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Kyoto University
\lambda \text{ Credibility}

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Abstract

This paper extends credibility measure in uncertainty theory to lambda credibility. Lambda credibility measure is a convex combination of possibility measure and necessity measure. We investigate differences between credibility measure and lambda credibility measure. Finally, we introduce lambda credibility expectation for a triangular fuzzy variable to have an insight on the differences between credibility and lambda credibility.

Keywords: uncertainty theory, credibility measure, possibility measure, necessity measure, fuzzy variable, expectation operator

1 Introduction

Fuzzy set was first introduced by Zadeh [16] in 1965. This notion has been very useful in human decision making under uncertainty. We can see lots of papers which use this fuzzy set theory in K.Iwamura and B.Liu [1] [2], B.Liu and K.Iwamura [9] [10] [11], X.Ji and K.Iwamura [4], X.Gao and K.Iwamura [5], G.Wang and K.Iwamura [14], M.Wen and K.Iwamura [15] and others. We also have some books on fuzzy decision making under fuzzy environments such as D.Dubois and H.Prade [17], H-J.Zimmermann [18], M.Sakawa [20], J.Kacprzyk [19], B.Liu and A.O.Esogbue [8].

Recently B.Liu has founded a frequentonist fuzzy theory with huge amount of applications in fuzzy mathematical programming. We see it in books such as B.Liu [3] [7] [6]. In his book [6] published in 2004, B.Liu [6] has succeeded in establishing an axiomatic foundation for uncertainty theory, where they have created a notion of credibility measure, which is a mean of possibility measure and necessity measure.

In this paper, we further define \lambda credibility measure which is a convex combination of possibility measure and necessity measure. Then we investigate differences between credibility measure and \lambda credibility measure. We will see that credibility measure is much better than \lambda credibility measure because the former can deal with many applications in mathematical programming in fuzzy environments. Yet, our \lambda credibility is also a special fuzzy measure in T.Murofushi and M.Sugeno [21]. Hence there could
come out some useful application of $\lambda$-credibility measure based on either Sugeno integral [13] or Choquet integral in the future, which is left unanswered in this paper.

The rest of the paper is organized as follows. The next section provides a brief review on the results of possibility, necessity and credibility. Section 3 presents the definition of $\lambda$-credibility and investigate its characteristics to recognize the importance of credibility measure. In section 4, we introduce $\lambda$-credibility expectation for a triangular fuzzy variable $(a, b, c)$ to see that its formula differs depending on cases $0 \leq a < b < c$, $a < 0 \leq b < c$, $a < b < 0 \leq c$, while they all reduces to the same expression $(a + 2b + c)/4$ if we set $\lambda = 1/2$. Finally in Section 5, we give a final conclusion on $\lambda$ credibility.

2 Possibility, Necessity and Credibility

We start with the axiomatic definition of possibility given by B. Liu [6] in 2004. Let $\Theta$ be an arbitrary nonempty set, and let $\mathcal{P}(\Theta)$ be the power set of $\Theta$.

The four axioms are listed as follows:

**Axiom 1.** $\text{Pos}\{\Theta\} = 1$.

**Axiom 2.** $\text{Pos}\{\emptyset\} = 0$.

**Axiom 3.** $\text{Pos}\{\bigcup_{i}A_{i}\} = \sup_{i} \text{Pos}\{A_{i}\}$ for any collection $\{A_{i}\}$ in $\mathcal{P}(\Theta)$.

**Axiom 4.** Let $\Theta_{i}$ be nonempty sets on which $\text{Pos}_{i}\{\cdot\}$ satisfy the first three axioms, $i = 1, 2, \ldots, n$, respectively, and $\Theta = \Theta_{1} \times \Theta_{2} \times \cdots \times \Theta_{n}$. Then

$$\text{Pos}\{A\} = \sup_{(\theta_{1}, \theta_{2}, \ldots, \theta_{n}) \in A} \text{Pos}_{1}\{\theta_{1}\} \wedge \text{Pos}_{2}\{\theta_{2}\} \wedge \cdots \wedge \text{Pos}_{n}\{\theta_{n}\}$$

for each $A \in \mathcal{P}(\Theta)$. In that case we write $\text{Pos} = \text{Pos}_{1} \wedge \text{Pos}_{2} \wedge \cdots \wedge \text{Pos}_{n}$.

We call $\text{Pos}$ a possibility measure if it satisfies the first three axioms. We call the triplet $(\Theta, \mathcal{P}(\Theta), \text{Pos})$ a possibility space.

**Note 2.1:** We have $\Theta_{1}$, $\text{Pos}_{1}$ and $\Theta_{2}$, $\text{Pos}_{2}$ which satisfy the first three axioms and $\Theta = \Theta_{1} \times \Theta_{2}$, $\text{Pos} = \text{Pos}_{1} \wedge \text{Pos}_{2}$ which satisfy the four axioms.

**Theorem 2.1 (B. Liu, 2004).** Let $(\Theta, \mathcal{P}(\Theta), \text{Pos})$ be a possibility space. Then we have

(a) 0-1 boundedness: $0 \leq \text{Pos}\{A\} \leq 1$ for any $A \in \mathcal{P}(\Theta)$;

(b) monotonicity: $\text{Pos}\{A\} \leq \text{Pos}\{B\}$ whenever $A \subset B$;

(c) subadditivity: $\text{Pos}\{A \cup B\} \leq \text{Pos}\{A\} + \text{Pos}\{B\}$ for any $A, B \in \mathcal{P}(\Theta)$.

(d) $\text{Pos}\{A \cup B\} = \max(\text{Pos}\{A\}, \text{Pos}\{B\})$ for any $A, B \in \mathcal{P}(\Theta)$. 

Note 2.2: We see that
\[ \text{Pos}(A \cup B) = \max(\text{Pos}(A), \text{Pos}(B)) \] (2)
naturally brings subadditivity in Theorem 2.1(c).

Theorem 2.2 (B. Liu, 2004). Let \((\Theta_i, \mathcal{P}(\Theta_i), \text{Pos}_i), i = 1, 2, \cdots, n\) be possibility spaces, \(\Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_n\) and \(\text{Pos} = \text{Pos}_1 \wedge \text{Pos}_2 \wedge \cdots \wedge \text{Pos}_n\). Then \((\Theta, \mathcal{P}(\Theta), \text{Pos})\) is a possibility space, which is called \(n\)-fold product possibility space of \((\Theta_i, \mathcal{P}(\Theta_i), \text{Pos}_i)\) \((i = 1, 2, \cdots, n)\).

Theorem 2.3 (B. Liu, 2004). Let \((\Theta, \mathcal{P}(\Theta), \text{Pos})\) be a possibility space and \(\text{Pos} = \text{Pos}_1 \wedge \text{Pos}_2 \wedge \cdots\), then the set function \(\text{Pos}\) is a possibility measure on \(\mathcal{P}(\Theta)\), and \((\Theta, \mathcal{P}(\Theta), \text{Pos})\) is a possibility space, which is called infinite product possibility space of \((\Theta_i, \mathcal{P}(\Theta_i), \text{Pos}_i)\) \((i = 1, 2, \cdots)\).

Let \((\Theta, \mathcal{P}(\Theta), \text{Pos})\) be a possibility space, and \(A\) a set in \(\mathcal{P}(\Theta)\). Then the necessity measure of \(A\) is defined by
\[ \text{Nec}(A) = 1 - \text{Pos}(A^c). \] (3)

We have

Theorem 2.4 (B. Liu, 2004). Let \((\Theta, \mathcal{P}(\Theta), \text{Pos})\) be a possibility space. Then we have
(a) \(\text{Nec}(\emptyset) = 1, \text{Nec}(\Theta) = 0\);
(b) \(\theta\)-1 boundedness: \(0 \leq \text{Nec}(A) \leq 1\) for any \(A \in \mathcal{P}(\Theta)\);
(c) monotonicity: \(\text{Nec}(A) \leq \text{Nec}(B)\) whenever \(A \subset B\);
(d) \(\text{Nec}(A) + \text{Pos}(A^c) = 1\) for any \(A \in \mathcal{P}(\Theta)\);
(e) \(\text{Nec}(A \cap B) = \min(\text{Nec}(A), \text{Nec}(B))\) for any \(A, B \in \mathcal{P}(\Theta)\);
(f) \(\text{Nec}(A) = 0\) whenever \(\text{Pos}(A) < 1\).

For a possibility space \((\Theta, \mathcal{P}(\Theta), \text{Pos})\) and \(A\) a set in \(\mathcal{P}(\Theta)\), B. Liu and Y-K. Liu [12] [6] have introduced the credibility measure of \(A\) by
\[ \text{Cr}(A) = \frac{1}{2} (\text{Pos}(A) + \text{Nec}(A)). \] (4)

We easily have

Theorem 2.5 (B. Liu, 2004).
\[ \text{Pos}(A) \geq \text{Cr}(A) \geq \text{Nec}(A) \] (5)
for any \(A \in \mathcal{P}(\Theta)\) of a possibility space \((\Theta, \mathcal{P}(\Theta), \text{Pos})\).

Theorem 2.6 (B. Liu, 2004). Let \((\Theta, \mathcal{P}(\Theta), \text{Pos})\) be a possibility space. Then we have
(a) \(\text{Cr}(\emptyset) = 1, \text{Cr}(\Theta) = 0\);
(b) \(\theta\)-1 boundedness: \(0 \leq \text{Cr}(A) \leq 1\) for any \(A \in \mathcal{P}(\Theta)\);
(c) monotonicity: \(\text{Cr}(A) \leq \text{Cr}(B)\) whenever \(A \subset B\);
(d) self duality: \(\text{Cr}(A) + \text{Cr}(A^c) = 1\) for any \(A \in \mathcal{P}(\Theta)\);
(e) subadditivity: \(\text{Cr}(A \cup B) \leq \text{Cr}(A) + \text{Cr}(B)\) for any \(A, B \in \mathcal{P}(\Theta)\).
We further have

**Theorem 2.7** Let \((\Theta, \mathcal{P}(\Theta), \text{Pos})\) be a possibility space. Let \(\text{Cr}\{ \}\) be the credibility measure on \(\mathcal{P}(\Theta)\). Then

(a) \(\text{Pos}\{A\} < 1\) implies \(\text{Cr}\{A\} = \frac{1}{2} \text{Pos}\{A\}\);
(b) \(\text{Cr}\{A\} \geq \frac{1}{2}\) implies \(\text{Pos}\{A\} = 1\).

A fuzzy variable is defined as a function from \(\Theta\) of a possibility space \((\Theta, \mathcal{P}(\Theta), \text{Pos})\) to the set of reals. Let \(\xi\) be a fuzzy variable defined on the possibility space \((\Theta, \mathcal{P}(\Theta), \text{Pos})\). Then the set

\[ \xi_\alpha = \{\xi(\theta) \mid \theta \in \Theta, \text{Pos}\{\theta\} \geq \alpha\} \]

is called the \(\alpha\)-level set of \(\xi\). The set

\[ \Theta^+ = \{\theta \in \Theta \mid \text{Pos}\{\theta\} > 0\} \]

is called the kernel of the possibility space and the set

\[ \{\xi(\theta) \mid \theta \in \Theta, \text{Pos}\{\theta\} > 0\} = \{\xi(\theta) \mid \theta \in \Theta^+\} \]

is called the support of \(\xi\).

Let \(\xi\) be a fuzzy variable defined on the possibility space \((\Theta, \mathcal{P}(\Theta), \text{Pos})\). Then its membership function is derived through the possibility measure \(\text{Pos}\) by

\[ \mu(x) = \text{Pos}\{\theta \in \Theta \mid \xi(\theta) = x\}, \quad x \in \mathbb{R}. \]  \hfill (6)

**Theorem 2.8** (B. Liu, 2004). Let \(\mu : \mathbb{R} \to [0, 1]\) be a function with \(\sup \mu(x) = 1\). Then there is a fuzzy variable whose membership function is \(\mu\).

### 3 \(\lambda\) Credibility

Let \(\text{Pos}\{ \}\) and \(\text{Nec}\{ \}\) be a possibility measure and the necessity measure on \(\mathcal{P}(\Theta)\). Let \(\lambda\) be a real number between 0 and 1, i.e., \(\lambda \in [0, 1]\). Define a set function \(\lambda \text{Cr}\{ \}\) on \(\mathcal{P}(\Theta)\) by

\[ \lambda \text{Cr}\{A\} = \lambda \text{Pos}\{A\} + (1 - \lambda) \text{Nec}\{A\}. \]  \hfill (7)

For \(\lambda = 1\), we have \(\lambda \text{Cr}\{A\} = \text{Pos}\{A\}\) and for \(\lambda = 0\), we have \(\lambda \text{Cr}\{A\} = \text{Nec}\{A\}\) and we see that 

\[ 0.5 \text{Cr}\{A\} = \text{Cr}\{A\}, \quad \text{for any } A \in \mathcal{P}(\Theta). \]

\(\lambda \text{Cr}\{ \}\) is not \(\text{Cr}\{ \}\) multiplied by \(\lambda\)!

We easily have

**Theorem 3.1** Let \(\lambda\) be a real number such that \(0 \leq \lambda \leq 1\). Then

(a) \(\lambda \text{Cr}\{\Theta\} = 1, \lambda \text{Cr}\{\emptyset\} = 0\);
(b) \(\text{Nec}\{A\} \leq \lambda \text{Cr}\{A\} \leq \text{Pos}\{A\}\) for any \(A \in \mathcal{P}(\Theta)\) and so 0-1 boundedness holds for \(\lambda \text{Cr}\{ \}\).
(c) monotonicity: \( \lambda \text{Cr}(A) \leq \lambda \text{Cr}(B) \) whenever \( A \subseteq B \);

(d) \( 2 \geq \lambda \text{Cr}(A) + \lambda \text{Cr}(A^c) \geq 1 \) whenever \( \lambda \geq \frac{1}{2} \)
while \( 0 \leq \lambda \text{Cr}(A) + \lambda \text{Cr}(A^c) \leq 1 \) whenever \( \lambda \leq \frac{1}{2} \) for any \( A \in \mathcal{P}(\Theta) \);

(e) restricted subadditivity: \( \lambda \text{Cr}(A \cup B) \leq \lambda \text{Cr}(A) + \lambda \text{Cr}(B) \) for any \( A, B \in \mathcal{P}(\Theta) \) provided that \( \lambda \geq \frac{1}{2} \).

**Corollary 3.1** Let \( \lambda \) be such that \( 0 < \lambda < 1 \). Then we get

(a) \( \lambda \text{Cr}(A) = 1 \) if and only if \( \text{Pos}(A) = \text{Nec}(A) = 1 \),

(b) \( \lambda \text{Cr}(A) = 0 \) if and only if \( \text{Pos}(A) = \text{Nec}(A) = 0 \).

**Theorem 3.2** Let \( 0 \leq \lambda \leq 1 \). Suppose that there exists a set \( A \in \mathcal{P}(\Theta) \) such that \( \text{Pos}(A) > \text{Nec}(A) \)
with \( \lambda \text{Cr}(A) + \lambda \text{Cr}(A^c) = 1 \). Then we get \( \lambda = \frac{1}{2} \).

**Corollary 3.2** Suppose that there exist \( s, t \) with \( 0 \leq s < t \leq 1 \) such that for any \( \lambda \in [s, t] \), any \( A \in \mathcal{P}(\Theta) \),
\( \lambda \text{Cr}(A) + \lambda \text{Cr}(A^c) = 1 \). Then we have \( \text{Pos}(A) = \text{Nec}(A) \) for any \( A \in \mathcal{P}(\Theta) \).

**Example 3.1.** Let \( \Theta = \{\theta_1, \theta_2\} \) with \( \text{Pos}(\theta_1) = 1.0, \text{Pos}(\theta_2) = 0.8 \). Then \( \text{Nec}(\theta_1) = 1 - \text{Pos}(\theta_2) = 0.2, \text{Nec}(\theta_2) = 1 - 1 = 0 \).

\( \lambda \text{Cr}(\Theta) = 1 \), \( \lambda \text{Cr}(\emptyset) = 0 \), \( \lambda \text{Cr}(\theta_1) = 0.8\lambda + 0.2 \), \( \lambda \text{Cr}(\theta_2) = 0.8\lambda \), \( \lambda \text{Cr}(\theta_1) + \lambda \text{Cr}(\theta_2) = 1.6\lambda + 0.2 \) for \( 0 \leq \lambda \leq 1 \). Therefore

\[
\lambda \text{Cr}(\theta_1) + \lambda \text{Cr}(\theta_2) = 1 \quad \text{if and only if} \quad \lambda = \frac{1}{2}.
\] (8)

For \( \lambda = 0.4 \), \( \lambda \text{Cr}(\theta_1) + \lambda \text{Cr}(\theta_2) = 0.84 < 1 \).

And for \( \lambda = 0.6 \), \( \lambda \text{Cr}(\theta_1) + \lambda \text{Cr}(\theta_2) = 1.16 > 1 \).

So, \( \lambda \text{Cr}(\emptyset) \) is not self dual for any \( \lambda, 0 < \lambda < 1 \) with \( \lambda \neq \frac{1}{2} \).

**Example 3.2.** Let \( \xi \) be a triangular fuzzy variable with its membership function

\[
\mu(x) = \begin{cases} 
x, & \text{if } 0 \leq x \leq 1 \\
-x + 2, & \text{if } 1 < x < 2 \\
0, & \text{otherwise.}
\end{cases}
\]

Then we get

\[
\text{Pos}(\xi \leq x) = \begin{cases} 
0, & \text{if } x \leq 0 \\
x, & \text{if } 0 < x \leq 1 \\
1, & \text{if } x > 1.
\end{cases}
\]

and

\[
\text{Pos}(\xi > x) = \begin{cases} 
1, & \text{if } x \leq 1 \\
-x + 2, & \text{if } 1 < x \leq 2 \\
0, & \text{if } x > 2.
\end{cases}
\]

For \( 0 < x < 1 \), \( \text{Pos}(\xi \leq x) + \text{Pos}(\xi > x) = x + 1 > 1 \). Therefore, we get \( \lambda = \frac{1}{2} \) provided that \( \lambda \text{Cr}(\emptyset) \) is self dual. This fact suggests the importance of \( 0.5 \text{Cr}(\emptyset) \), i.e., credibility measure. Furthermore, we have
Theorem 3.3 Let $\xi$ be a triangular fuzzy variable with its membership function

$$
\mu(x) = \begin{cases} 
  x - a, & \text{if } a \leq x \leq b \\
  b - a, & \text{if } b < x \leq c \\
  x - c, & \text{if } a \leq x \leq b - c \\
  b - c, & \text{otherwise} 
\end{cases}
$$

where $a \leq b \leq c$. Suppose that $\text{Pos}\{\xi \leq x\} + \text{Pos}\{\xi > x\} = 1$ holds for any real number $x$, i.e., $\lambda \text{Cr}\{\}$ is self dual for fuzzy events $\{\xi \leq x\}$. Then we have $a = b = c$.

Example 3.3: Let $\xi$ be a fuzzy variable with its membership function $\mu(x) = \delta_a(x)$, where

$$
\delta_a(x) = \begin{cases} 
  1, & x = a \\
  0, & x \neq a 
\end{cases}
$$

We call $\delta_a$ a Dirac measure. Then we get

$$
\text{Pos}\{\xi \leq x\} = \begin{cases} 
  0, & x < a \\
  1, & x \geq a
\end{cases}
$$

Hence we get $\text{Pos}\{\xi \leq x\} + \text{Pos}\{\xi > x\} = 1$ for any number $x$. Therefore $\lambda \text{Cr}\{\xi \leq x\} + \lambda \text{Cr}\{\xi > x\} = 1$ for any number $x$. $\lambda \text{Cr}\{\}$ is self dual for fuzzy events $\{\xi \leq x\}$, where $x$ is a real number.

4 $\lambda$-Credibility Expectation

Let $\xi$ be a triangular fuzzy variable $(a, b, c)$ with its membership function $\mu(x)$ as follows;

$$
\mu(x) = \begin{cases} 
  x - a, & \text{if } a \leq x \leq b \\
  b - a, & \text{if } b < x \leq c \\
  x - c, & \text{if } a \leq x \leq b - c \\
  b - c, & \text{otherwise} 
\end{cases}
$$

Then we get

$$
\lambda \text{Cr}\{\xi \leq x\} = \begin{cases} 
  1, & \text{if } x \leq a \\
  b - \lambda a - \frac{1 - \lambda}{b - a} x, & \text{if } a < x \leq b \\
  -\lambda c + \frac{\lambda}{b - c} x, & \text{if } b < x \leq c \\
  0, & \text{if } c < x.
\end{cases}
$$

Define $\lambda$-credibility expectation $\lambda \text{Cr}\{\}$ of $\xi$ by

$$
E_{\lambda \text{Cr}}[\xi] = \int_{-\infty}^{+\infty} \lambda \text{Cr}\{\xi > x\} \, dx - \int_{-\infty}^{0} \lambda \text{Cr}\{\xi \leq x\} \, dx,
$$
where integrals are defined through Lebesgue integral, i.e., well defined in case either of the two takes finite value.

Then we see the followings, in case $0 \leq a < b < c$, we get

$$E_{\lambda \text{Cr}}[\xi] = \int_{0}^{+\infty} \lambda \text{Cr}\{\xi \geq x\} \, dx$$

$$= \int_{0}^{a} 1 \, dx + \int_{a}^{b} \left\{ \frac{b - \lambda a - 1 - \lambda}{b - a} \right\} dx$$

$$= \frac{b + a + (c - a)\lambda}{2},$$

in case $a < b < 0 \leq c$, we get

$$E_{\lambda \text{Cr}}[\xi] = \left( \frac{-c^2}{2(b - c)} + \frac{a - b}{2} + \frac{b^2 - 2bc}{2(b - c)} \right) \lambda + \frac{b^2}{2(b - c)}.$$ (10)

If we set $\lambda = \frac{1}{2}$ in equation (9),(10) we get $E_{0.5 \text{Cr}}[\xi] = \frac{a + 2b + c}{4} \cdot \frac{1}{4}$, which is identical to the credibility expectation for a triangular fuzzy variable $\xi$. This fact shows us the importance and adequateness of the credibility expectation operator. Finally let us see the case of $a = 0, b = 1, c = 3$. In this case we get

$$E_{\lambda \text{Cr}}[\xi] = \frac{1}{2} + \frac{3}{2},$$

which coincides with the credibility expectation of $\xi$, $E[\xi] = 0 + 2 \cdot 1 + 3 = \frac{5}{4}$, if we take $\lambda = \frac{1}{2}$.

We further see that $\frac{1}{2} \leq E_{\lambda \text{Cr}}[\xi] \leq 2$ and $\frac{1}{2}(\frac{1}{2} + 2) = \frac{5}{4} = E[\xi]$, where $E_{\text{Nec}}[\xi] = \frac{1}{2}$ and $E_{\text{Pos}}[\xi] = 2$.

5 Conclusion

We have introduced $\lambda$ credibility $\lambda \text{Cr}\{\cdot\}$ on a possibility space $(\Theta, \mathcal{P}(\Theta), \text{Pos})$ and have shown that $\lambda \text{Cr}\{\cdot\}$ is 0-1 bounded, monotone for set inclusion, over self dual or under self dual according to $\lambda \geq \frac{1}{2}$ or $\lambda \leq \frac{1}{2}$ and that it satisfies restricted subadditivity. We further have shown that for a non-trivial possibility space, where there exists a set $A \in \mathcal{P}(\Theta)$ such that $\text{Pos}\{A\} > \text{Nec}\{A\}$, self duality of $\lambda \text{Cr}\{\cdot\}$ naturally leads to $\lambda = \frac{1}{2}$. We have given some examples for which self duality naturally holds when $\lambda = \frac{1}{2}$, or a triangular fuzzy variable $\xi$ reduces to a Dirac measure if we demand $\lambda \text{Cr}\{\cdot\}$ be self dual for fuzzy events $\{\xi \leq x\}$. Finally we have defined $\lambda \text{Cr}\{\cdot\}$ expectation of a fuzzy variable $\xi$. For a triangular fuzzy variable $(a, b, c)$, the $\lambda \text{Cr}\{\cdot\}$ expectation $E_{\lambda \text{Cr}}[\xi]$ has different formula in $\lambda$, dependent on $0 < a$ or $a < 0 < b$ or $a < b < 0 < c$. But if we let $\lambda = \frac{1}{2}$ then they all reduce to $(a + 2b + c)/4$, the expectation of $\xi$ in the
sense of B.Liu [6] in 2004. So, we believe that $0.5\text{Cr}\{} = \text{Cr}\{}$ is the most useful one among $\lambda\text{Cr}\{}$, $0 < \lambda < 1$.

References


