde{\pi}_i^j(v_2)(W)$$

For Axiom MC_4, the same discussion is valid with Axiom MC_3.

Theorem 1. The new function $\tilde{f}^j : MG_0(N,R) \to (\mathbb{R}_+^n)^{P(N,R)}$ defined by (1) is the unique function which satisfies *Axiom* MC_1 through MC_4.

4. The new value for multi-alternative fuzzy games and its axioms system

Generally speaking, it is not easy to give the explicit form of the new function on any class of fuzzy games. Tsurumi et al. [10] introduces a class $G_C(N)$ which is the set of fuzzy games with Choquet integral forms and proves that any $v \in G_C(N)$ is both monotone nondecreasing and continuous with regard to rates of players' participation. Then we use this concept. We define a generalization of $G_C(N)$ which is denoted by $MG_C(N,R)$.

Definition 9. For $S \in FA(N,R)$, we put

$Q(S) = \{S_j(i) \mid S_j(i) > 0, i \in N, j \in R\}$. We write the elements of $Q(S)$ in the increasing order as $h_1 < \cdots < h_{q(S)}$ where $q(S)$ is the cardinality of the set $Q(S)$. Then a game $v \in MG(N,R)$ is called to be a multi-alternative fuzzy game 'with Choquet integral form' if and only if the following holds:

$$v(S_j, S) = \sum_{l=1}^{q(S)} v'([S_j]_{h_l}, [S]_{h_l}) \cdot (h_l - h_{l-1}) \quad \forall S \in FA(N,R) \tag{3}$$
$$\textit{where } h_0 = 0,\ v' \in MG_0(N,R).$$

Let $MG_C(N,R)$ be the set of all multi-alternative fuzzy games with Choquet integral forms.

Finally, we define the new value function on $MG_C(N,R)$

$f^j : MG_C(N,R) \to (\mathbb{R}_+^n)^{L(N,R)}$ $(j = 1, \cdots, r)$ as follows:

$$f_i^j(v)(U)=\sum_{l=1}^{q(U)}\tilde{f}_i^j(v)([[U]]_{h_l})\cdot(h_l-h_{l-1}). \tag{4}$$

where $\tilde{f}_i^j$ is defined by the equation (1). U is a grade matrix which is defined on Section 2.

Note that (5) is a Choquet integral of the function U with regard to $\tilde{f}_i^j(v)$.

We give some definitions before proposing an axioms system.

Definition 10. Given $U\in L(N,R)$ *and* $k,l\in N$, for any $S\in L(U)$ we define a new grade matrix $P_{kl}[S]$ as follows: its (i,j) element is given by

$$P_{kl}[S](i,j)=\begin{cases} s_{lj} & \text{if } i=k; j=1,\cdots,r \\ s_{kj} & \text{if } i=l; j=1,\cdots,r \quad (i=1,\cdots,n; j=1,\cdots,r) \\ s_{ij} & \text{otherwise}\end{cases}$$

, that is, the matrix $P_{kl}[S]$ is obtained by exchanging the k^{th} low for the l^{th} low in the grade matrix S.

Definition 11. Given $U\in L(N,R), v\in MG(N,R)$ and $j\in R$, we define $\bar{v}_j(U)$ and $\bar{v}_j^i(U)$ as follows:

$$\bar{v}_j(U)=\sum_{l=1}^{q(U)}\sum_{\Gamma\in[[U]]_{h_l}}p(\Gamma)v(\Gamma_j,\Gamma)(h_l-h_{l-1}) \tag{5}$$

$$\bar{v}_j^i(U)=\sum_{l=1}^{q(U)}\sum_{\substack{\Gamma\in[[U]]_{h_l}\\ i\in\Gamma_j}}p(\Gamma)v(\Gamma_j,\Gamma)(h_l-h_{l-1}).$$

Note that the equation (6) denotes a Choquet integral of the function U with regard to an expectation of v.

Definition 12. *(j-zero player* on multi-alternative fuzzy games*)* Let $U\in L(N,R)$ and $i\in Supp\, U_j$, and let p be a probability distribution on $CA(N,R)$. When $\bar{v}_j^i(U)=0$

whenever $U_j(i) = \max_{i_1 \in N, j_1 \in R} U_{j_1}(i_1)$, player i is called a *j-zero player* on U.

Definition 13. Let $v \in MG_C(N,R)$ and $U \in L(N,R)$. If the following holds, i and k are called *symmetric* in the set (U,v,j,p).

$$\bar{v}_j(S) = \bar{v}_j(P_{ik}[S]) \quad (\forall S \in L(U))$$

In the following, we give an axioms system which a value for multi-alternative fuzzy games should satisfy. Let π^j be a function from $MG_C(N,R)$ into $(\mathbb{R}^n_+)^{L(N,R)}$ $(j=1,\cdots,r)$. Note that for any $v,w \in MG_C(N,R)$, $v+w \in MG_C(N,R)$ holds. It can be proved that Axiom MF_1 through MF_4 is a generalization of Axiom MC_1 through MC_4 to multi-alternative fuzzy games respectively. Thus, the same interpretation is valid for Axiom MF_1 through MF_4 as that of Axiom MC_1 through MC_4 respectively.

Axiom MF_1. Given $v \in MG_C(N,R), U \in L(N,R)$ and a probability distribution p on $CA(N,R)$, the following holds.

$$\sum_{i \in N} \pi_i^j(v)(U) = \bar{v}_j(U)$$

$$\pi_i^j(v)(U) = 0 \quad if\ i \notin Supp\, U_j$$

Axiom MF_2. Given $v \in MG_C(N,R), U \in L(N,R), i \in Supp\, U_j$, player i is a *j-zero player* on U if and only if $\pi_i^j(v)(U) = 0$.

Axiom MF_3. Let $v \in MG_C(N,R)$ and $U \in L(N,R)$, and let p be a probability distribution on $CA(N,R)$. If $i \in N$ and $k \in N$ are symmetric in the set (U,v,j,p), the following holds.

$$\pi_i^j(v)(U) = \pi_k^j(v)(U)$$

Axiom MF_4. For any $v_1, v_2 \in MG_C(N,R)$ *and* $U \in L(N,R)$, the following holds.

$$\pi^j(v_1+v_2)(U) = \pi^j(v_1)(U) + \pi^j(v_2)(U)$$

Theorem 2. The function $f^j : MG_C(N,R) \to (R^n_+)^{L(N,R)}$ defined by (5) is the unique function which satisfies *Axiom* MF_1 through MF_4.

Proposition 1. Let $v \in MG_C(N,R)$, $U=[u_{kl}], U'=[u'_{kl}] \in L(N,R)$ *where* $u'_{kl} \begin{Bmatrix} = \\ < \end{Bmatrix} u_{kl}$ *if* $(k,l) \begin{Bmatrix} \neq \\ = \end{Bmatrix} (i,j)$. Then $f_i^j(v)(U) > f_i^j(v)(U')$.

5. The Comparison of the New Value with the Others

Evaluating the influence power of each player in a numerical example which is called "Three Alternative Job Game", we compare the new function with the MDP value by Masuya et al. [7], the Bolger value and the MBZ value.

Three Alternative Job Game :

There are three working students A, B and C. There are three jobs 1, 2 and 3 and they are about to perform one job respectively. If each student performs a different job each other, student A gets payoff 8 and student B gets payoff 6 and student C gets payoff 4. If student A performs a job by himself and student B, C perform their jobs together, student A gets payoff 5 and the group of students B, C gets payoff 18. If student B performs a job by himself and student A, C perform their jobs together, student B gets payoff 3 and the group of students A, C gets payoff 25. If student C performs a job by himself and student A, B perform their jobs together, student C gets payoff 1 and the group of students A, B gets payoff 30. If all students perform their jobs together, the group of students A, B and C gets payoff 50.

This game can be represented by a multi-alternative crisp game v as follows:

$N=\{A,B,C\}, R=\{1,2,3\}$,

$v(\{A\},(\{A\},\{B\},\{C\}))=8, v(\{B\},(\{A\},\{B\},\{C\}))=6, v(\{C\},(\{A\},\{B\},\{C\}))=4$,

$v(\{A\},(\{A\},\{B,C\},\{\phi\}))=5, v(\{B,C\},(\{A\},\{B,C\},\{\phi\}))=18$,

$v(\{B\},(\{B\},\{A,C\},\{\phi\}))=3, v(\{A,C\},(\{B\},\{A,C\},\{\phi\}))=25$,

$v(\{C\},(\{C\},\{A,B\},\{\phi\}))=1, v(\{A,B\},(\{C\},\{A,B\},\{\phi\}))=30$,

$v(\{A,B,C\},(\{A,B,C\},\{\phi\},\{\phi\}))=50$, $v(T,\Gamma)=0$ *for each* (T,Γ) *if* $T=\phi$,

$v(\Gamma_j,\Gamma)=v(\Gamma_j,P(\Gamma))$ *where* $P(\Gamma)$ is an arbitrary permutation of Γ,

e.g. $v(\{A\},(\{A\},\{B\},\{C\}))=v(\{A\},(\{B\},\{C\},\{A\}))$.

We assume that players are homogeneous in this game. Furthermore, we assume that the crisp arrangement satisfying $\Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \subset N$ is not formed because we would like to compare the new function with other values which is defined on traditional multi-alternative games. Then the probability distribution of crisp arrangements p is assumed to be uniformed as follows:

$$p(\Gamma)=\begin{cases}\frac{1}{27} & \text{if } \Gamma_1\cup\Gamma_2\cup\Gamma_3=N\\ 0 & \text{otherwise.}\end{cases}$$

Also, we assume the grade matrix U is given by

$$U=\begin{bmatrix}1 & 1 & 1\\ 1 & 0.5 & 0.5\\ 1 & 0.5 & 0.25\end{bmatrix}.$$

Each participation rate represents each player's rate of not loafing on his job if he performs the job. That is to say, we assume that player B and C decrease their participation rate for the coalition choosing alternative 2 and 3.

In this situation, how much influence power does each student have ?

In Table1, 2, 3, we show the value for each player in each solution with respect to alternative 1, 2 and 3 respectively. In order to compare them we normalize each solution. The grade matrix U for the crisp game in Table 1, 2, and 3 can be regarded to $[u_{ij}]=1 \quad \forall i, \forall j$.

Table1. The value of each solution for each player for alternative 1

Player	Value			
	Fuzzy	Crisp		
	NEW	MDP	Bolger	MBZ
Player A	0.3236	0.3906	0.4166	0.4173
Player B	0.3397	0.3306	0.3266	0.3279
Player C	0.3365	0.2786	0.2566	0.2547
Total	1	1	1	1

Table2. The value of each solution for each player for alternative 2

Player	Value			
	Fuzzy	Crisp		
	NEW	MDP	Bolger	MBZ
Player A	0.4602	0.3906	0.4166	0.4173
Player B	0.2717	0.3306	0.3266	0.3279
Player C	0.2679	0.2786	0.2566	0.2547
Total	1	1	1	1

Table3. The value of each solution for each player for alternative 3

Player	Value			
	Fuzzy	Crisp		
	NEW	MDP	Bolger	MBZ
Player A	0.4496	0.3906	0.4166	0.4173
Player B	0.3589	0.3306	0.3266	0.3279
Player C	0.1913	0.2786	0.2566	0.2547
Total	1	1	1	1

We compare the new function with the MDP value, the Bolger value and the MBZ value from Table1, 2 and 3. These four values evaluate player B at the similar level. That is to say, difference among these four values is shown by the difference among powers of player A and C. The comparison of the value of A and C for the MDP value, the Bolger value and the MBZ value has been completed in our previous paper [7]. Then we compare the new function with the MDP value because the new function is proportional to the MDP value when the game is crisp and the probability distribution is a uniform distribution. The MDP value evaluates player A better than player C. On the other hands, the new value evaluates C better than A for alternative 1. For alternative 2, the new value evaluates B and C lower than the MDP value. For alternative 3, the new value evaluates player C much lower than the MDP value.

In traditional cooperative fuzzy games, when the rate of participation of a player for his coalition decreases, his influence power decreases too. However, this example shows that by decreasing the rate of participation for coalitions which choose particular alternatives, a player can increase his influence power for other alternatives. Then his influence power decreases for the alternative for which he decreases the rate of participation for the coalition. These phenomena are not observed in the framework of traditional cooperative games. It is seemed to be very interesting result.

6. Conclusion

We developed fuzzy games with n players and r alternatives called multi-alternative fuzzy games. Furthermore, we propose a new value on a class of multi-alternative fuzzy games. The new value considers players' non-homogeneity. Furthermore, we show an axioms system on which the new value is based. The numerical example shows interesting results which are not observed in the framework of traditional cooperative games.

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