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The Resale-Proof Trades of Information as a Stable Standard of Behavior

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1. Introduction.

The notion of a resale-proof trade has been introduced in [10] to formulate a trade of information under the following environments: resales of information are freely allowed and players can freely communicate and make agreements with each other; but agreements are not binding. The agreement on never reselling acquired information was called *self-enforcing* if no single player has an incentive to violate it when every other player keeps it. A resale-proof coalition is one in which the agreement can be self-enforcing. It turned out that any minimal-size resale-proof coalition attains the maximal aggregate profit over all resale-proof coalitions, and that the original seller can exploit the surplus produced by selling information to the buyers in the coalition (see, [10,9]). This is the main property of the resale-proof trade.

The same result has been obtained so far in a variety of game models. The core is adopted in the coalitional form in [10,9]; and the bargaining set is considered in Muto and Nakayama [8] with appropriate modifications to that of Aumann and Maschler [1]. In a noncooperative model in Muto [7], in which trading is formulated as an extensive-form game, a modified version of his previous model [6], the coalition-proof Nash equilibrium due to Bernheim, Peleg and Whinston [2] has been applied. In the present paper, we shall characterize the resale-proof trade as a stable standard of behavior in the theory of social situations of Greenberg [3,4].

The self-enforcing agreement in a resale-proof trade is a condition which was exogenously imposed on the behavior of players. A behavioral basis from which it can be derived has been lacking thus far. In the extensive form in [7], this condition holds at an equilibrium; but the analysis required an additional assumption on a trading manner which was unnecessary in the coalitional-form approach. In contrast, the theory of social situations does not require any such assumptions on the behavior of players. It only requires us to describe a model in the form of situation. The behavior of players can then be analyzed from a single, basic standpoint, the stable standard of behavior. We will show that trading in a minimal-size resale-proof coalition with the self-enforcing agreement is derived as a unique stable standard of behavior for our information trading situation. In addition, one may show that it is also given as a unique vN-M stable set for an abstract system associated with our information trading situation. Thus the approach to be taken in this paper will overcome the shortcoming remained in the former analyses.

There is another point in the present model of information trading. Information is assumed, as in [7, 8, 9 and 10], to have the property that the profit from it is non-increasing in the number of its holders. Besides, it is assumed here that a profit to a non-holder is also non-increasing in the number of its holders. Since a technological innovation can be thought of as a typical example possessing this property (see, e.g., [5]), this assumption may extend the scope of our analysis. In addition, the above mentioned result in Muto [7] crucially depends on the assumption that the profit to a non-holder is invariantly zero, so that his extensive-form game may not provide a basis for considering the resale-proof trade of information with the above property.

In the next section, we state assumptions and rules on the trading, define strategies and the resale-proof trade, and state a result characterizing the resale-proof trade. In section 3, the trading model is set out in the form of a social situation, and the main theorem is proved. Finally, in section 4, we conclude with several remarks including the abstract vN-M stable set.

2. The Resale-Proof Trade of Information

Let $N=\{1,2,\ldots,n\}$ be the set of all players, where player 1 is a seller of information and players $2,\ldots,n$ are buyers. The information held by player 1 is completely replicable without costs, so that for any player it is useless to acquire more than one unit of it. Every player has a profit function depending on how many players have acquired the information. When the information is held by h players, the profit to each holder and nonholder is given by $W(h)$ and $L(h)$, respectively, in terms of money. $W(h)$ and $L(h)$ satisfy the following conditions:

Assumption 2.1 (i) $W(h) > L(h)$ for all $h=1,2,\ldots,n-1$.

(ii) $W(1)\geqq W(2)\geqq\cdots\geqq W(n)\geqq 0$ and $L(1)\geqq L(2)\geqq\cdots\geqq L(n-1)\geqq 0$.

Thus the profit of each holder is uniformly greater than that of each non-holder, and each profit is non-increasing in the number of holders. We assume that values of $W(h)$ and $L(h)$ are common knowledge.

We now describe trading environments. There exists no external authority that can regulate and enforce agreements on trading information. Reproduction and resales of the information are free. Players can communicate and make agreements with each other. In particular, the seller and buyers can agree, in advance, not to resell the information, but the agreement is not binding. Under such environments and Assumption 2.1, the seller may seek to make a self-enforcing agreement which maximizes his total profit. The notion of resale-proofness has been proposed to deal with this problem (see [10]).

Before stating its definition in detail, strategic opportunities open to players need to be given. Let $1 \in H \subseteq N$. When we assume that every $i \in H$ has acquired the information, we express this as state (H). Then, at state (H), player $i \in H$ can choose buyers and a uniform price; and the designated buyers decide to buy or not at that price. Formally, local strategies at state (H) are given as follows.

Decision s_i of player $i \in H$:

$$s_i \in S_i(H) := \{\phi\} \cup \{(T_i, p_i) \mid \phi \neq T_i \subseteq N-H,\ p_i \in [0, \infty)\},$$

where $s_i = \phi$ means that i does not sell.

Put $s_H := (s_i)_{i \in H}$ and $J(s_H) := \{j \in N-H \mid j \in T_i \text{ for some } s_i \neq \phi\}$.

Decision $b_j(s_H)$ of player $j \in J(s_H)$, given s_H with $J(s_H) \neq \phi$:

$$b_j(s_H) \in B_j(s_H) := \{0\} \cup \{i \in H \mid j \in T_i\},$$

where $b_j(s_H) = 0$ means that j does not buy.

Put $b(s_H) := (b_j(s_H))_{j \in J(s_H)}$.

Thus, for each player $i \in N$, local strategies are defined dependent on state (H).

Let $(s_H, b(s_H))$ be a local strategy combination at state (H), and define

$$T(s_H, b(s_H)) := \bigcup_{i \in H^+} T_i,$$

where $H^+ := \{i \in H | s_i \neq \phi$, and $b_j(s_H) = i$ for all $j \in T_i\}$.

Then, H^+ is the set of those players in H whose buyers all agree to buy. When $H^+ = \phi$, we let $T(s_H, b(s_H)) = \phi$. Note that every T_i with $i \in H^+$ is disjoint with each other. We assume the rule that $T(s_H, b(s_H))$ is the set of new holders, that is, that any player $i \in H$, having chosen $T_i \neq \phi$, does not carry out the trade unless all buyers in T_i agree to buy. Buyers in T_i are assumed to buy when and only when:

$$W(|H \cup T(s_H, b(s_H))|) - p_i \geqq L(|H|)$$

where $|\cdot|$ denotes the number of players in a set.

The special local strategies s°_H at state (H) is called the <u>agreement at (H) not to resell the information</u>, or the agreement, for short, if $s^\circ_i = \phi$ for all $i \in H$. Although $b(s_H)$ is undefined at $s_H = s^\circ_H$, we write, for convenience, that $s^\circ_H = (s^\circ_H, b(s^\circ_H))$ for all $b_j(\cdot)$ and $j \in N-H$. Note that the agreement s°_H is tentative in nature, since it cannot be binding.

Let $u_i(s^\circ_H)$ denote the tentative payoff to i at state (H).[1] For $i \in N-H$, let $u_i(s^\circ_H) = L(|H|)$. We shall say the agreement s°_H at (H) is <u>self-enforcing</u> if no player $i \in H$ has an incentive to deviate from s°_H by choosing a set of buyers $T_i \neq \phi$ such that the agreement at state $(H \cup T_i)$ becomes self-enforcing. Formally, we define the self-enforcing agreement in a backward inductive way as follows:

<u>Definition 2.2</u>. Let $1 \in H \subseteq N$, and let s°_H be the agreement at (H). Then: (i) If H=N, we say s°_H is self-enforcing. (ii) Suppose

the definition is completed for all H with $|H|\geqq h+1$. Then for H with $|H|=h$, we say $s°_H$ is self-enforcing iff

$$u_i(s°_H) \geqq u_i(s°_{H\cup T}) \quad ^{2)}$$

$$\text{where } T=T(s°_{H-\{i\}},s_i,b(s°_{H-\{i\}},s_i)),$$

for all $i\in H$, for all $s_i\in S_i(H)$ and for all $b(s°_{H-\{i\}},s_i)$ such that the agreement $s°_{H\cup T}$ at $(H\cup T)$ is self-enforcing.

This definition includes the case where $s°_1$ itself is self-enforcing. This is the case where the original seller does not sell to any buyer. When $s°_1$ is not self-enforcing, the definition states that there exist $s_1=(T_1,p_1)\neq s°_1$ and $b(s_1)$ such that for $H=\{1\}\cup T(s_1,b(s_1))$, $s°_H$ is self-enforcing and $u_1(s°_1)<u_1(s°_H)$. Note that $T(s_1,b(s_1))=T_1$, because $T(s_1,b(s_1))\neq\phi$ and our rule imply that $b_j(s_1)=1$ for all $j\in T_1$. Then, if $s°_1$ is not self-enforcing, the seller may choose such $s_1=(T_1,p_1)$ that maximizes his payoff.

Definition 2.3. A local strategy combination $(s^*_1,b^*(s^*_1))$ at state $(\{1\})$ is a resale-proof trade iff for $H^*=\{1\}\cup T(s^*_1,b^*(s^*_1))$, $s°_{H*}$ is self-enforcing and

$$u_1(s°_{H*}) \geqq u_1(s°_H)$$

for all $(s_1,b(s_1))$ such that for $H=\{1\}\cup T(s_1,b(s_1))$, $s°_H$ is self-enforcing.

When $s°_1$ is self-enforcing, the unique resale-proof trade $(s°_1, b(s°_1))$ is an empty trade.

We conclude this section by showing a theorem that states that in a resale-proof trade the seller chooses the minimum number of buyers necessary to form the self-enforcing agreement, and sells to them at the maximum price. Let $\mathscr{H}:=\{H\subseteq N \mid 1\in H$ and s°_H is self-enforcing$\}$. $\mathscr{H}$ is nonempty since $N\in\mathscr{H}$. Then:

Theorem 2.4. Let $H^*\in\mathscr{H}$ satisfy that $|H^*|\leqq|H|$ for all $H\in\mathscr{H}$. Then, a local strategy combination $(s^*_1, b^*(s^*_1))$ at state $(\{1\})$ is a resale-proof trade iff

(i) $H^*=\{1\}$ and $s^*_1=s^\circ_1$, or

(ii) $\{1\}\subset H^*, T^*_1=H^*-\{1\}$, $p^*_1=W(|H^*|)-L(1)$ and $b^*_j(s^*_1)=1$ for all $j\in T^*_1$.

To show this, we need the following two lemmas.

Lemma 2.5. $H\in\mathscr{H}$ iff $W(|H|)+|T|L(|H|) \geqq (1+|T|)W(|H\cup T|)$ for all $T\subseteq N-H$ such that $H\cup T\in\mathscr{H}$.

Lemma 2.6. Let $H, H'\in\mathscr{H}$ satisfy that $\{1\}\subseteq H\subset H'$. Then,

$$|H|W(|H|)-(|H|-1)L(1) > |H'|W(|H'|)-(|H'|-1)L(1).$$

The proofs of these lemmas will be proved in the appendix. Lemma 2.5 is a re-statement of the inequality in Definition 2.2 in terms of $W(\cdot)$, $L(\cdot)$ and $p_i=W(|H\cup T|)-L(|H|)$, the maximal price that i can require at (H). Lemma 2.6 states that $H\in\mathscr{H}$ with a smaller size gives a greater maximal payoff to 1. Theorem 2.4 can be proved from these lemmas.

Proof of Theorem 2.4. If $s^\circ_1=s^*_1$ is self-enforcing, then $(s^*_1, b^*(s^*_1))$ is the unique resale-proof trade. Otherwise, there exist $s_1=(T_1,p_1)\neq s^\circ_1$ and $b(s_1)$ such that $H=\{1\}\cup T(s_1,b(s_1))\in\mathscr{H}$ and that $u_1(s^\circ_1) < u_1(s^\circ_H)$. It suffices to show that $s^*_1=(T^*_1,p^*_1)$ gives the maximum of $u_1(s^\circ_H)$. Assumption 2.1 and Lemma 2.5 imply that $0 \leqq W(1)-W(|\{1\}\cup T|)<|T|(W(|\{1\}\cup T|)-L(1))$ for all $\{1\}\cup T \in \mathscr{H}$. Hence, $0 \leqq p_1 \leqq W(|H|)-L(1)$ for all $s_1=(T_1,p_1)$ such that $H=\{1\}\cup T(s_1,b(s_1))\in\mathscr{H}$ because $u_1(s^\circ_H)-u_1(s^\circ_1)=W(|H|)-p_1-L(1)\geqq 0$ for all $i\in T(s_1,b(s_1))$. Then, Lemma 2.6 implies that for all such $s_1=(T_1,p_1)$ we have

$$
\begin{aligned}
u_1(s^\circ_H) &= W(|H|)+(|H|-1)p_1 \\
&\leqq W(|H|)+(|H|-1)(W(|H|)-L(1)) \\
&= |H|W(|H|)-(|H|-1)L(1) \\
&\leqq |H^*|W(|H^*|)-(|H^*|-1)L(1) \\
&= W(|H^*|)+(|H^*|-1)p^*_1 \\
&= u_1(s^\circ_{H^*})
\end{aligned}
$$

Hence, $u_1(s^\circ_H)$ is maxmized at $s^*_1=(T^*_1,p^*_1)$, which completes the proof.

3. The Stable Standard of Behavior

The resale-proof trade is defined to obtain the trade in which the payoff to the seller is maximal in all such trades that the agreement with the buyers is self-enforcing. This is an assumption we made on the behavior of the seller and buyers. In this section, we shall base our argument on the theory of socialsituations due to Greenberg [3] and show that the resale-proof trade can be derived from a standard of behavior. We only review below a part of the theory necessary for the later discussion. For the details of the theory, refer to Greenberg [3].

The theory of social situations first requires us to completely describe the environment as a <u>situation</u> (γ,Γ), where Γ is the set of <u>positions</u> and γ is a mapping called <u>the inducement correspondence</u>. A position $G\in\Gamma$ describes "the current state of affairs" in terms of a triple $(N(G), X(G), \{u_i(G)\}_{i \in N(G)})$, where $N(G)$ is the set of players, $X(G)$ is the set of all potentially feasible outcomes, and $u_i(G)$ is the utility function of player i <u>in position G</u> over the outcomes. The inducement correspondence γ specifies, for each $G \in \Gamma$, $x \in X(G)$ and $S \subseteq N(G)$, the set $\gamma(S|G,x) \subseteq \Gamma$ of alternative positions that coalition S can induce when it rejects x in position G. Thus, a situation (γ, Γ) gives a complete specification of the social interactions.

We now describe a situation representing our information trading, called hereafter <u>the information trading situation</u>, or the IT situation for short. The IT situation is similar, at least in the spirit, to what is called in Greenberg [3] <u>the individual contingent threats situation</u>. The IT situation describes an open negotiation process, in which every single individual is free to object to a currently proposed outcome, and the outcome is adopted only when all individuals concent to accept it.

Recall that in our trading there is only one seller, player 1. Therefore, a set of all potentially feasible outcomes available to all players corresponds to the set of all agreements between player 1 and buyers. Thus, initially, we define the position:

$G_N = (N(G_N), X(G_N), \{u_k(G_N)\}_{k \in N(G_N)})$, where

$N(G_N) = N$,

$X(G_N) = \{s^\circ_{\{1\} \cup T} \mid s_1 = (T_1, p_1) \in S_1(\{1\})\}$,

where $T = T(s_1, b(s_1))$,

$u_k(G_N)(s^\circ_{\{1\} \cup T}) = u_k(s^\circ_{\{1\} \cup T})$ for all $k \in N(G_N)$.

For each outcome $s^\circ_{\{1\} \cup T} \in X(G_N)$ in position G_N, every player $i \in \{1\} \cup T$ speculates if there is a resale at state $(\{1\} \cup T)$ that makes i better off. Let $(s_i|H)$ represent that at state (H) player i proposes to choose $s_i \in S_i(H)$. Then, for all $(s_i|H)$ such that

$$T(s^\circ_{H-\{i\}}, s_i, b(s^\circ_{H-\{i\}}, s_i)) \neq \phi$$

we define the position:

$$G_i(s_i|H) = (N(G_i(s_i|H)), X(G_i(s_i|H)), \{u_k(G_i(s_i|H))\}_{k \in N(G_i(s_i|H))}), \quad \text{where}$$

$$N(G_i(s_i|H)) = N,$$

$$X(G_i(s_i|H)) = \{s^\circ_{H \cup T}\},$$

$$\text{where } T = T(s^\circ_{H-\{i\}}, s_i, b(s^\circ_{H-\{i\}}, s_i)),$$

$$u_k(G_i(s_i|H))(s^\circ_{H \cup T}) = u_k(s^\circ_{H \cup T})^{3)} \text{for all } k \in N(G_i(s_i|H)).$$

Thus, $G_i(s_i|H)$ is a position describing that at state (H) player $i \in H$ openly declares that he will choose $s_i \in S_i(H)$ and generate the alternative outcome $s^\circ_{H \cup T}$ if every other player $j \in H$ sticks to s°_j. Note that the outcome set of every position $G_i(s_i|H)$ is a singleton, and that $G_i(s_i|H)$ is undefined for $H=N$, $s_i = s^\circ_i$ or $s_i = (T_i, p_i)$ with p_i large enough.

Put $\Gamma := \{G_N\} \cup \{G_i(s_i|H) \mid 1 \in H \subset N,\ i \in H,\ s_i \in S_i(H)$ and $T(s^\circ_{H-\{i\}}, s_i, b(s^\circ_{H-\{i\}}, s_i)) \neq \phi\}$.

Then, the inducement correspondence $\gamma \subseteq \Gamma$ is defined for all $G \in \Gamma$ as follows:

$$\gamma(\{j\}|G_N, s^\circ_{\{1\} \cup T})$$

$$= \{G_1(s_1|\{1\}) \mid s_1 \in S_1(\{1\})\} \cup \{G_1(s_1|\{1\} \cup T) \mid s_1 \in S_1(\{1\} \cup T)\} \qquad \text{if } j=1,$$

$$= \{G_j(s_j|\{1\} \cup T) \mid s_j \in S_j(\{1\} \cup T)\} \qquad \text{if } j \in T,$$

$$= \phi \qquad \text{if } j \notin \{1\} \cup T,$$

$$\gamma(\{j\}|G_i(s_i|H), s^\circ_{H \cup T})$$

$$= \{G_j(s_j|H \cup T) \mid s_j \in S_j(H \cup T)\} \qquad \text{if } j \in H \cup T,$$

$$= \phi \qquad \text{if } j \in H \cup T,$$

and for all $G \in \Gamma$, $x \in X(G)$ and $S \subseteq N(G)$ with $|S|>1$, we define $\gamma(S|G,x)=\phi$.

In the IT situation (γ,Γ), the irreversibility inherent in information trading is embodied in the inducement correspondence γ. Namely, for any outcome $s^\circ_{H \cup T}$, every player j in $H \cup T$ can only induce positions $G_j(s_j|H \cup T)$ taking as given the state $(H \cup T)$. This is so because when reselling the information every player must assume that every other player in $H \cup T$ has acquired it. Only player 1, the original seller, can induce positions $G_1(s_1|\{1\})$ from G_N and revise his proposal $s^\circ_{\{1\} \cup T}$ made in G_N. For this reason, our IT situation is not the same to the individual contingent threats situation in Greenberg [3].

Another important distinction is that the IT situation is hierarchical (Greenberg [3, p.43, Definition 5.11]). [4] In fact, letting $\Gamma_0=\{G_N\}$ and $\Gamma_h=\{G_i(s_i|H) \mid |H|=h\}$, $h=1,\ldots,n-1$, one may show that the following two conditions characterizing the hierarchical situation are satisfied:

H.1. For each $h \in \{0,1,\ldots,n-1\}$ and $G \in \Gamma_h$, $(\gamma, \{G\} \cup \Gamma_{h+1} \cup \cdots \cup \Gamma_n)$ is a situation, where $\Gamma_n := \phi$.

H.2. For every $G \in \Gamma$, if $G \in \gamma(S|G,x)$ for some $x \in X(G)$, then S is unique in G.

We now turn to the solution theory. Let (γ,Γ) be the IT situation. For each position $G \in \Gamma$, a subset $\sigma(G)$ of $X(G)$ is called a solution for the position G, and a mapping σ that assigns to each $G \in \Gamma$ a solution $\sigma(G)$ is called a standard of behavior for Γ, or SB, for short. An SB σ for Γ is said to be internally stable for (γ,Γ) if for all $G \in \Gamma$, $x \in \sigma(G)$ implies that there do not exist $j \in N(G)$, $G' \in$

$\gamma(\{j\}|G,x)$ and $y\in\sigma(G')$ such that $u_j(G')(y)>u_j(G)(x)$; and it is said to be <u>externally stable for (γ,Γ)</u> if for all $G\in\Gamma$, $x\in X(G)-\sigma(G)$ implies that there exist $j\in N(G)$, $G'\in\gamma(\{j\}|G,x)$ and $y\in\sigma(G')$ such that $u_j(G')(y)>u_j(G)(x)$. An SB σ for Γ is said to be <u>stable</u> if it is both internally and externally stable.

We are now ready to state our main result.

<u>Theorem 3.1</u> There exists a unique stable SB σ for the IT situation (γ,Γ). The stable SB σ is given by the following (i) and (ii).

(i) $\sigma(G_N)=\{s^\circ{}_{H*}\in X(G_N)|(s^*{}_1,b^*(s^*{}_1))$ is a resale-proof trade$\}$.

(ii) For each $G_i(s_i|H)\in\Gamma-\{G_N\}$,

$$\begin{aligned}\sigma(G_i(s_i|H))&=\{s^\circ{}_{HUT}\} && \text{if } s^\circ{}_{HUT} \text{ is self-enforcing,}\\ &=\phi && \text{otherwise.}\end{aligned}$$

<u>Proof</u>. By Theorem 5.2.1 in Greenberg [3], if there exists a stable SB for a hierarchical situation, then it is unique. Hence, it suffices to show that the SB σ given by (i) and (ii) is in fact a stable SB.

Internal stability: Let $s^\circ{}_{H*}=s^\circ{}_{\{1\}UT*}\in\sigma(G_N)$. Suppose that there exist $j\in\{1\}UT^*$, $G=G_j(s_j|\{1\}UT^*)$ and $s^\circ{}_{\{1\}UT*UR}\in\sigma(G)$ such that $u_j(G)(s^\circ{}_{\{1\}UT*UR})>u_j(G_N)(s^\circ{}_{\{1\}UT*})$, where $R=T(s^\circ{}_{(\{1\}UT*)-\{j\}}, s_j,b(s^\circ{}_{(\{1\}UT*)-\{j\}},s_j))$. Then:

$$u_j(s^\circ{}_{\{1\}UT*UR}) > u_j(s^\circ{}_{\{1\}UT*}).$$

Since $s^\circ{}_{\{1\}UT*}$ is self-enforcing by assumption, $s^\circ{}_{\{1\}UT*UR}$ must not be self-enforcing. But this contradicts (ii). If for $j=1$ there exist $G=G_1(s_1|\{1\})\in\gamma(\{1\}|G_N,s^\circ{}_{\{1\}UT*})$ and $s^\circ{}_{\{1\}UQ}\in\sigma(G)$ such that $u_1(G)(s^\circ{}_{\{1\}UQ})>u_1(G_N)(s^\circ{}_{\{1\}UT*})$, where $Q=T(s_1,b(s_1))$, then we have:

$$u_1(s^\circ_{\{1\}\cup Q}) > u_1(s^\circ_{\{1\}\cup T*}).$$

But this contradicts the maximality of the resale-proof trade $(s^*_1, b^*(s^*_1))$.

Next, let $x \in \sigma(G_i(s_i|H))$. Then we can show in a similar way as the former part of the proof above that there exist no $j \in N(G_i(s_i|H))$, $G \in \gamma(\{j\}|G_i(s_i|H), x)$ and $y \in \sigma(G)$ such that $u_j(G)(y) > u_j(G_i(s_i|H))(x)$. This completes the proof that the SB σ is internally stable.

External stability: Let $s^\circ_{\{1\}\cup T} \in X(G_N) - \sigma(G_N)$, where $T = T(s_1, b(s_1))$. Then, $(s_1, b(s_1))$ is not a resale-proof trade. Then, either one of the following (a) and (b) must be true:

(a) $s^\circ_{\{1\}\cup T}$ is not self-enforcing.

(b) $s^\circ_{\{1\}\cup T}$ is self-enforcing, but $u_1(s^\circ_{\{1\}\cup T})$ is not maximal over all $u_1(s^\circ_{\{1\}\cup Q})$ such that $s^\circ_{\{1\}\cup Q}$ is self-enforcing, where $Q = T(s'_1, b'(s'_1))$.

If (a) is true, then by definition there exsit $j \in \{1\} \cup T$ and $s''_j \in S_j(\{1\} \cup T)$ such that $u_j(s^\circ_{\{1\}\cup T\cup R}) > u_j(s^\circ_{\{1\}\cup T})$, where $s^\circ_{\{1\}\cup T\cup R}$ is self-enforcing and $\phi \neq R = T(s^\circ_{(\{1\}\cup T)-\{j\}}, s''_j, b''(s^\circ_{(\{1\}\cup T)-\{j\}}, s''_j))$. But this implies that j can induce a position $G'' = G_j(s''_j|\{1\} \cup T) \in \gamma(\{j\}|G_N, s^\circ_{\{1\}\cup T})$ such that for $s^\circ_{\{1\}\cup T\cup R} \in \sigma(G'')$:

$$u_j(G'')(s^\circ_{\{1\}\cup T\cup R}) > u_j(G_N)(s^\circ_{\{1\}\cup T}).$$

If (b) is true, then for j=1 there exists $s'_1 \in S_1(\{1\})$ such that $u_1(s^\circ_{\{1\}\cup Q}) > u_1(s^\circ_{\{1\}\cup T})$ and $s^\circ_{\{1\}\cup Q}$ is self-enforcing, where $\phi \neq Q = T(s'_1, b'(s'_1))$. Hence 1 can induce a position $G' = G_1(s'_1|\{1\})$ such that for $s^\circ_{\{1\}\cup Q} \in \sigma(G')$:

$$u_1(G')(s^\circ_{\{1\}\cup Q}) > u_1(G_N)(s^\circ_{\{1\}\cup T}).$$

Next, let $s^\circ_{H\cup T} \in X(G_i(s_i|H)) - \sigma(G_i(s_i|H))$. Then, $s^\circ_{H\cup T}$ is not self-enforcing. The rest of the proof is similar to the case (a) above. Hence the SB σ is externally stable, and it is the unique stable SB.

4. Concluding Remarks

As we have argued in [10], modelling information trading as a game in extensive form is not the only way to analyze it without a basis of binding agreements or contracts. Any extensive-form trading, modelled on a take-it-or-leave-it basis in particular, can easily lead to a price competition once the replicable information is sold at least to one buyer, benefitting only final buyers (see, e.g., Muto [6]). In contrast, the resale-proof trade provides a more sensible outcome at least to the seller, or the innovator under the environment where legal protections are absent or imperfect.

The theory of social situations has proved useful in locating our solution concept on a game-theoretical construct. We have shown that the resale-proof trade is derived as a unique stable standard of behavior for our information trading situation. The IT situation is only a special case of social situations;yet we believe it is a sensible application of the theory.

It should also be noted that in any social situation, its stable standard of behavior can be characterized as a vN-M abstract stable set for a system associated with the situation (Greenberg [3, p.38, Theorem 4.5]). The abstract system, denoted by $(D, \angle)$, can be defined for the IT situation (γ, Γ) as follows: D is the set of all pairs of possible positions and their outcomes, i.e.,

$$D = \{(G_N, s^\circ_{\{1\}\cup T_1}) \mid s_1 \in S_1(\{1\})\}$$
$$\cup\{(G_i(s_i \mid H), s^\circ_{H\cup T} \mid \{1\} \subseteq H \subset N,\ i \in H,\ s_i \in S_i(H)\},$$

where $T_1 = T(s_1, b(s_1))$ and $T = T(s^\circ_{H-\{i\}}, s_i, b(s^\circ_{H-\{i\}}, s_i)) \neq \phi$. The domination relation $\angle$ over the set D can be given by:

$(G, x) \angle (G', x')$ iff there exists an $i \in N$ such that
$$G' \in \gamma(\{i\} \mid G, x) \text{ and } u_i(G')(x') > u_i(G)(x).$$

Then, we can conclude from the above Greenberg's theorem that the resale-proof trades and self-enforcing agreements are obtained as a vN-M abstract stable set of the system $(D, \angle)$.

Finally, we conclude with a remark on Nash equilibria and the resale-proof trade. Greenberg has shown that given a game in strategic form and the ICT (individual contingent threats) situation constructed on it, if the set of Nash equilibria is nonempty, it is contained in the solution $\sigma(G^N)$ where G^N is the (grand) position in his notation [3, Theorem 7.4.1]. A similar relation holds in our IT situation. To see this, let $S^\circ := \{s^\circ_H \mid u_i(s^\circ_H) \geqq u_i(s^\circ_{H\cup T})$ for all $i \in H$ and $T = T(s^\circ_{H-\{i\}}, s_i, b(s^\circ_{H-\{i\}}, s_i))\}$. Then, we say a local strategy combination $(s_1, b(s_1))$ is a Nash equilibrium if $s^\circ_H \in S^\circ$ for $H = \{1\} \cup T(s_1, b(s_1))$; and

$$u_1(s^\circ_H) \geqq u_1(s^\circ_K) \text{ for all } s^\circ_K \in S^\circ,$$
$$\text{where } K = \{1\} \cup T(s'_1, b'(s'_1)).$$

Note that $s^\circ_{H\cup T}$ is not restricted here to the class of self-enforcing agreements. Thus, if $(s^*_1, b(s^*_1))$ is a Nash equilibrium, it is a resale-proof trade; i.e., $s^\circ_{H*} \in \sigma(G_N)$ for $H^* = \{1\} \cup T(s^*_1, b(s^*_1))$. In terms of $W(\cdot)$ and $L(\cdot)$, we can show that $s^\circ_H \in S^\circ$ iff

$$W(|H|) + |T| L(|H|) \geqq (1 + |T|) W(|H \cup T|) \text{ for } \underline{\text{all}}\ T \subseteq N - H.$$

However, this condition is rather stringent since it requires that the value $W(\cdot)$ diminish rapidly beyond the size $|H|$. The following numerical example illustrates the case in which the resale-proof trade is not a Nash equilibrium:

$n=5$; $W(1)=17$, $W(2)=15$, $W(3)=13$, $W(4)=10$ and $W(5)=8$; and

$L(1)=11$, $L(2)=7$, $L(3)=4$, $L(4)=2$.

It is easy to see that $W(|H|)+(5-|H|)L(|H|)<(1+[5-|H|])W(5)$ for $|H|=4$ and 3. Hence, s°_H is not self-enforcing because s°_N is self-enforcing. Then, letting $H^*=\{1,i\}$ for $i\in N-\{1\}$, s°_{H*} is self-enforcing because $W(2)+3L(2)\geq 4W(5)$. Thus, $s^*_1=(\{i\}, W(2)-L(1))$ generates a resale-proof trade, since $W(1)+L(1)<2W(2)$. But, this resale-proof trade is not a Nash equilibrium because $s^\circ_{H*}\notin S^\circ$, i.e., $W(2)+tL(2)<(1+t)W(2+t)$ for $t=1$ or 2.

Appendix

Lemma 2.5. $H\in\mathscr{H}$ iff $W(|H|)+|T|L(|H|) \geq (1+|T|)W(|H\cup T|)$ for all $T\subseteq N-H$ such that $H\cup T\in\mathscr{H}$.

Proof. Assume that $W(|H|)+|T|L(|H|)<(1+|T|)W(|H\cup T|)$ for some T such that $H\cup T\in\mathscr{H}$. Note that $T\neq\phi$. For $i\in H$, define $s_i=(T,p)$ where

$$p=[W(|H|)-L(|H|)]/(1+|T|)\geq 0.$$

Then, for each $j\in T$,

$$\begin{aligned} u_j(s^\circ_{H\cup T}) &= W(|H\cup T|)-p \\ &= [(1+|T|)W(|H\cup T|)-W(|H|)+L(|H|)]/(1+|T|) \\ &> L(|H|) = u_j(H|s^\circ_H). \end{aligned}$$

Hence $b_j(s^\circ_{H-\{i\}},s_i)=i$, or $T=T(s^\circ_{H-\{i\}},s_i,b(s^\circ_{H-\{i\}},s_i))$.

Then,

$$u_i(s^\circ_H)-u_i(s^\circ_{H\cup T})$$

$$= W(|H|)-[W(|H\cup T|)+|T|p]$$
$$= [(1+|T|)(W(|H|)-W(|H\cup T|)) - |T|(W(|H|)-L(|H|))]/(1+|T|)$$
$$= [W(|H|)+|T|L(|H|)-(1+|T|)W(|H\cup T|)]/(1+|T|)$$
$$< 0.$$

Hence, s°_H is not self-enforcing, implying that $H\notin\mathscr{H}$.

Conversely, assume that $H\notin\mathscr{H}$. Then, $u_i(s^\circ_H)-u_i(s^\circ_{H\cup T})<0$ for some $i\in H$ and for some $s_i=(T,p)$ such that $H\cup T\in\mathscr{H}$. Since $\phi\neq T= T(s^\circ_{H-\{i\}},s_i,b(s^\circ_{H-\{i\}},s_i))$, $b_j(s^\circ_{H-\{i\}},s_i)=i$ for all $j\in T$. But,

$$0 > u_i(s^\circ_H) - u_i(s^\circ_{H\cup T}) = W(|H|) - [W(|H\cup T|)+|T|p],$$

which implies

$$p > [W(|H|)-W(|H\cup T|)]/|T|$$
$$\geq [(1+|T|)W(|H\cup T|)-|T|L(|H|)-W(|H\cup T|)]/|T|$$
$$= W(|H\cup T|)-L(|H|).$$

Then, for all $j\in T$, we have

$$u_j(s^\circ_{H\cup T}) = W(|H\cup T|)-p$$
$$< L(|H|) = u_j(s^\circ_H).$$

Hence $b_j(s^\circ_{H-\{i\}},s_i)=0$, which is a contradiction. Therefore $H\in\mathscr{H}$.

Lemma 2.6. Let $H,H'\in\mathscr{H}$ satisfy that $\{1\}\subseteq H\subset H'$. Then,

$$|H|W(|H|)-(|H|-1)L(1) > |H'|W(|H'|)-(|H'|-1)L(1).$$

Proof. Since $H,H'\in\mathscr{H}$ and $H\subset H'$, it follows from Lemma 2.5 that

$$W(|H|)+|H'-H|L(|H|) \geq (1+|H'-H|)W(|H'|).$$

Then, by the monotonicity of $W(\cdot)$ and $L(\cdot)$, we have

$$|H|W(|H|) - (|H|-1)L(1) - [|H'|W(|H'|)-(|H'|-1)L(1)]$$
$$= W(|H|) + (|H|-1)W(|H|)$$

$$
\begin{aligned}
&\qquad\qquad - |H'|W(|H'|) + (|H'|-|H|)L(1) \\
&\geqq W(|H|) + (|H|-1)W(|H'|) \\
&\qquad\qquad - |H'|W(|H'|) + (|H'|-|H|)L(1) \\
&= W(|H|) - (1+|H'|-|H|)W(|H'|) + (|H'|-|H|)L(1) \\
&\geqq -|H'-H|L(|H|) + (|H'|-|H|)L(1) \\
&= (|H'|-|H|)(L(1)-L(|H|)) \\
&\geqq 0.
\end{aligned}
$$

All the inequalities become the equality only when $L(1)=L(|H|)=W(|H|)=W(|H'|)$. But, by Assumption 2.1, $W(|H|)>L(|H|)$ for all H ($1\in H\subseteq N$). Hence the inequality must be strict, and the desired inequality follows.

Footnotes

1) The value of $u_i(s^\circ{}_H)$ depends upon the way how the members in H has come to share the information. For example, if $H=\{1\}\cup T(s_1,b(s_1))$ for some $s_1=(T_1,p_1)$ then $u_i(s^\circ{}_H)=W(|H|)+|T(s_1,b(s_1))|p_1$ if $i=1$; and $u_i(s^\circ{}_H)=W(|H|)-p_1$ if $i\in T(s_1,b(s_1))$. In general, $u_i(s^\circ{}_H)=W(|H|)+q_i$ for some $q=(q_i)_{i\in H}$ where $\Sigma_{i\in H}q_i=0$. However, this insufficiency of the notation $s^\circ{}_H$ causes no problem in the analysis. See also footnote 2).

2) Since $T=T(s^\circ{}_{H-\{i\}},s_i,b(s^\circ{}_{H-\{i\}},s_i))$, i is the only player to resell at state (H). Hence, $u_i(s^\circ{}_{H\cup T})-u_i(s^\circ{}_H)=W(|H\cup T|)+|T|p_i-W(|H|)$ so that $u_i(s^\circ{}_H)\geqq u_i(s^\circ{}_{H\cup T})$ iff $W(|H|)\geqq W(|H\cup T|)+|T|p_i$. This shows that the self-enforcingness of $s^\circ{}_H$ is determined independently of how payoffs have been made up to state (H).

3) Since $T=T(s^\circ{}_{H-\{i\}},s_i,b(s^\circ{}_{H-\{i\}},s_i))\neq\phi$, $u_k(s^\circ{}_{H\cup T})$ is given

by:

$$
\begin{aligned}
u_k(s^\circ_{H\cup T}) &= u_i(s^\circ_H) - W(|H|) + W(|H\cup T|) + |T|p_i && \text{for } k=i, \\
&= u_k(s^\circ_H) - W(|H|) + W(|H\cup T|) && \text{for } k\in H-\{i\}, \\
&= u_k(s^\circ_H) - L(|H|) + W(|H\cup T|) - p_i = W(|H\cup T|) - p_i && \text{for } k\in T, \\
&= u_k(s^\circ_H) - L(|H)| + L(|H\cup T|) = L(|H\cup T|) && \text{for } k\in N-(H\cup T).
\end{aligned}
$$

4) The authors wish to acknowledge a referee for pointing out this fact. Our original proof of the main theorem (Theorem 3.1) has been considerably shortened by the use of this fact.

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