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# グラフの上のランダムな衝突モデル
# A RANDOM COLLISION MODEL ON GRAPHS

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We investigate a random collision model on graphs which is an idealized situation for interacting types. The types may represent species, genotypes, brands, factions or other classifications.

Consider a graph $G$ of a finite nonempty set $V = V(G)$ of $p$ points together with a specified set $X(G)$ of $q$ unordered pairs of distinct points. A pair $x = \{u, v\}$ of points in $X$ is called a line of $G$ and $x$ is said to join $u$ and $v$. The points $u$ and $v$ are adjacent, $u$ and $v$ are incident with each other, as are $v$ and $x$.

$W(u_1, u_2, \cdots, u_k)$ is the set of all pairs of distinct points $u_1, u_2, \cdots, u_k$. We consider a population of paricles of $p$ types, $1, 2, \cdots, p$ which consist of $N_1(t), N_2(t), \cdots, N_p(t)$ respectively at time $t$. We write

$$\vec{N}(t) = (N_1(t), N_2(t), \cdots, N_p(t)) \tag{1}$$

It is assumed that initially $\vec{N}(0) = \vec{\alpha}$, that in each unit of time a pair of particles are chosen at random from the $n$ particles , and that all possible pairs are equiprobably chosen. There exist adjacency relations between the types as defined by the graph $G$, with points $V(G) = \{1, 2, \cdots, p\}$ and adjacencies $X(G)$. Further it is assumed that a choice of a pair of particles of different types $i$ and $j$, for which $(i, j) \in X(G)$, changes the pair of particles of the type $i$ with probability $\frac{1}{2}$ and of the type $j$ with probability $\frac{1}{2}$. A choice of a pair of particles of different types $(i, j) \in W(V) - X(G)$ does not make any effect, as well as a choice of a pair of the same type .

For each point $i \in V(G)$, consider variable $x_i$. $S_k(x_1, x_2, \cdots, x_p)$ is $k$-th order elementary symmetric function. $W(u_1, u_2, \cdots, u_k)$ is the set of all pairs of distinct points $u_1, u_2, \cdots, u_k$. Let $\mid S \mid$ be the number of the elements of the set $S$. Consider

$$T_{k,l}(x_1, x_2, \cdots, x(p)) = \sum_{|W(u_1,u_2,\cdots,u_k)\cap X(G)|=l} x_{u_1} x_{u_2} \cdots x_{u_k} \tag{2}$$

$\{N(t), t = 0, 1, 2, \cdots\}$ is a Markov chain with transition probabilities defined by

$$Pr(\vec{N}(t+1) = \vec{n}_{ij} \mid \vec{N}(t) = \vec{n}) = \frac{n_i n_j}{n(n-1)} \tag{3}$$

for $(i, j) \in X(G)$, where $\vec{n} = (n_1, n_2, \cdots, n_p)$, $n = \sum_{i=1}^{p} n_i$, $\vec{n_{ij}}$ is a vector with $i$-th component $n_i + 1$, $j$-th component $n_j - 1$, and all other components equal to those of $\vec{n}$. We have

$$Pr(N(t \vec{+} 1) = \vec{n} \mid \vec{N}(t) = \vec{n}) = \sum_{i=1}^{p} \frac{n_i^2}{n(n-1)} + \sum_{(i,j) \in W(V) - X(G)} \frac{2 n_i n_j}{n(n-1)} \tag{4}$$

Theorem. Let $Q(t)$ be the number of existing types at time $t$.

$$Pr(Q(t) \geq k) \geq \frac{T_{k,0}(\vec{N}(0))}{{}_M C_k (\frac{n}{M})^k} \tag{5}$$

where $M = max\{k \mid\mid W(u_1, u_2, \cdots, u_k) \cap X(G) \mid= 0\}$.

## 参考文献


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