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Blow-up at space infinity for nonlinear heat equations

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1 Introduction and main theorems

In this paper we gather the papers [5], [6] and [12] for our talk at Kyoto University. In particular we make the proofs of theorems in [5] easier by using the methods in [12] and other.

We consider solutions of the initial value problem for the equation

$$\begin{cases} u_t = \Delta u + f(u), & x \in \mathbf{R}^n, t > 0, \\ u(x,0) = u_0(x), & x \in \mathbf{R}^n. \end{cases} \tag{1}$$

The nonlinear term $f \in C^1(\bar{\mathbf{R}}_+)$ saitsfies that

$$\int_C^\infty \frac{d\xi}{f(\xi)} < \infty \text{ with some } C \geq 0, \tag{2}$$

and

$$\begin{cases} \text{there exists a function } \Phi \in C^2(\mathbf{R}_+) \text{ such that} \\ \Phi(v) > 0,\ \Phi'(v) > 0 \text{ and } \Phi''(v) \geq 0 \text{ for } v > 0, \\ \displaystyle\int_1^\infty \frac{d\xi}{\Phi(\xi)} < \infty, \\ \text{and } f'(v)\Phi(v) - f(v)\Phi'(v) \geq c\Phi(v)\Phi'(v) \text{ for } v > b \\ \text{with some } b \geq 0 \text{ and } c \geq 0. \end{cases} \tag{3}$$

Remark. *The conditions* (2) *and* (3) *were used in [12]. They are weaker than the conditions used in [5] and [6]:*

$$f(\delta b) \leq \delta^p f(b)$$

for all $b \geq b_0$ *and for all* $\delta \in (\delta_0, 1)$ *with some* $b_0 > 0$*, some* $\delta_0 \in (0,1)$ *and some* $p > 1$.

The initial data u_0 is assumed to be a measureable function in $\mathbf{R}^n$ satisfying

$$0 \le u_0(x) \le M \text{ a.e. in } \mathbf{R}^n \tag{4}$$

for some positive M. We are interested in initial data such that $u_0 \to M$ as $|x| \to \infty$ for x in some sector of $\mathbf{R}^n$. We assume that there exists a sequence $\{x\}_{m=1}^{\infty} \subset \mathbf{R}^n$ such that

$$\lim_{m\to\infty} u_0(x + x_m) = M \qquad \text{a.e. in } \mathbf{R}^n. \tag{5}$$

Remark. *The condition* (5) *was given in [12]. This condition is equivalent to the condition in [5] with [6]:*

$$\operatorname{essinf}_{x\in\tilde{B}_m}(u_0(x) - M_m(x - x_m)) \ge 0 \quad \textit{for} \quad m = 1, 2, \dots ,$$

where $\tilde{B}_m = B_{r_m}(x_m)$ *with a sequence* $\{r_m\}_{m=1}^{\infty}$, *a sequence of functions* $\{M_m(x)\}_{m=1}^{\infty}$ *satisfying*

$$\lim_{m\to\infty} r_m = \infty, \quad M_m(x) \le M_{m+1}(x) \quad \textit{for } m \ge 1$$
$$\lim_{m\to\infty} \inf_{s\in[1,r_m]} \frac{1}{|B_s|} \int_{B_s(0)} M_m(x)dx = M,$$

and some sequence of vectors $\{x_m\}_{m=1}^{\infty}$. *Here* $B_r(x)$ *denotes the opened ball of radius* r *centered at* x.

Problem (1) has a unique bounded solution at least locally in time. However, the solution may blow up in finite time. For a given initial value u_0 and nonlinear term f let $T^* = T^*(u_0, f)$ be the maximal existence time of the solution. If $T^* = \infty$, the solution exists globally in time. If $T^* < \infty$, we say that the solution blows up in finite time. It is well known that

$$\limsup_{t\to T^*} \|u(\cdot, t)\|_\infty = \infty, \tag{6}$$

where $\|\cdot\|_\infty$ denotes the L^∞-norm in space variables.

In this paper we are interested in behavior of a blowing up solution near space infinity as well as location of blow-up directions defined below. A point $x_{BU} \in \mathbf{R}^n$ is called a *blow-up point* if there exists a sequence $\{(x_m, t_m)\}_{m=1}^{\infty}$ such that

$$t_m \uparrow T^*, \quad x_m \to x_{BU} \quad \text{and} \quad u(x_m, t_m) \to \infty \quad \text{as} \quad m \to \infty.$$

If there exists a sequence $\{(x_m, t_m)\}_{m=1}^{\infty}$ such that

$$t_m \uparrow T^*, \quad |x_m| \to \infty \quad \text{and} \quad u(x_m, t_m) \to \infty \quad \text{as} \quad m \to \infty,$$

then we say that the solution blows up to at space infinity.

A direction $\psi \in S^{n-1}$ is called a *blow-up direction* if there exists a sequence $\{(x_m, t_m)\}_{m=1}^{\infty}$ with $x_m \in \mathbf{R}^n$ and $t_m \in (0, T^*)$ such that $u(x_m, t_m) \to \infty$ as $m \to \infty$ and

$$\frac{x_m}{|x_m|} \to \psi \quad \text{as} \quad m \to \infty. \tag{7}$$

We consider the solution $v(t)$ of an ordinary differential equation

$$\begin{cases} v_t = f(v), & t > 0, \\ v(0) = M. \end{cases} \tag{8}$$

Let $T_v = T^*(M, f)$ be the maximal existence time of solutions of (8), i. e.,

$$T_v = \int_M^{\infty} \frac{ds}{f(s)}.$$

We are now in position to state our main results.

Theorem 1. *Assume that $f \in C^1(\mathbf{R}_+)$ is nondecreasing function and locally Lipschitz in $\bar{\mathbf{R}}_+$. Let u_0 be a continuous function satisfying* (4) *and* (5). *Then there exists a subsequence of $\{x_m\}_{m=1}^{\infty}$, independent of t such that*

$$\lim_{m \to \infty} u(x + x_m, t) = v(t) \quad \text{in } \mathbf{R}^n. \tag{9}$$

The convergence is uniform in every compact subset of $\mathbf{R}^n \times [0, T_v)$. Moreover, the solution blows up at T_v.

For this theorem we should introduce the results of Gladkov [7]. In his paper there is the result [7, Theorem 1] relative to our first theorem. He considered the initial-boundary value problem:

$$\begin{cases} u_t = u_{xx} + f(x, t, u), & x > 0, 0 < t < T_0, \\ u(x, 0) = u_0(x), & x > 0, \\ u(0, t) = \mu(t) & 0 < t < T_0, \end{cases}$$

and the ordinary differential equation

$$\begin{cases} v_t = \tilde{f}(t, u), & 0 < t < T_0, \\ v(0) = M, \end{cases}$$

where $T_0 \in (0,\infty]$, $0 \le f(x,t,u) \le \tilde{f}(t,u)$, $\lim_{x\to\infty} f(x,t,u) = \tilde{f}(t,u)$, $0 \le u_0 \le M$ and $\lim_{x\to\infty} u_0(x) = M$. For the equations he had $u(x,t) \to v(t)$ as $x \to \infty$ uniformly for $[0,T]$ with $T < T_0$. For the proof of this result, he used the fundamental solution of the heat equation.

In [5] the expression (9) was the weak sense:

$$\lim_{n\to\infty} u(x_m,t) = v(t). \tag{10}$$

After [5], (9) was used in [12]. However, for proving Theorems 2 and 3, we can select even the expression (10).

Our second main result is on the location of blow-up points.

Theorem 2. *Assume the same hypotheses of Theorem 1 and that f satisfies* (2) *and* (3). *Let $u_0 \not\equiv M$ a.e. in $\mathbf{R}^n$. Then the solution of* (1) *has no blow-up points with ∞ in $\mathbf{R}^n$.* (*It blows up only at space infinity.*)

There is a huge literature on location of blow-up points since the work of Weissler [15] and Friedman-McLeod [1]. (We do not intend to list references exhaustively in this paper.) However, most results consider either bounded domains or solutions decaying at space infinity; such a solution does not blow up at space infinity [2].

As far as the authors know, before the result of [4] the only paper discussing blow-up at space infinity is the work of Lacey [8]. He considered the Dirichlet problem in a half line. He studied various nonlinear terms and proved that a solution blows up only at space infinity. His method is based on construction of suitable subsolutions and supersolutions. However, the construction heavily depends on the Dirichlet condition at $x = 0$ and does not apply to the Cauchy problem even for the case $n = 1$.

As previously described, the Giga-Umeda [4] proved the statement of Theorems 1 and 2 assuming that $\lim_{|x|\to\infty} u_0(x) = M$ for positive solutions of $u_t = \Delta u + u^p$. Later, Simojō[13] had the same results as in [4] by relaxing the assumptions of initial data $u_0 \ge 0$ which is similar to that in the present paper. His approach is a construction of a suitable supersolution which implies that $a \in \mathbf{R}^n$ is not a blow-up point. Although he restricted himself for $f(s) = s^p$, his idea works our f under slightly strong assumption on u_0. Here we give a different approach.

By Simojō's results[13] it is natural to consider a problem of "blow-up direction" defined in (7). We next study this "blow-up direction" for the value ∞.

Theorem 3. *Assume the same hypotheses of Theorem 1. Let a direction $\psi \in S^{n-1}$. If and only if there exists sequences $\{y_m\}_{m=1}^{\infty}$ and satisfying*

$\lim_{m\to\infty} y_m/|y_m| = \psi$ *such that*

$$\lim_{m\to\infty} u_0(x+y_m) = M \ \ a.e. \ \ in \ \mathbf{R}^n, \tag{11}$$

then ψ *is a blow-up direction.*

After [5] there are some results in this field. Shimojō had the result of the upperbound and the lowerbound:

$$v(t-\eta(x,t)) \le u(x,t) \le v(t-c\eta(x,t))$$

with some function η and $c \in (0,1)$. Moreover, he proved the complete blow-up of the solution. Seki-Suzuki-Umeda [12] and Seki [11] improved the results of [5] for the quasilinear parabolic equation:

$$u_t = \Delta\varphi(u) + f(u).$$

In particular they had more results for more general case. In [3] some of the proofs of theorems in [5] were corrected.

This paper is organized as follows. In section 2 we prove Theorem 1 by using the fundamental solution of the heat equation. The proof of Theorem 2 is given in section 3 by using the argument used in [12]. In section 4 we show Theorem 3 using Theorem 1 and Lemma 3.2.

2 Behavior at space infinity

In this section we prove Theorem 1. We give proof of Theorem 1 which is inspired in private communication with Y. Seki and M. Shimojō.

Proof of Theorem 1. Put $w = v - x$. Then, we have for $t \in (0, T_0]$ with $T_0 \in (0, T(M))$,

$$w_t = \Delta w + f(v(t)) - f(u(\cdot,t)) \le \Delta w + C(v-u),$$

where

$$C = \sup_{t\in[0,T_0]} \left\| \int_0^1 f'(\theta v(t) + (1-\theta)u(\cdot,t))d\theta \right\|_\infty .$$

Then, by comparison we obtain

$$w(x,t) \le e^{CT_0} e^{\Delta t}(M - u_0(x)) = \frac{1}{(4\pi t)^{n/2}} \int_{\mathbf{R}^n} e^{-|x-y|^2/4t}(M - u_0(y))dy.$$

From (5) we have

$$\lim_{m\to\infty} u(x+x_m,t) = v(t) \quad \text{in } \mathbf{R}^n. \tag{12}$$

It remains to prove that u blows up at $t = T_v$. For this purpose it suffices to prove that $\lim_{m\to\infty} u(x_m,t_m) = \infty$ for some sequence $t_m \to T_v$. We argue by contradiction. Suppose that $\lim_{m\to\infty} u(x_m,t_m) \le C$ for some $C \in [M,\infty)$. Then we could take $t_0 \in (0,T_v)$ satisfying $v(t_0) \ge C$ and $v_t(t) > 0$ for $t \ge t_0$. By (12) we have

$$\lim_{m\to\infty} u\left(x_m, \frac{t_0+T_v}{2}\right) = v\left(\frac{t_0+T_v}{2}\right) > C,$$

which yields a contradiction. We thus proved that $\lim_{m\to\infty} u(x_m,t_m) = \infty$, so that $u(x,t)$ blows up at T_v. □

3 No blow-up point in $\mathbf{R}^n$

In this section we prove Theorem 2. We use three lemmas for proving the theorem..

Lemma 3.1. *Assume the same hypothesis of Theorem 1. Let u and v be solutions of* (1) *and* (8) *with u_0, M and f satisfying* (2), (3) *and* (4). *Then there exist $\delta = \delta(a,t_0,u_0,f) \in (0,1)$ such that for $(x,t) \in B_1(a) \times [t_0,T_v)$,*

$$u(x,t) \le \delta v(t)$$

with $t_0 \in [0,T_v)$.

Proof. By (2) there exist $M_f = M_f(f) > M$ and $\delta_f = \delta_f(f) \in (0,1)$ satisfying for $r \ge M_f$ and $\delta \in (\delta_f,1)$,

$$f(\delta r) \le \delta f(r). \tag{13}$$

Let $T_0 = T_0(u_0,f) \in (0,T_v)$ such that $v(T_0) = M_f$. Since $u_0 \le M$ and $u_0 \not\equiv M$ a.e. in $\mathbf{R}^n$, we have $u(x,T_0) < v(T_0)$. Note that $u(x,t) < v(t)$ for $t \in (0,T_0]$. Let w be the solution of

$$\begin{cases} w_t = \Delta w, & x \in \mathbf{R}^n, t \in (T_0,T^*), \\ w(x,T_0) = \max\{u(x,T_0)/v(T_0), \delta_f\}, & x \in \mathbf{R}^n. \end{cases}$$

Put $\bar{u} = vw$. Then we have

$$\begin{cases} \bar{u}_t = \Delta\bar{u} + wf(v), & x \in \mathbf{R}^n, t \in (T_0,T^*), \\ \bar{u}(x,T_0) = \max\{u(x,T_0), \delta_f v(T_0)\}, & x \in \mathbf{R}^n. \end{cases}$$

Since $w(x,t) \in [\delta_f, 1)$ and $v(t) \geq M_f$, we have

$$wf(v) \geq f(wv) = f(\bar{u})$$

by (13). This $\bar{u}$ is supersolution of (1).

Since for any $x \in \mathbf{R}^n$, $\sup_{t\in[T_0,T^*)} w(x,t) < 1$, we can take $\delta = \delta(a, T_0, u_0, f) \in (0,1)$ satisfying $w(x,t) \leq \delta$ for $(x,t) \in B_1(a) \times [T_0, T_v)$. Thus, we obtain

$$u(x,t) \leq \bar{u}(x,t) = w(x,t)v(t) \leq \delta v(t)$$

and Lemma 3.1 is proved. □

For any $a \in \mathbf{R}^n$, we consider the solution $\phi = \phi_a$ of the equation:

$$\begin{cases} \phi_t = \Delta\phi + f(\phi), & x \in B_1, t \in (t_1, T_v), \\ \phi(x,0) = \phi_0(x), & x \in B_1, \\ \phi(x,t) = v(t), & x \in \partial B_1, t \in (t_1, T_v), \end{cases} \tag{14}$$

where $\phi_0(x) = v(t_1)(1 - \varepsilon \cos \frac{\pi|x|}{2})$ with $\varepsilon = \varepsilon(u_0, f, a) > 0$ sufficiently small satisfying

$$\phi_0(x) \geq u(x+a, t_1) \tag{15}$$

and B_1 denotes the open ball of radius 1 and centered at 0. It is easily seen that

$$\Delta\phi_0(x) + f(\phi_0(x)) \geq 0.$$

By the maximum principle [10] we have

$$\phi(x,t) \geq u(x+a,t) \quad \text{and} \quad \phi_t \geq 0 \quad \text{for } x \in \bar{B}_1,\ t \in [t_1, T_v). \tag{16}$$

If w has no blow-up point in $\mathbf{R}^n$, the u has no blow-up point in $\mathbf{R}^n$, neither. We should show that w has no blow-up point.

Lemma 3.2. *Assume the same hypotheses of Lemma 3.1. Let $\Omega \in B_1$ be a domain. If $\partial_t\phi(x,t) \geq 0$ in $\Omega \times (t_1, T_v)$ and there exist $\nu \in S^{n-1}$ and $\delta > 0$, such that*

$$\nu \cdot \nabla\phi(x,t) \leq -\delta|\nabla\phi(x,t)| < 0 \quad \text{in } \Omega \times (t_1, T_v),$$

then ϕ does not uniformly blow-up in Ω:

$$\inf_{x\in\Omega} \phi(x,t) \leq L < \infty \quad \text{for } t \in (t_1, T_v).$$

Proof of Lemma 3.2. This lemma is proved in [9] (See [9, Lemma 4.1]). □

Proof of Theorem 2. Put $r \in (0,1)$. Define

$$\mu(x,t) = \phi(2r - x_1, x', t) - \phi(x_1, x', t),$$

where $x = (x_1, x')$ with $x' = (x_2, x_3, \ldots, x_n) \in \mathbf{R}^{n-1}$. Then, we obtain

$$\begin{cases} \mu_t \geq \Delta\mu + C(x,t)\mu, & x \in D_r, t \in (t_1, T_v), \\ \mu(x,0) = \phi_0(2r - x_1, x') - \phi_0(x_1, x') \geq 0, & x \in D_r, \\ \mu(x,t) \geq 0, & x \in \partial D_r, t \in (t_1, T_v), \end{cases}$$

where

$$C(x,t) = \int_0^1 \{\theta\phi(2r - x_1, x', t) + (1-\theta)\phi(x_1, x', t)\}\, d\theta$$
$$D_r = \{x : x_1 < r\} \cap \{x : (x - 2r)^2 < 1\}.$$

Thus, by the maximum principle [10] we have

$$\mu \geq 0 \quad \text{in} \quad D \times [t_1, T_v)$$

and

$$\phi(2r - x_1, x', t) \geq \phi(x_1, x', t) \quad \text{in} \quad D \times [t_1, T_v).$$

Since $r \in (0,1)$ is arbitrary, we obtain that $\phi_{x_1} \geq 0$ for $x \in \{x | x_1 > 0\}$ and

$$-e_1 \cdot \nabla\phi \leq -\phi_{x_1} \leq -\frac{\delta x_1}{|x|}|\nabla\phi|, \quad \text{in } D \cup \{x | x_1 \geq 0\}$$

with some $\delta > 0$, where $e_1 = {}^t(1,0,0,\ldots,0)$. Since $\phi_t \geq 0$ and $\inf_{x \in B_1} \phi(x,t) = \phi(0,t)$, by Lemma 3.2 we have

$$\lim_{t \to T_v} \phi(0,t) \leq L \text{ with some } L < \infty.$$

Thus

$$\lim_{t \to T_v} u(a,t) \leq L \text{ with same } L.$$

Since $a \in \mathbf{R}^n$ is arbitrary, u does not blow up at $t = T_v$ in $\mathbf{R}^n$. □

4 On blow-up direction

We shall prove Theorem 3 which gives a condition for blow-up direction.

Proof of Theorem 3. We first prove that if u_0 satisfies (11), then ψ is a blow-up direction. By assumption we obtain that $u_0(x)$ satisfies (5) with some sequences $\{x_m\}_{m=1}^{\infty}$ satisfying $\lim_{m\to\infty} x_m/|x_m| = \psi$. Then, from the proof of Theorem 1 it follows that

$$\lim_{m\to\infty} u(x_m, t_m) = \infty$$

with the sequence $\{t_m\}_{m=1}^{\infty}$ satisfying $\lim_{m\to\infty} t_m = T_v$. Since $\lim_{m\to\infty} x_m/|x_m| = \psi$ by the assumption we obtain that ψ is a blow-up direction.

We next show that if ψ is a blow-up direction, then there exist $\{x_m\}_{m=0}^{\infty} \subset \mathbf{R}^n$ such that $x_m/|x_m| \to \psi$, $t_m \to T_v$ and $u(x_m,, t_m) \to \infty$ as $m \to \infty$. In contrary it says that if for any sequences $\{x_m\}_{m=1}^{\infty} \subset \mathbf{R}^n$ satisfying $\lim_{m\to\infty} x_m/|x_m| = \psi$, u_0 does not satisfy (11), then ψ is not a blow-up direction.

Since $\lim_{m\to\infty} u_0(x + x_m) = M$ a.e. in $\mathbf{R}^n$, we have

$$\lim_{m\to\infty} \sup_{x\in B_3(x_m)} \frac{1}{(4\pi t)^{n/2}} \int_{\mathbf{R}^n} e^{-(x-y)^2/4t} u_0(y)dy < M \tag{17}$$

for $t > 0$. Since the solution of (1) satisfies the integral equation

$$u(x,t) = e^{\Delta t}u_0(x) + \int_0^t e^{\Delta(t-s)} f(u(x,s))ds,$$

we have

$$u(x,t) \le e^{\Delta t}u_0(x) + \int_0^t f(v(s))ds = v(t) - M + e^{\Delta t}u_0(x)$$

for $(x,t) \in \mathbf{R}^n \times [0, T^*)$.

Let M_f, δ_f and T_0 be the same as proof of Lemma 3.1. We consider the solution w of

$$\begin{cases} w_t = \Delta w, & x \in \mathbf{R}^n, t \in (T_0, T_v), \\ w(x, T_0) = \max\{\{v(T_0) - M + e^{\Delta T_0}u_0(x)\}/v(T_0), \delta_f\}, & x \in \mathbf{R}^n. \end{cases}$$

We now introduce $\tilde{u} = vw$. From the proof of Lemma 3.1, it follows that $\tilde{u} \ge u$ for $(x,t) \in \mathbf{R}^n \times [T_0, T^*)$. Then we have

$$u(x,t) \le v(t)e^{\Delta(t-T_0)} \max\{\{v(T_0) - M + e^{\Delta T_0}u_0(x)\}/v(T_0), \delta_f\}$$

for $(x,t) \in \mathbf{R}^n \times [T_0, T_v)$.

Put $U_m = \sup_{x\in B_2(x_m)} e^{T_0}u(x)$. From (17), there exists $M_0 \in (0, M)$ such that

$$\lim_{m\to\infty} U_m \le M_0 (< M).$$

There exists a sequence $\{V_k\}_{k=1}^{\infty}$ such that $V_k = (M_0+M)/2$, $\lim_{k\to\infty} V_k = M_0$ $V_{k+1} \le V_k$ and $V_k \ge U_{m_k}$ with a sequence $\{m_k\}_{k=1}^{\infty}$ satisfying $u_{k+1} > u_k$ for $k \in \mathbf{N}$. Thus, since $(x-y)^2 \le 2x^2 + 2y^2$, we obtain

$$\begin{aligned} &\sup_{x\in B_1(\tilde{x}_k)} w(x,t) \le W_k(t) \\ &= e^{\Delta(t-T_0)} \max\left\{ \frac{v(T_0) - (M - V_k)e^{-|x|^2/2t}\int_{|y|<2} e^{-|y|^2/2t}u_0(y)dy}{(4\pi T_0)^{-n/2}v(T_0)}, \delta_f \right\} < 1 \end{aligned}$$

for $t \in [T_0, T_v)$, where $\tilde{x}_k = x_{m_k}$. By comparison we have $W_{k+1}(t) \le W_k(t)$ for $t \in [T_0, T_v)$ and $k \in \mathbf{N}$. From Lemma 3.2 and comparison it follows that there exist the sequence $\{\eta_k\}_{k=1}^{\infty}$ satisfying $0 < \eta_{k+1} \le \eta_k < \infty$ such that

$$\lim_{t\to T_v} u(x_{m_k}, t) \le \eta_k.$$

Since the sequence $\{x_m\}_{m=1}^{\infty}$ is arbitrary, we obtain that ψ is not blow-up direction. □

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