Multiple stable patterns in a balanced bistable equation with heterogeneous environments (Variational Problems and Related Topics)

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1 Introduction and Main Result

There are several results on the studies of solutions to the following equation with a balanced bistable nonlinearity:

\[ \varepsilon^2 \Delta u + h(x)^2(a(x)^2 - u^2)u = 0, \quad x \in \Omega, \quad \frac{\partial u}{\partial n} = 0, \quad x \in \partial \Omega, \]

where \( \Omega \) is a bounded domain in \( \mathbb{R}^n, n \geq 1 \) with smooth boundary, \( \varepsilon > 0 \) is a parameter, and \( h(x) \) and \( a(x) \) are positive functions on \( \Omega \). Solutions \( u \) of the boundary value problem above is corresponding to critical points of the functional

\[ J(u) = \frac{1}{2} \varepsilon^2 \int_{\Omega} |\nabla u(x)|^2 \, dx + \frac{1}{4} \int_{\Omega} h(x)^2(a(x)^2 - u(x)^2)^2 \, dx \]

on \( H^1(\Omega) \). The global minimizer \( u(x) \) of \( J(u) \) on \( H^1(\Omega) \) has an asymptotic behavior \( u(x) \to a(x) \) (or \( u(x) \to -a(x) \)) as \( \varepsilon \to 0 \). In general, to find a nontrivial local minimizer \( u(x) \) with inner transition layers is a delicate problem.

If the dimension is one, there are several results. Let \( \Omega = (0,1) \). When \( h(x) \equiv 1 \), Nakashima [8] proved by using a delicate construction of a subsolution and a supersolution that if \( a \in C^2[0,1] \) takes a nondegenerate local minimum at \( x_0 \in (0,1) \), then there exists a stable solution which has the asymptotic behavior \( u_\varepsilon(x) \sim -a(x) \) on \( (0,x_0) \) and \( u_\varepsilon(x) \sim a(x) \) on \( (x_0,1) \) as \( \varepsilon \to 0 \). Later, Matsuzawa [7] extended her result in a degenerate setting. On the other hand, when \( a(x) \equiv 1 \), Nakashima [9] also constructed a stable solution which has an inner transition layer near a local minimal point of \( h(x) \) and studied the location of inner transition layers of solutions in details.

For the studies in the higher dimensional case and $a(x) = 1$, we refer to [3], [6], [11], [12]. In these previous results, the effect of domain geometry or the effect of $h(x)$ have been studied for the existence of stable solutions with inner transition layers. However, it seems that there exist few studies on the effect of $a(x)$ to this problem in the higher dimensional case.

In this paper, we consider the special case $a(x) = \chi_{D}(x)$ with a subdomain $D \subset \Omega$ and show existence of stable solutions with inner transition layers to

$$
\epsilon^{2}\Delta u + (a(x)^{2} - u^{2})u = 0, \quad x \in \Omega, \quad \frac{\partial u}{\partial n} = 0, \quad x \in \partial\Omega.
$$

Assume that $D = D_{1} \cup D_{2}, \overline{D_{1}} \cap \overline{D_{2}} = \emptyset, \overline{\partial D \cap \Omega} \subset \Omega$ and $\partial D_{1}, \partial D_{2}$ belong to the $C^{2}$ class. Then we have the following.

**Theorem 1.** For sufficiently small $\epsilon > 0$, there exists a local minimizer $u_{\epsilon}$ of $J(u)$ on $H^{1}(\Omega)$ which has the following asymptotic behavior: $u_{\epsilon}$ converges to 1 uniformly on any compact subset of $D_{1}$, converges to $-1$ uniformly on any compact subset of $D_{2}$, and converges to 0 uniformly on any compact subset of $\Omega \setminus (\overline{D})$.

**Remark 1.** The same result holds under the homogeneous Dirichlet boundary condition.

**Remark 2.** When $D$ consists of several components, by choosing $D_{1}$ and $D_{2}$ suitably, Theorem says the existence of local minimizers which have different asymptotic behavior, i.e. are close to 1 on some components and are close to $-1$ on other components.

**Remark 3.** Although we think the smoothness of $\partial D_{i}, i = 1, 2$, is not necessary, we need at least $C^{2}$ regularity from a technical reason.

## 2 Useful Lemmas

We recall two useful lemmas.

**Lemma 1 (Asymptotic behavior).** Let $D = \{x \in \mathbb{R}^{n} \mid |x| < \delta\}$, $g \in C^{1}(\mathbb{R}^{1})$, and there exists a constant $T > 0$ such that $g(t) > 0$ ($t < 0$), $g(T) = 0$, $g(t) < 0$ ($t > T$). Suppose that $G(t) = \int_{0}^{t} g(s) ds$ has a unique maximum at $t = T$. Then, for a minimizer $u_{\epsilon} \in H^{1}_{0}(D)$ of

$$
\inf\{J_{\epsilon}(u; D) \mid u \in H^{1}_{0}(D)\},
$$

where

$$
J_{\epsilon}(u; D) = \frac{\epsilon^{2}}{2} \int_{D} |\nabla u|^{2} dx - \int_{D} G(u) dx,
$$

we have $0 \leq u_{\epsilon}(x) \leq T$, ($x \in D$), $u_{\epsilon}(x) = u_{\epsilon}(|x|)$. Moreover, $u_{\epsilon}(x)$ converges to $T$ uniformly on any compact subset $K \subset D$. 

Next, let \( g_1(x, t), g_2(x, t) \) be \( C^1 \)-functions with respect to \( t \) and let

\[
G_i(x, t) = \int_0^t g_i(x, s) \, ds, i = 1, 2.
\]

For \( \eta_i \in H^1(D), i = 1, 2 \), consider the minimizing problem:

\[
\inf\{ J_i(u; D) \mid u - \eta_i \in H_0^1(D) \}, \quad J_i(u; D) = \frac{\epsilon^2}{2} \int_D |\nabla u|^2 \, dx - \int_D G_i(x, u) \, dx.
\]

**Lemma 2 (Energy comparison).** \( u_i \in H^1(D), i = 1, 2 \) be minimizers to the minimization problem above. Assume that there exist constants \( m < M \) such that

(a) \( m \leq u_i(x) \leq M \) for \( i = 1, 2, x \in D \).

(b) \( g_1(x, t) \geq g_2(x, t) \) for \( x \in D, t \in [m, M] \).

(c) \( \eta_1(x) \geq \eta_2(x) \) for \( x \in D \).

Suppose \( \eta_j \in C(D), \eta_1(x) \not\equiv \eta_2(x) \) on \( \partial D \). Then, we have \( u_1(x) \geq u_2(x), x \in D \).

Although the proofs of these lemmas are known (see [3], [14]), we present it for reader's convenience.

**Proof of Lemma 1.** \( u_\epsilon \) satisfies

\[
\begin{aligned}
-\epsilon^2 \Delta u &= g(u), \quad \text{for } x \in D = \{x \mid |x| < \delta\}, \\
u &= 0, \quad \text{on } \partial D.
\end{aligned}
\]

By the maximum principle and the condition on \( g(t) \), we have \( 0 \leq u_\epsilon(x) \leq T, x \in D \). Gidas-Ni-Nirenberg's theorem implies

\[
u_\epsilon(x) = u_\epsilon(|x|), \quad \nu_\epsilon'(r) < 0, \quad (r = |x| > 0).
\]

For sufficiently small \( \epsilon > 0 \), define \( w_\epsilon \in H_0^1(D) \) as follows:

\[
w_\epsilon(x) = \begin{cases} 
T, & (|x| \leq \delta - \epsilon) \\
-\frac{T}{\epsilon}(|x| - \delta), & (\delta - \epsilon < |x| \leq \delta).
\end{cases}
\]

Since \( u_\epsilon \) is a minimizer,

\[-\int_D G(u_\epsilon) \, dx \leq J_\epsilon(u_\epsilon; D) \leq J_\epsilon(w_\epsilon; D).\]

There exists a constant \( C_0 \) such that

\[
J(w_\epsilon; D) \leq \frac{\epsilon^2}{2} \int_{\{x \mid \delta - \epsilon < |x| \leq \delta\}} |\nabla w_\epsilon|^2 \, dx - G(T)|B(0, \delta)| + 2 \max_{0 \leq t \leq T} G(t)\{x \mid \delta - \epsilon < |x| \leq \delta\} \\
\leq -G(T)|D| + C_0 \epsilon.
\]
where $|A|$ denotes the Lebesgue measure of a set $A \subset \mathbb{R}^n$. Thus

\[ \int_D (G(T) - G(u_\epsilon)) \, dx \leq C_0 \epsilon. \]

Since $G(t)$ takes its maximum only at $t = T$, we have $G(T) - G(u_\epsilon) \geq 0$ on $D$.

Take arbitrary $r_0 \in (0, \delta)$ and fix. For $\sigma \in (0, \delta - r_0)$,

\[ \int_D (G(T) - G(u_\epsilon)) \, dx \geq \int_{\{r_0 \leq |x| \leq r_0 + \sigma\}} (G(T) - G(u_\epsilon)) \, dx \]

\[ = (G(T) - G(u_\epsilon(r_\epsilon)))|\{x| r_0 \leq |x| \leq r_0 + \sigma\}| \]

holds for some $r_\epsilon \in (r_0, r_0 + \sigma)$.

Because the measure $|\{x| r_0 \leq |x| \leq r_0 + \sigma\}|$ is positive and independent of $\epsilon$, as $\epsilon \to 0$ we have

\[ 0 \leq G(T) - G(u_\epsilon(r_\epsilon)) \leq C_1 \epsilon. \]

Since $G(t)$ takes its maximum only at $t = T$, we obtain $u_\epsilon(r_\epsilon) \to T$ as $\epsilon \to 0$. Noting $u_\epsilon(x) = u_\epsilon(|x|)$ and $u'_\epsilon(r) < 0$, we see

\[ u_\epsilon(r_\epsilon) \leq u_\epsilon(r) = u_\epsilon(|x|) \leq T, \quad r = |x| \leq r_0 \leq r_\epsilon. \]

In particular, it follows

\[ \max_{\{x| |x| \leq r_\epsilon\}} |u_\epsilon(x) - T| \to 0 \quad \text{as} \quad \epsilon \to 0. \]

By using a compactness argument, $u_\epsilon(x)$ converges to $T$ uniformly on any compact subset of $D$.

**Proof of Lemma 2.** Let $M = \{x \in D| u_2(x) > u_1(x)\}$. Assume $M \neq \emptyset$. Then $D \setminus M$ contains nonempty open set. Put $\phi(x) = (u_2 - u_1)^+$. Then $\phi \in H_0^1(D)$, $\phi \not\equiv 0$ on $D$, and $\phi(x) = 0$ on $D \setminus M$. Since $u_1, u_2$ are minimizers respectively,

\[ 0 \leq J_1(u_1 + \phi) - J_1(u_1) \]

\[ = \frac{\epsilon^2}{2} \int_M (|\nabla(u_1 + \phi)|^2 - |\nabla u_1|^2) \, dx - \int_M \int_{u_1(x)}^{u_1(x) + \phi(x)} g_1(x, s) \, ds \, dx \]

\[ \leq \frac{\epsilon^2}{2} \int_M (|\nabla(u_1 + \phi)|^2 - |\nabla u_1|^2) \, dx - \int_M \int_{u_1(x)}^{u_1(x) + \phi(x)} g_2(x, s) \, ds \, dx \]

\[ = J_2(u_2) - J_2(u_2 - \phi) \leq 0. \]

This means that $u_1 + \phi$ is also a minimizer of $J_1$, and hence

\[-\epsilon^2 \Delta(u_1 + \phi) = g_1(x, u_1 + \phi).\]

Therefore, there exists a bounded function $c(x)$ such that

\[-\epsilon^2 \Delta \phi = g_1(x, u_1 + \phi) - g_1(x, u_1) = c(x) \phi.\]

The maximum principle or the unique continuation property leads a contradiction. Thus we can conclude $M = \emptyset$. 
3 Proof of Theorem 1

In this section we use the notation
\[
J_{\epsilon}(u; G) = \frac{1}{2}\epsilon^{2} \int_{G} |\nabla u(x)|^{2} dx + \frac{1}{4} \int_{G} (a(x)^{2} - u(x)^{2})^{2} dx
\]
for \( u \in H^{1}(G) \) with \( G \subset \Omega \). Let \( u_{\epsilon} \) be a positive global minimizer of
\[
\inf_{v \in H_{0}^{1}(D_{1})} J_{\epsilon}(v; D_{1}).
\]
Existence of \( u_{\epsilon} \) follows from the standard argument. Moreover, by the maximum principle we have \( 0 < u_{\epsilon}(x) < 1 \) on \( D_{1} \). By Lemma 1, \( u_{\epsilon}(x) \) converges to 1 uniformly on any compact subset \( K \subset D_{1} \). Let \( w_{\epsilon} \) be a negative global minimizer of
\[
\inf_{v \in H_{0}^{1}(\Omega \setminus \overline{D_{1}})} J_{\epsilon}(v; \Omega \setminus \overline{D_{1}}).
\]
By Lemma 1, \( w_{\epsilon}(x) \) converges to \(-1\) uniformly on any compact subset \( K \subset D_{2} \) and to 0 uniformly on any compact subset \( K \subset \Omega \setminus \overline{D_{1}} \cup \overline{D_{2}} \). Define \( u_{\epsilon} \in H^{1}(\Omega) \) as follows:
\[
u_{\epsilon}(x) = \begin{cases} 
    u_{\epsilon}(x), & x \in D_{1} \\
    w_{\epsilon}(x), & x \in \Omega \setminus \overline{D_{1}}.
\end{cases}
\]

**Lemma 3.** Let \( \nu \) be the outward unit normal vector on \( \partial D_{1} \). Then there exist positive constants \( \delta_{0}, C_{0} \) independent of \( \epsilon \) such that
\[
\frac{\partial \nu_{\epsilon}}{\partial \nu}(x) \leq -\delta_{0}, \quad (x \in \partial D_{1}),
\]
\[
\frac{\partial w_{\epsilon}}{\partial \nu}(x) \geq -C_{0}\epsilon, \quad (x \in \partial D_{1}).
\]

**Proof.** For simplicity, we assume \( \overline{D} \subset \Omega \). Let \( u_{\epsilon_{0}} \) be a positive global minimizer of
\[
\inf_{v \in H_{0}^{1}(D_{1})} J_{\epsilon_{0}}(v; D_{1}).
\]
Then, it is easy to see that \( u_{\epsilon_{0}} \) is a subsolution of the equation with \( \epsilon(<\epsilon_{0}) \) on \( D_{1} \). Since \( v \equiv 1 \) is a supersolution and the uniqueness of a positive solution, we have
\[
0 \leq u_{\epsilon_{0}}(x) \leq u_{\epsilon}(x) \leq 1, \quad (x \in D_{1}).
\]
This implies
\[
\frac{\partial u_{\epsilon}}{\partial \nu}(x) \leq \frac{\partial u_{\epsilon_{0}}}{\partial \nu}(x) \leq -\delta_{0} < 0, \quad (x \in \partial D_{1}).
\]
Let \( w = -w_{\epsilon} > 0 \) be a positive minimizer of
\[
\inf_{v \in H_{0}^{1}(\Omega \setminus \overline{D_{1}})} J_{\epsilon}(v; \Omega \setminus \overline{D_{1}}).
\]
It suffices to show
\[
\frac{\partial w}{\partial \nu}(x) \leq C_{0}\epsilon, \quad (x \in \partial D_{1}),
\]
where $\nu$ be the outward (from $D_1$) unit normal vector on $\partial D_1$.

Take a smooth domain $(\Omega \supset) \tilde{D}_1 \supset \overline{D}_1$ s.t. $\overline{D}_1 \cap D_2 = \emptyset$. Let $\bar{w}$ be a global minimizer of

\[ \inf\{J_\epsilon(v; \tilde{D}_1 \setminus D_1); v \in H^1(\tilde{D}_1 \setminus D_1), v = 0 \text{ on } \partial D_1, v = 1, \text{ on } \partial \tilde{D}_1.\} \]

By Lemma 2, we have

\[ w(x) \leq \bar{w}(x), \ (x \in \tilde{D}_1 \setminus D_1). \]

Since $\tilde{D}_1 \setminus D_1 \subset \Omega \setminus \overline{D}$, $\bar{w}$ satisfies

\[ \epsilon^2 \Delta \bar{w} = \bar{w}^3. \]

Let $W_\epsilon(x) = \epsilon^{-1} \bar{w}(x)$. Then

\[ \Delta W_\epsilon = W_\epsilon^3, \ x \in \tilde{D}_1 \setminus D_1, \]

\[ W_\epsilon(x) = 0, \ (x \in \partial D_1), \ W_\epsilon(x) = \frac{1}{\epsilon}, \ (x \in \partial \tilde{D}_1). \]

It is well-known (e.g., [5], [1], [13] and the references therein) that under the assumption $\partial D_1$ and $\partial \tilde{D}_1$ are of $C^2$ class there exists a unique positive solution to

\[ \Delta V_\infty = V_\infty^3, \ x \in \tilde{D}_1 \setminus D_1, \]

\[ V_\infty(x) = 0, \ (x \in \partial D_1), \ V_\infty(x) = +\infty, \ (x \in \partial \tilde{D}_1). \]

Moreover, by comparison's theorem (see, e.g. [4]) we have

\[ W_\epsilon(x) \leq V_\infty(x), \ (x \in \tilde{D}_1 \setminus D_1). \]

Thus, we have

\[ w(x) \leq \bar{w}(x) = \epsilon W_\epsilon(x) \leq \epsilon V_\infty(x), \ (x \in \tilde{D}_1 \setminus D_1). \]

For any compact subset $K \subset \tilde{D}_1 \setminus D_1$, where $K$ include a neighborhood of $\partial D_1$,

\[ \frac{\partial w}{\partial \nu}(x) \leq \epsilon \frac{\partial V_\infty}{\partial \nu}(x) \leq \epsilon C_0, \ x \in K. \]

This completes the proof of Lemma 3.

As an easy consequence of Lemma 3, we have the following.

**Proposition 1.** There exists a sufficiently small $\epsilon_0 > 0$ such that, $w_\epsilon$ is a subsolution for $0 < \epsilon < \epsilon_0$.

**Proof.** We show that

\[ \int_{\Omega} \left( \epsilon^2 \nabla u_\epsilon \cdot \nabla \phi - (a(x)^2 - u_\epsilon^2) u_\epsilon \phi \right) dx \leq 0 \]
holds for any $\phi \in C_0^\infty(\Omega)$ with $\phi(x) \geq 0$ in $\Omega$. Note that by the elliptic regularity theorem we have $v_\epsilon \in W^{2,p}(D_1)$ for any $p > n$ and hence $v_\epsilon \in C^1(\overline{D_1})$. Also we have $w_\epsilon \in W^{2,p}(\Omega \setminus D_1)$ for any $p > n$ and hence $w_\epsilon \in C^1(\overline{\Omega \setminus D_1})$. Thus we obtain

$$
\int_\Omega (\epsilon^2 \nabla u_\epsilon \cdot \nabla \phi - (a(x)^2 - u_\epsilon^2)u_\epsilon \phi) \, dx
= \int_{D_1} (\epsilon^2 \nabla v_\epsilon \cdot \nabla \phi - (a(x)^2 - v_\epsilon^2)v_\epsilon \phi) \, dx
+ \int_{\Omega \setminus D_1} (\epsilon^2 \nabla w_\epsilon \cdot \nabla \phi - (a(x)^2 - w_\epsilon^2)w_\epsilon \phi) \, dx
= \int_{\partial D_1} \epsilon^2 \frac{\partial v_\epsilon}{\partial \nu} \phi \, dS - \int_{D_1} (\epsilon^2 \Delta v_\epsilon \phi + (a(x)^2 - v_\epsilon^2)v_\epsilon \phi) \, dx
- \int_{\partial D_1} \epsilon^2 \frac{\partial w_\epsilon}{\partial \nu} \phi \, dS - \int_{\Omega \setminus D_1} (\epsilon^2 \Delta w_\epsilon \phi + (a(x)^2 - w_\epsilon^2)w_\epsilon \phi) \, dx
= \epsilon^2 \int_{\partial D_1} (\frac{\partial v_\epsilon}{\partial \nu} - \frac{\partial w_\epsilon}{\partial \nu}) \phi \, dS
\leq \epsilon^2 \int_{\partial D_1} (-\delta_0 + C_0 \epsilon) \phi \, dS \leq 0.
$$

This completes the proof of Proposition 1.

In a similar way, let $\overline{v}_\epsilon$ be a negative global minimizer of

$$
\inf_{v \in H^1_0(D_2)} J_\epsilon(v; D_2).
$$

Let $\overline{w}_\epsilon$ be a positive global minimizer of

$$
\inf_{v \in H^1_0(\Omega \setminus D_2)} J_\epsilon(v; \Omega \setminus D_2).
$$

Define $\overline{u}_\epsilon \in H^1(\Omega)$ as follows:

$$
\overline{u}_\epsilon(x) = \begin{cases} 
\overline{v}_\epsilon(x), & x \in D_2 \\
\overline{w}_\epsilon(x), & x \in \Omega \setminus D_2.
\end{cases}
$$

Then we have the following lemma which can be proved in the same way as in the proof of Lemma 3.

**Lemma 4.** Let $\nu$ be the outward unit normal vector on $\partial D_2$. Then there exist positive constants $\delta_1, C_1$ independent of $\epsilon$ such that

$$
\frac{\partial \overline{u}_\epsilon}{\partial \nu}(x) \geq \delta_1, \ (x \in \partial D_2),
$$

$$
\frac{\partial \overline{w}_\epsilon}{\partial \nu}(x) \leq C_1 \epsilon, \ (x \in \partial D_2).
$$

By Lemma 4, we have the following proposition as in the proof of Proposition 1.
Proposition 2. There exists a sufficiently small $\epsilon_0 > 0$ such that, $\overline{u}_\epsilon$ is a supersolution for $0 < \epsilon < \epsilon_0$.

The following lemma is a consequence of the energy comparison lemma.

Lemma 5. For $0 < \epsilon < \epsilon_0$, we have $\overline{u}_\epsilon(x) > \underline{u}_\epsilon(x)$, $(x \in \Omega)$.

Proof. By using Lemma 2, we have $\overline{w}_\epsilon(x) \geq \underline{v}_\epsilon(x)$ on $D_1$. Moreover, by the strong maximum principle we have $\overline{w}_\epsilon(x) > \underline{v}_\epsilon(x)$ on $D_1$. In a similar way, we have $\overline{w}_\epsilon(x) > \underline{v}_\epsilon(x)$ on $D_2$. By the construction, these yield the desired result.

Proof of Theorem 1. By Lemma 5 and Brezis-Nirenberg's argument (see e.g. [2], [7], [12]), we have a local minimizer $u_\epsilon$ of $J_\epsilon(u; \Omega)$ on $H^1(\Omega)$ such that

$$\overline{u}_\epsilon(x) \geq u_\epsilon(x) \geq \underline{u}_\epsilon(x), (x \in \Omega).$$

The asymptotic behavior of $u_\epsilon$ follows from the constructions of $\overline{u}_\epsilon, \underline{u}_\epsilon$, Lemma 1 and the proof of Lemma 3.

4 Some Extensions and Questions

In this section we discuss about possible extensions and open questions. First, for a given positive function $b(x)$, when $a(x) = b(x) \chi_D(x)$ with the same assumptions on $D$ as in Theorem 1, we have a similar result. Moreover, it is possible to extend our result for the equation on compact manifolds, since the proof of Theorem 1 depends on simple minimizing problems, a comparison theorem and a solvability of solutions which blow up at the boundary.

Finally, we mention that the following questions remain open.

1. For a technical reason, in Theorem 1 we assume the condition $\partial D \cap \Omega \subset \Omega$. It is an open question to show the same statement as in Theorem 1 for the case that $\partial D \cap \Omega$ intersects $\partial \Omega$.

2. When $a(x) = \chi_D$, $\overline{D} \subset \Omega$, and $D$ is a dumbbell like domain with a thin channel, can one still have a stable solution with inner transition layers? (cf. [2])

3. Without the smallness of $\epsilon > 0$, under certain assumption on $D$ as in Theorem 1, can one show the existence of solutions which change sign?

References


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