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Kyoto University
Model for one Style of Self-defending System
founded in Non-articulated Coralline Algae

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Key Words: Oscillator function, Periodic sequence, Behavior

ABSTRACT
The self-repairing of living systems is explained by the diffusion-reaction model in which diffusion and reaction is operated regardless of the progress of metabolism, the difference of situation (boundary condition) and of the material density. We proposed a model of cell's behavior that involves metabolism, growth, information transmission and self-repairing. The essential feature of this model is that the change of state causes change of function and function's change brings about further change and such succession follows one after another. Such model would be closer to autopoietic system which decides itself or constitutes its own causality.

1. Introduction

The concept of autopoiesis, proposed by Maturana and Varela(1973), is defined as characteristic feature of living things. The point is that it is self-decided system and that it decides its own outside by itself and chooses the interaction with outside. That is, when one can observe living things and may see material flow or change of condition, the causality is autonomously constituted. Hence, the causal relation is operationally closed in the inside of living things. A molecule, cell or other biomaterials can behave as condition (to be changed) in one case and as function (to make change) in others.

In general mathematical model which applies a priori
causality to phenomena, condition and function are separately defined. Owing to the essential independence of them, we can enjoy the convenience to obtain exactly one output for one input. It is true that the role of it in our life is to be glorified, but at the same time this dependence turns to be the restriction. Let us consider in material science. Materially measured state or imagined one would be to express mass, volume, electric charge, or so on. Then, for the orderly working of function, the process which maps condition from domain to codomain would never be varied so long as the condition is within the domain. But the agent of the process(mapping) would also consist of materials and there would be certainly some influence for the variation of mass, volume or other condition in the world of material science. Therefore, to make the function work orderlily, there would have to be the agent to cut off the influence and to guarantee the working of function(e.g. the independence) in phenomenon².

There are two ways to search the agent of the process. One is to search within the phenomenon. If there is the agent in the phenomenon, it would be also invariant however condition varies. So, the agent ,who guarantee the independence of condition and function, would have to be independent of function and state which could vary. To guarantee the new independence, there would be the second agent independently in phenomenon, and after all this procedure would follows infinitely. If this procedure is interrupted in finite times, there would have to be absolute agent which have no guarantee or no logical basis. The other is to owe to initial condition and/or boundary condition. In such case what to be intrinsically substituted to function a priori would take up the role to guarantee the very independence between condition and function. Furthermore, if boundary condition make the mapping work orderlily, the process would depends on external situation and then it could not reflect the concept of auto-poietic system which decides its own causality and selects external objects as its own outside(e.g. boundary condition).

So, if one observes phenomenon that many objects interact with each other and describes it using one function over all objects(e.g. diffusion-reation system), each objects would have to be supported by the absolute agent or by initial condition and/or boundary condition. In other words, each of them would be in the same situation such that the variation of condition within
domain is left as condition's variation and never affect the progress of the mapping. In such case that objects in the phenomenon can be regarded as being in the same situation, this description is good enough. But in the case that it is almost hard to look upon phenomenon as like that and if we would try to describe autopoietic system, we might have to introduce another approach. In next section we would introduce one motif for it.

2. Self-defending in Non-articulated Coralline Algae

We investigated self-defending of *Lithophyllum pallescens* which is a species of non-articulated coralline algae. The body of this species consists of many series of cells and divided into three sections, Hypothallium, Perithallium and Epithallium. Hypothallium is ordinarily constituted in one layer and sets its body on rock or coral body, and it does not obviously appear in this experiment. Epithallium is also in one layer and covers its hole body from outside. Most of whole body is occupied by perithallium. The growing to upper direction is performed by one perithallial cell directly under epithallium in each series of cells, which is called meristem. This species dwells in eulittoral zone and the body of the plant is very hard because of calcium carbonate involved in the cell wall. This plant would be very suitable for the experimental study firstly because it can be collected easily and secondly because the cells around the wound never move to cover it when the body is injured. The second reason is much more important in observing the change of behavior(both condition and function) of the cell there from the viewpoint of self-repairing.

The method of experiment is following; We scraped out the top of collected branches of *L. pallescens* with standardization and put them back into a tank. The point of this operation was removal of meristem. After that we took out a branch every day for fourteen days and saw the wound under a microscope. The observation result is thus; Within a few days later meristem was appeared in the perithallial cell which is shortly distant from the wound. We saw one or more cells between the distance(Fig.1).

We could describe this phenomenon with diffusion-reaction system, in which the material transmitting information diffuse in cells through pit connection and a certain density of it is a
signal to change cell's behavior into meristem. This description is very clear and probably we could get a tolerable approximation by management of diffusion constant. One could be satisfied with this success if he interprets and treats livingthings like machines. But if we want to describe autopoietic system as it is, we should think about two things in this description. One is that in each cell one mapping is operated in spite of the difference of condition, or the material density. What is the agent to dissolve the material effect of the density difference to mapping in cell and to keep orderly mapping? The other is the difference of the situation. The cell which is exposed by injure is in obviously different situation from the cell directly under it or the cell in more deeper location. Of course this difference of situation could not be substituted before the condition is operated solong as there is not another information transmitter with infinite velocity (even if there was the another transmitter, the progress of transmission and the reception of information in each cell must be guaranteed regardless of the amount of information).

Then we would select another standpoint. That is, the behavior (both states and function) would be changed by the absence of the neighbor cell and this change would cause the change of the interaction with the other neighbors. And its behavior's change would occur the change of interaction with others and this succession of change might follow further and then finally cause the appearance of meristem. To describe this succession formally, we would use oscillator function (Varela (1979)).

3. Oscillator function forced by external oscillation.

Spencer-Brown (1969) constructed Brownian algebra in which operator and operand are not obviously discriminated. He expressed that as for the proposition,

$$a = \overline{a}$$

which involves contradiction for the axiom, we could dissolve it with introduction of time like that

$$a^t = \overline{a^{t+1}}$$.
Varela (1979) extended the idea and constructed oscillator function. However his introduction of the relation between periodic sequence and function was ambiguous and did not make initial value dependency clear. Nakamura et al. (1991) pointed it out and revealed the dependence in internal orbit using flow diagram (Fig. 2). And he introduced forced oscillator function affected by external oscillation and expressed some dependence generated in forced orbit which complicate the relation between external sequence (it could be regarded as external input) and forced orbit (it is fit path according to external input). They are internal initial value dependence, external initial value dependence and external sequence dependence (Fig. 3).

As we will explain in next chapter, forced orbit might be changed by absence or modulation of external oscillation. Then, we have to think about removal value dependence which occurs in the change of orbit (Fig. 4). Because of these dependence we could not construct rigorous reference for output sequence under the forced orbit. If we want to control forced orbit, we would have to accidentally find the special form to make them have no effect.

Here we will introduce two forms (Fig. 5) for next chapter. They dissolve those dependence and are called R&A and R&I respectively. If we arrange R&A oscillators in one dimension and oscillation of them are their neighbor's external one, each of them oscillates in same period as boundary oscillators. R&I is modulator that oscillates in $2\pi$ period for external oscillation in $\pi$ period.

4. Model for a self-defending system

In the second section we made certain of our intention to describe self-defending as succession of the behavior of cells. In order to construct model, each cell is interpreted as self-exciting oscillator forced by neighbor ones and its behavior as periodic sequence.

Both inner- and inter-cells material flow or other signals we can detect, but they are received and recognized in their own way, not ours. It is true there are many cases that the causality between signals is obvious for us, but most cases assumes the
existence of the agent to guarantee the causality as boundary states. Our standpoint is that in autopoietic system change of condition might cause change of function and such change of succession follows one after another and naturally boundary condition could be changed. So, movement of material could be both states and function and Our aim is to find such succession of change. Forced oscillator function as our tool has some of dependence, then our approach would start from finding one of the simplest forms to explain the phenomenon in which causality could be changed in succession. In order to simplify, each behavior is applied to periodic sequence $S$ as such,

\[
\begin{align*}
S_e & \quad \text{(epithallium)} \quad = 01. \\
S_{p1} & \quad \text{(perithallium in self-maintenance)} \quad = 00101011. \\
S_{p2} & \quad \text{(perithallium as meristem)} \quad = 0011. \\
S_{p3} & \quad \text{(perithallium exposed)} \quad = 01. 
\end{align*}
\]

The period of $S_{p1}$ or $S_{p2}$ is a multiple of $S_{p3}$ because of controllability and dissolution of dependence. Here the period $S_e$ is same as $S_{p3}$ for the sake of simplification and we could think about the case of $S_e, S_{p3}$ or more complicated cases.

Now we would construct concrete flow diagram or oscillator. Using R&A and R&I, one example is shown in fig.6. Here only one figure represents external oscillation and each notation on arrows $\{0\}, \{1\}$ or $\{\bullet\}$ shows neighbor oscillator expressed $P_{n+1}$ (The symbol $\bullet$ means that external value is 0 or 1). To fit the circumstance and result of experiment, each cell treats its neighbor in growth direction. The absence of neighbor oscillator (e.g. scraped out) is corresponded to the following procedure,

\[
f(a,b) \rightarrow f(a,_)f(a,\bar{\_})
\]

in which $a$ expresses internal oscillator and $b$ does external one.

Perithallium(P-cell) neighboring epithallium(E-cell) oscillates in periodic sequence 1100(period 4) as meristem, and P-cell neighboring is meristem oscillates in 11010100(period 8) then being in self-maintenance, and P-cell whose both neighbors is in self-maintenance oscillates in period 8. When P-cell in self-maintenance happens to be exposed by its neighbor's abrupt absence, it is no longer forced by the external and oscillates in
internal orbit whose sequence is 10. Then P-cell whose neighbor is the exposed P-cell is modulated in sequence 1100, which is oscillation of meristem. Eventually the external sequence of meristem's other neighbor is successively modulated. So, P-cell's behavior, self-maintenance, growth, information-transmission and appearance of meristem as self-defending are expressed in the flow-diagram.

Oscillator function is following,

Epithallium
\[ a_t^{t+1} = a_t^t \]

Perithallium
\[
\begin{align*}
&Ps_t^{t+1} = Ps_t^t Ps_t^{t+1} Qs_t^t Rs_t^t Ps_t^t Qs_t^t Rs_t^t, \\
&Qs_t^{t+1} = Ps_t^t Ps_t^{t+1} Qs_t^t Rs_t^t Ps_t^t Qs_t^t Rs_t^t, \\
&Rs_t^{t+1} = Ps_t^t Ps_t^{t+1} Qs_t^t Ps_t^t Rs_t^t Ps_t^t Qs_t^t Rs_t^t.
\end{align*}
\]

5. Discussion

Our approach to autopoiesis starts at the question what guarantees the independence between states and functions in livingthings. The movement of planets or the thermodynamical circumstance assumes infinite velocity of information transmission and the independence is guaranteed by initial and/or boundary condition. This assumption is convenient enough to describe the motion of material particle but autopoietic system, in which transmission velocity is very slow(matsumo(1986,1990)). We can no longer prepare constant boundary condition through the change of state in autopoietic systems, that is, the form of function could be changed. The change causes new change and such sequential succession would follows.

For example, let us think about embryogenesis. An aggregate of material could generate causal relation(e.g. material flow) in its own body by change of boundary condition. Execution of the causality causes one or more causality in it and the new causality could change prior one and generate next one. If we
take microscopic viewpoint, boundary condition would perpetually change and if we observe whole body, it would enlarge its own body and operate to its boundary condition. Our description could be applied to such phenomenon with corresponding each behavior to periodic sequence.

Our way of description is quite complicated because of such dependence, and mathematical sophistication is needed. We expect that mathematical structure would give some nature of autopoiesis.

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Reference


Fig. 1: It shows experiment and observation results. The branch of L. pallescens, in which we could saw epithallium one layer and many series of perithallial cells, grows to upper direction. Older perithallial cells are not activated and only one cell directly under epithallium or meristem is divided into meristem and an older cell. We filed the top of the branch with standardization and scraped out epithallium, meristem, and some perithallial cells. A few days after, the perithallial cell which is located one or more layers under the wound became meristem.

Fig. 2: Each vertex in flow diagram shows vector s.t. \( V_k = (a_1, \ldots, a_i, \ldots, a_n) \) where \( a_i = 1 \) or 0 in Brownian algebra. For simple description, 1 and 0 are exchanged to 1 and 0 respectively. This figure shows case of \( n = 2 \) and for example, vertexes is assumed as \( V_1 = (0, 0), V_2 = (0, 1), V_3 = (1, 0) \) and \( V_4 = (1, 1) \). The periodic sequence of \( a_i \) which is could obtained from the flow diagram (FD) is always 001 in FD-A, FD-B, FD-C. FD-D has the initial value dependence. If initial vertex is \( V_1, V_2 \) or \( V_3 \), periodic sequence of \( a_1 \) is 001 but if \( V_4 \) is initial, 1 is the output and the period is 1. This dependence has affected much more in forced oscillator function.
Fig. 3: It shows forced oscillator function in the case of $n=4$. External value $E$, which affects internal one but is not affected, is assumed to be $e_1$ or $e_2$. '(*),' on the arrow between $V_1$ and $V_2$ shows that the vertex in the next step is always $V_2$ regardless of the external value if contemporary vertex is $V_1$. In the case that contemporary vertex is $V_2$, the next one is $V_3$ if the external value is $e_1$ and the next is $V_1$ if $e_2$ is external value. $\{a_1\}$ expresses the course to next vertex corresponding to contemporary external value. Let's consider the case that initial vertex is $V_1$. If external periodic sequence is $e_1$ then output sequence is sequence is $V_1V_2V_3V_4V_5V_6V_7V_8$, and if external's is $e_2$ then vertex's is $V_1V_2$, so forced orbit is decided not by the period of external oscillation but by external sequence. That is external sequence dependence. Furthermore consider the case initial vertex is $V_1$ and external sequence is $e_1e_2$. If initial external value is $e_1$ then output sequence is $V_1V_2$, and initial is $e_2$ then output sequence is $V_4V_5V_6V_7$ so forced orbit is decided by initial external value. That is external initial value dependence. The internal initial value dependence occurs in the case that initial vertex is selected within $V_3\sim V_{18}$. Output sequence is always $V_9V_{13}V_{12}V_{13}V_{14}V_{15}V_{16}$ regardless of external sequence or external initial value.

Fig. 4: As same as fig.3, the left figure is FD of forced oscillator function with $n=3$ and right figure is FD of internal oscillator function which is not affected by external value. When external sequence is $e_1$ then output sequence is $V_1V_2V_3V_4V_5V_6$ whatever
internal initial value is. Consider external oscillation happens to be absent and oscillation follows in internal orbit. If the removal vertex is \( V_1 \sim V_2 \) or \( V_4 \sim V_5 \) then internal oscillation is \( V_1 V_2 V_3 V_4 V_5 V_6 \), and if the removal occurs in \( V_7 \sim V_8 \) then internal oscillation is \( V_7 V_8 \). Next consider the case external sequence is modulated from \( e_1 \) to \( e_1 e_2 \) in forced FD. If modulation happens in \( V_1 \) then output sequence is not changed and modulation does in \( V_2 \) then output sequence modulated in period 6. That is removal sequence dependence.

\[ \text{Fig. 5} \]

\begin{align*}
\{ I(V_1') \} & \quad \{ I(V_2') \} \\
V_1 & \quad V_2 \\
\{ I(V_3') \} & \quad \{ I(V_4') \} \\
V_3 & \quad V_4 \\
\{ I(V_5') \} & \quad \{ I(V_6') \} \\
V_5 & \quad V_6 \\
\{ I(V_7') \} & \quad \{ I(V_8') \} \\
V_7 & \quad V_8 \\
\{ I(V_9') \} & \quad \{ I(V_{10}') \} \\
\vdots & \quad \vdots \\
\{ I(V_2^{n-1+1}) \} & \quad \{ I(V_2^n) \}
\end{align*}

where

\begin{align*}
I(V_2') &= I(V_3') \\
I(V_4') &= I(V_5') \\
I(V_6') &= I(V_7') \\
I(V_{2^{n-1+1}}) &= I(V_2^n)
\end{align*}

Fig. 5: \( V_k \) is vector s.t \( \{ a_1, \ldots, a_i, \ldots, a_n \} | a_j = 0 \text{ or } 1 \) and \( V_k' \) represents m component vector whose components are \( a_j (m \leq n, 1 \leq j \leq n) \). \( I(V_k') \) shows an element of an arbitrary subset of \( V_k' \) and \( I(V_k') \) does of complementary set of \( I(V_k') \). Let's consider the situation that a series of oscillators are located in a dimension and each vertex is forced by neighbor oscillators receiving \( I(V_k') \) as
external value and the oscillator on the boundary is forced by external and it outputs periodic sequence. In R&A vertex sequence on the boundary is \( V_1 \cdots V_1 V_2 \) whatever external sequence is and external one for neighbor oscillator is \( I(V_1') \cdots I(V_1') I(V_2') \). Eventually the neighbor oscillator outputs the same sequence and oscillates as external value similarly. If both boundary oscillates in the same period, all of oscillators comes to oscillate in the same period because in R&A each dependence is dissolved by double loop. Consider the situation that each oscillator in one dimension is forced by one side neighbor and affects the other. If the oscillator on the boundary (site 1) is forced by external sequence \( I(V_1') \) then the neighbor (site 2) oscillates in period 2 and outputs sequence \( V_1 V_2 \). When \( I(V_2') = I(V_1') \) is assumed, the oscillators at site 3 is affected by \( I(V_1') I(V_2') \) and outputs \( V_1 V_2 V_4 V_4 V_6 \). When \( I(V_2') = I(V_1') \) is assumed, the oscillators at site 4 oscillates in period 8 forced by external sequence and the one at site 5 oscillates in period 16 and so on.

Fig. 6

---

Forced orbit

\[
\begin{array}{c}
V_1 \xymatrix{ \{1\} & V_2 \ar[r]^{\{0\}} & \{1\} \\
\{0\} & V_4 \ar[l]^{\{1\}} & \{1\} \\
\{0\} & V_6 \ar[r]^{\{1\}} & \{0\} \\
\{1\} & V_7 \ar[l]^{\{1\}} & \{0\} \\
\{0\} & V_8 \ar[r]^{\{1\}} & \{0\} \\
\end{array}
\]

Internal orbit

\[
\begin{array}{c}
V_1 \xymatrix{ \{1\} & V_2 \ar[r]^{\{0\}} & \{1\} \\
\{0\} & V_3 \ar[l]^{\{1\}} & \{1\} \\
\{0\} & V_5 \ar[r]^{\{1\}} & \{0\} \\
\{1\} & V_6 \ar[l]^{\{1\}} & \{0\} \\
\{0\} & V_7 \ar[r]^{\{1\}} & \{0\} \\
\{0\} & V_8 \ar[l]^{\{1\}} & \{0\} \\
\end{array}
\]

where \( V_1 = (000) \), \( V_2 = (111) \), \( V_3 = (001) \), \( V_4 = (100) \), \( V_5 = (101) \), \( V_6 = (010) \), \( V_7 = (110) \), \( V_8 = (011) \).

Fig. 6: Here in forced orbit \( I(V_1') \) as external value for neighbor oscillator is decided such that \( I(V_k') \) is 1 when \( a_1 \)'s value in \( V_k \) is 1 and \( I(V_k') \) is 0 when \( a_1 \)'s is 0. If the boundary oscillator (\( s = 1 \)) is forced by periodic sequence \( 01 \) which is oscillation of epithallium, then vertex sequence is \( V_1 V_3 V_4 V_2 \) and external sequence for the neighbor (\( a_1 \)'s periodic sequence) is 0011 which is of meristem. So, the neighbor at \( s = 2 \) outputs vertex sequence \( V_1 V_3 V_5 V_6 V_7 V_8 V_4 V_2 \) and then \( a_1 \)'s sequence 00101011, which expresses perithallial cell's self-maintenance. The oscillator at \( s = 2 \) affects other one at \( s = 3 \) by external sequence 00101011, and oscillator at \( s = 4 \) also outputs the same sequence as one at \( s = 3 \) and then the others at \( s = 4 \) oscillate similarly. As for each oscillator at site \( S(\geq 4) \), if the neighbor oscillator at \( s = S - 1 \) is absent, the orbit of the oscillator at \( s = S \) is changed from forced one to internal one. Whatever the removal value is, \( a_1 \)'s sequence is 01 and then the oscillator at \( s = S + 1 \) oscillates in period 4 and outputs \( a_1 \)'s value 0011. Then the oscillator \( s = S + 2 \) oscillates in period 8 as it did and the very oscillator \( s = S + 1 \) outputs the oscillation of meristem.