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<tr>
<td>タイトル</td>
<td>A Fano 3-fold with the 1-dimensional locus of non-rational singularities (Analytic varieties and singularities)</td>
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<tr>
<td>著者</td>
<td>ISHII, Shihoko</td>
</tr>
<tr>
<td>引用</td>
<td>数理解析研究所講究録 (1992), 807: 188-197</td>
</tr>
<tr>
<td>発行日</td>
<td>1992-09</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/2433/82949">http://hdl.handle.net/2433/82949</a></td>
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<tr>
<td>タイプ</td>
<td>Departmental Bulletin Paper</td>
</tr>
<tr>
<td>テキストバージョン</td>
<td>publisher</td>
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Kyoto University
A Fano 3-fold with the 1-dimensional locus of non-rational singularities

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The structure of this paper

$$Fano \ 3 \ fold \ with \ \Sigma \neq \phi$$

$$dim\Sigma = 0$$
Theorem 1A, 1B

$$dim\Sigma = 1$$
Theorem 2

base dim = 2
Theorem 3

$\Delta$ has no vertical component

$\Delta$ has vertical component
Theorem 4A, 4B

$E_0$ is a section
Theorem 5A, 5B

pictures of these models §4

is accomplished here

is not treated here

Introduction

In this paper a Fano 3-fold means a normal projective variety of dimension three over C whose anticanonical sheaf is ample and invertible. During the past fifteen years, there has been big progress in the investigation of a non-singular Fano 3-fold owing to Iskovskih, Mori, Mukai and Shokulov. And it is still developing. On the other hand, in singular Fano 3-folds, progress seems to have started recently. Here we study the structure of a Fano 3-fold with non-rational singularities.
Let $\Sigma$ be the locus of non-rational singular points of a Fano 3-fold $X$. As $X$ is normal, $\dim \Sigma \leq 1$. If $\dim \Sigma = 0$, then $X$ is isomorphic to a projective cone over a normal K3-surface or an Abelian surface (Theorem 1A, 1B). The proof of this theorem also works in the case that $\Sigma$ contains an isolated point. So what we should study next is the case that $\Sigma$ has pure dimension one. Such a Fano 3-fold is classified in three families according to the maximal basis-dimension of its $\mathbb{Q}$-factorial terminal modification (Theorem 2, Definition 1). We obtain the fact that a Fano 3-fold with the maximal basis-dimension 2 admits a projective bundle over a non-singular surface as a $\mathbb{Q}$-factorial terminal modification (Theorem 3). We try to make clear the stucture of a Fano 3-fold in this family: what kind of surface occurs as a basis, what kind of projective bundle appears as a $\mathbb{Q}$-factorial terminal modification and which parts on the projective bundle are contracted in a Fano 3-fold.

The author would like to thank Professors Nakayama and Kei-ichi Watanabe and also other members of Waseda Seminar for their stimulating discussion during the preparation of this article. In particular Nakayama's proof of Proposition 2 helped her very much and also K-i. Watanabe's comment "a Weil divisor on a $\mathbb{Q}$-factorial terminal singularity is Cohen-Macaulay" was very helpful in the proof of Theorem 2.

§1. The case $\dim \Sigma = 0$

**Theorem 1A([I]).** Let $X$ be a Fano 3-fold with $\dim \Sigma = 0$. Then there exist a normal surface $S$ which is either an Abelian surface or a normal K3-surface and an ample invertible sheaf $\mathcal{L}$ on $S$ such that $X$ is the contraction of the negative section of a projective bundle $\mathbb{P}(\mathcal{O}_S \oplus \mathcal{L})$. Here a normal K3-surface implies a normal projective surface with the trivial canonical sheaf and has only rational singularities.

**Theorem 1B([I]).** Let $X$ be a projective cone over a surface $S$ which is either an Abelian or a normal K3-surface. Then $X$ is a Fano 3-fold with $\Sigma = \{\text{the vertex}\}$.

§2. Basic structure theorem of $\mathbb{Q}$-factorial terminal modifications for the case $\dim \Sigma = 1$

**Theorem 2.** Let $X$ be a Fano 3-fold with $\Sigma$ of pure dimension one. Let $g : Y \to X$ be a $\mathbb{Q}$-factorial terminal modification whose existence is proved by Mori ([M]). Denote $K_Y = g^* K_X - \Delta$. Then we have a sequence of projective morphisms:
$Y = Y_0 \xrightarrow{\varphi_0} Y_1 \xrightarrow{\varphi_1} Y_2 \ldots \xrightarrow{} Y_r \xrightarrow{\varphi_r} Z$, where for each $i$, $\varphi_i$ is the contraction of an extremal ray $R_i$ on $Y_i$ such that $R_i\Delta_i > 0$ (here, $\Delta_0 = \Delta$, and $\Delta_i = (\varphi_{i-1})_*\Delta_{i-1}$).

For $i \leq r - 1$, $\varphi_i$ is a birational contraction of a divisor isomorphic to $F_{a,0}$ ($a \geq 1$) to a non-singular point and $\varphi_r$ is a fibration to a lower dimensional variety $Z$.

**Definition 1.** The variety $Z$ above is called a basis of $X$. And each $\varphi_i$ is called a $\Delta$-extremal contraction. Of course a basis of $X$ is not unique for $X$. It depends on the choice of a Q-factorial terminal modification $Y$ and also on the choice of extremal rays $R_i$'s.

From now on, we devote to study $X$ which has a two dimensional basis $Z$. In this case, the last contraction $\varphi_r : Y_r \rightarrow Z$ satisfies the assumption of the following proposition. So we can see that it is a $\mathbb{P}^1$-bundle over a non-singular surface $Z$.

**Proposition 1 (Nakayama).** Let $\varphi : Y \rightarrow Z$ be a contraction of an extremal ray on a 3-fold $Y$ with at worst Q-factorial terminal singularities on it to a surface $Z$. Assume there exists an invertible sheaf on $Y$ whose degree on a general fiber is 1. Then $Z$ is non-singular and $Y$ is a $\mathbb{P}^1$-bundle over $Z$.

**Theorem 3.** Let $X$ be a Fano 3-fold with one dimensional $\Sigma$ and a two dimensional basis. Then there exists a Q-factorial terminal modification $g : Y \rightarrow X$ such that a $\Delta$-extremal contraction $\varphi_0 : Y \rightarrow Z$ gives a $\mathbb{P}^1$-bundle over a non-singular surface $Z$.

This theorem is proved by applying the following lemma successively.

**Lemma.** Let $X$ be as above and $Y = Y_0 \xrightarrow{\varphi_0} Y_1 \xrightarrow{\varphi_1} Y_2 \ldots \xrightarrow{} Y_r \xrightarrow{\varphi_r} Z$ be a sequence of $\Delta$-extremal contractions of Q-factorial terminal modification $Y$ of $X$ with 2-dimensional basis $Z$. If $r > 0$, then there is a flop $Y'_i$ of $Y_i$ for each $i$ ($i \leq r - 1$) such that $g' : Y' = Y'_0 \rightarrow X$ is a Q-factorial terminal modification of $X$ and $Y' = Y'_0 \xrightarrow{\varphi'_0} Y'_1 \xrightarrow{\varphi'_1} Y'_2 \ldots \xrightarrow{} Y'_{r-1} \xrightarrow{\varphi'_{r-1}} Z'$ is a sequence of $\Delta'$-extremal contractions with 2-dimensional basis $Z'$, where $\Delta'$ is a Q-divisor such that $K_{Y'} = g'^*K_X - \Delta'$.

§3. Fano 3-folds which have $\mathbb{P}^1$-bundles as Q-factorial terminal modi-
Let $X$ be a Fano 3-fold with a 2-dimensional basis. Then, by Theorem 3, we can take a $\mathbb{Q}$-factorial terminal modification $g : Y \to X$ such that a $\Delta$-extremal contraction $\varphi : Y \to Z$ gives a $\mathbb{P}^1$-bundle over a non-singular surface $Z$. Then we have the following facts:

(i) $-g^*K_X \ell = \Delta \ell = 1$, where $\ell$ is a fiber of $\varphi : Y \to Z$.
(ii) $\Delta$ is denoted by $E_0 + \varphi^*(\Delta')$, where $E_0$ is an irreducible component with $E_0 \ell = 1$ and $\Delta' \in \text{Pic}(Z)$.

The case $\text{Supp} \Delta$ contains a vertical component

We call an irreducible divisor $D$ in $Y$ a vertical divisor for $g$, if $D$ is mapped to a point of $X$ by $g$.

**Theorem 4A.** Let $X,g : Y \to X, \Delta$ and $\varphi : Y \to Z$ be as in the beginning of this section. Assume $\text{Supp} \Delta$ contains a vertical component.

Then, (i) a vertical component is unique and coincides with $E_0$ and it is a section of the projection $\varphi$,
(ii) there exists a normal surface $Z_0$ with at least one non-rational singular point on it whose canonical sheaf is trivial and whose minimal resolution is $h : Z \to Z_0$ and
(iii) the $\mathbb{P}^1$-bundle $\varphi : Y \to Z$ is a pull back of a $\mathbb{P}^1$-bundle $\varphi_0 : Y_0 \to Z_0$ by $h$ and $g : Y \to X$ factors as $Y \xrightarrow{h} Y_0 \xrightarrow{g_0} X$, where $g_0$ is a contraction of the negative section $h(E_0)$.

**Theorem 4B.** Let $S$ be a normal surface with trivial canonical sheaf and at least one non-rational singular point on it. Then an arbitrary projective cone $X$ over $S$ is a Fano 3-fold and $\Sigma$ is generating lines over a non-rational singular points of $S$.

Remark. Normal surfaces with the trivial canonical sheaf and at least one non-rational singular point are studied in [U] among others. The number of non-rational singular points is less than or equal to 2. It is 2, if and only if both of them are simple elliptic singularities [U, Theorem 1].

The case $\text{Supp} \Delta$ contains no vertical component
In the previous case, $E_0$ is a section of $\varphi$. But in this case, it is not necessarily true. First we consider the case that $E_0$ is a section. Since $E_0$ is not a vertical component, $g|_{E_0} : E_0 \to C$ is a fibration to a curve $C$.

**Proposition 2.** The possible triples $(E_0, g|_{E_0}, \Delta')$ are the following:

(i) $(\mathbb{P}^1 \times \text{elliptic curve}, \text{the first projection} \ p_1, \phi),$ 
(ii) $(\text{a rational elliptic surface, the elliptic fibration}, \phi),$
(iii) $E_0$ is the composite of $r$-blowing ups $E_0 \xrightarrow{\sigma_r} \ldots \xrightarrow{\sigma_1} \mathbb{P}^1 \times \text{elliptic curve}$, where $\sigma_1$ is the blow up at a point on the fiber $C = p_1^{-1}(z)$ of a point $z \in \mathbb{P}^1$ and $\sigma_i (i > 1)$ is the blow up at the intersection of the proper transform of $C$ and the exceptional curve of $\sigma_{i-1}$. The morphism $g|_{E_0}$ is $p_1 \sigma_1 \sigma_2 \ldots \sigma_r$ and $\Delta'$ is the proper transform of $C$.
(iv) $E_0$ is a ruled surface $p : E_0 \to S$ such that there exist a covering $\pi : S \to \mathbb{P}^1$ and a member $D$ in $|-K_{E_0}|$ of type $D = (\pi p)^{\ast}(z) + \Delta'$, where $z \in \mathbb{P}^1$ and $\Delta'$ is an effective divisor with $K_{E_0}C \geq 0$ for every component $C \subset \Delta'$. The morphism $g|_{E_0}$ is $\pi p$.

**Theorem 5A.** Let $X, g : Y \to X, \Delta, \Delta'$ and $\varphi : Y \to Z$ be as in the beginning of this section. Assume $\text{Supp} \Delta$ contains no vertical component and $E_0$ is a section of $\varphi$. Denote $-g^\ast K_X = E_0 + \varphi^\ast L$ for $L \in \text{Pic}Z$. Then the triple $(E_0, g|_{E_0}, \Delta')$ is as one of (i)~(iv) in Proposition 2 and the $\mathbb{P}^1$-bundle $\varphi : Y \to Z$ is obtained by an sheaf $\mathcal{E}$ which satisfies the following properties:

(I) $\mathcal{E}$ is an extension of $\mathcal{N}$ by $\mathcal{O}_Z$, where $\mathcal{N} = \mathcal{O}_Z(-K_Z - \Delta' - L)$ such that $\mathcal{E}|_{\Delta'} = \mathcal{O}_{\Delta'}(-L) \oplus O_{\Delta'}(-L)$ and $(L \otimes \mathcal{E})_y$ is generated by its global sections for each $y \in \Delta'$.

(II) $L - \Delta'$ is semi-ample and $(L - \Delta')(L - \Delta' - K_Z) > 0$.

**Theorem 5B.** Let a triple $(Z, \tilde{g}, \Delta')$ be as one of (i)~(iv) in Proposition 2 and $L \in \text{Pic}Z$ and $\mathcal{E}$ be as in (I) and (II) in Theorem 5A.

Let $Y = \mathbb{P}(\mathcal{E}) \xrightarrow{\pi} Z$ be the projective bundle defined by $\mathcal{E}$ and $E_0$ be a section of $\varphi$ defined by the surjection $\mathcal{E} \to \mathcal{N}$. Denote $E_0 + \varphi^\ast L$ by $H$.

Then $|mH|$ is base point free for $m \gg 0$ and the image $X$ of the morphism $g = \Phi_{|mH|} : Y \to \mathbb{P}^M$ becomes a Fano 3-fold with one dimensional $\Sigma$ and $g|_{E_0} = \tilde{g}$ under the identification of $E_0$ with $Z$.

Now we give an example of a Fano 3-fold with $E_0$ not a section.
Example. Let $Z$ be the projective plane $\mathbb{P}^2$, $C$ and $C'$ be two general curves of degree 3 on $Z$. Let $\sigma : \tilde{Z} \rightarrow Z$ be the blowing up at 9-distinct points $\{p_1, p_2, \ldots, p_9\} = C \cap C'$, then $\tilde{Z}$ becomes an elliptic surface with elliptic fibers $[C], [C']$, where $[C]$ is the proper transform of $C$ on $\tilde{Z}$. Denote the fiber $\sigma^{-1}(p_i)$ by $\ell_i$. Let $L$ be $\sigma^*L_0 + \sum_{i=1}^{9}\ell_i$ where $L_0$ is an ample divisor on $Z$.

Since $H^1(\tilde{Z}, L - [C]) \cong \oplus H^1(\ell_i, L - [C]|_{\ell_i}) \cong C^{\oplus 9}$, we can take an extension sheaf $\tilde{\mathcal{E}}$ of $\mathcal{O}([C]-L)$ by $\mathcal{O}_Z$ such that the restriction $\tilde{\mathcal{E}}|_{\ell_i}$ is not zero for every $i$ ($i = 1, 2, \ldots, 9$). Now $0 \rightarrow \mathcal{O}_Z|_{\ell_i} \rightarrow \tilde{\mathcal{E}}|_{\ell_i} \rightarrow \mathcal{O}([C]-L)|_{\ell_i} = \mathcal{O}_{p1}(2) \rightarrow 0$ does not split, so $\tilde{\mathcal{E}}|_{\ell_i}$ is trivial for each $i$. By Schwarzenberger's Theorem, $\tilde{\mathcal{E}}' = \sigma^*\mathcal{E}$ for some locally free sheaf $\mathcal{E}$ on $Z$.

Let $Y$ be the projective bundle $\mathbb{P}(\mathcal{E})$ and $\tilde{Y}$ be $\mathbb{P}(\tilde{\mathcal{E}}')$. Then we have the diagram of a fiber product

\[
\begin{array}{ccc}
Y & \xrightarrow{\phi} & Y \\
\downarrow \sigma & & \downarrow \varphi \\
\tilde{Z} & \xrightarrow{\sigma} & Z
\end{array}
\]

Let $E_0$ be the section of $\sigma$ defined by the surjection $\tilde{\mathcal{E}} \rightarrow \mathcal{O}([C]-L)$ and $E_0$ be the image $\sigma(E_0)$. Then $H = E_0 + \varphi^*L_0$ is a semipositive divisor on $Y$. The image $X$ of the morphism $\Phi_{|mH|} : Y \rightarrow \mathbb{P}^M$ becomes a Fano 3-fold with $\Sigma \cong \mathbb{P}^1$ and $Y$ is a Q-factorial terminal modification of $X$. It is easy to see that $\Delta = E_0$ and $E_0$ contains the fibers of $\varphi$ over $p_1, p_2, \ldots, p_9 \in Z$.

§4. Pictures of $Y$ and $X$ of Theorem 5

(i) In the case the triple is $(\mathbb{P}^1 \times C$, the first projection $p_1, \phi)$, where $C$ is an elliptic curve. Then $\Delta = E_0$. If we denote $L = p_1^*\mathcal{O}_{\mathbb{P}^1}(a) \otimes p_2^*B$, then $a \geq 0$ and $B$ is ample.

(i-1) $a > 0$. $g|_{Y-E_0} : Y - E_0 \cong X - \Sigma$.

\[
\begin{array}{ccc}
Y & \xrightarrow{g} & X \\
\downarrow g|_{E_0=p_1} & & \downarrow g|_{E_0=p_2} \\
E_0 & \cong & \Sigma
\end{array}
\]

(i-2) $a = 0$ and the exact sequence $(\mathcal{E}) : 0 \rightarrow \mathcal{O}_Z \rightarrow \mathcal{E} \rightarrow \mathcal{N} \rightarrow 0$ splits. $g|_{Y-E_0-E_\infty} : Y - E_0 - E_\infty \cong X - \Sigma - \Sigma_0$, and $g|_{E_\infty} = p_2$, where $\Sigma_0$ is the locus of canonical singularities.

\[
\begin{array}{ccc}
Y & \xrightarrow{g} & X \\
\downarrow g|_{E_0=p_1} & & \downarrow g|_{E_0=p_2} \\
E_0 & \cong & \Sigma
\end{array}
\]
(i-3) $a = 0$ and the exact sequence $(\mathcal{E}) : 0 \to \mathcal{O}_Z \to \mathcal{E} \to \mathcal{N} \to 0$ does not split. There exists a divisor $\sum_{i=1}^{s} m_iq_i \in |B|$ such that the restriction $(\mathcal{E})_{|f_i}$ splits for each $i,(i = 1, \ldots, s)$, where $f_i = p_2^{-1}(q_i)$. For a general fiber $f = p_2^{-1}(q), q \in C$, $Y_f$ is $\mathbb{P}^1 \times \mathbb{P}^1$ and for $f_i$, $Y_f \simeq \mathbb{P}(\mathcal{O} \oplus \mathcal{O}(2))$. $E_0|_{Y_f}$ is an ample section for general $f$ and is the disjoint section from the negative section for $f = f_i$. Denote the negative section of $Y_{f_i}$ by $\tilde{f}_i$. Then the restriction $g|_{Y-E_0-\cup\tilde{f}_i}$ is an isomorphism, $g|_{E_0} = p_1$, and each $\tilde{f}_i$ is contracted to a canonical singularity in $X$.

(ii) The case that the triple $(E_0, g|_{E_0}, \Delta')$ is (rational elliptic surface, the elliptic fibration, $\phi$). Then $E_0 = \Delta$ in this case too. If $L$ is big then the exact sequence $(\mathcal{E})$ splits and if $L$ is not big $|L|$ gives a fibration $\Phi = \Phi|_{|L|} : Z \to \mathbb{P}^1$ with a general fiber $\mathbb{P}^1$. Let $C_i (i = 1, 2, \ldots, r)$ be $(-2)$-curves on $Z$ with $LC_i = 0$ and $f_j (j = 1, \ldots, s)$ be $(-1)$-curves on $Z$ with $Lf_j = 0$. Then $E_0|_{Y_f}$ is the section disjoint from the negative section. Denote the negative section of $Y_{f_i}$ by $\tilde{f}_j$. Then the normal bundle of $\tilde{f}_j$ in $Y$ is $\mathcal{O}_{\mathbb{P}^1}(1) \oplus \mathcal{O}_{\mathbb{P}^1}(1)$.

(ii-1) $L$ is big. Then the restriction $g|_{Y-E_0-\cup\tilde{f}_j}$ is an isomorphism, $g|_{Y_{C_i}} : Y_{C_i} \simeq C_i \times \mathbb{P}^1 \to \mathbb{P}^1$ is the projection to the second factor and $g(\tilde{f}_j)$ is an isolated canonical singular point for each $j$. A point of $g(Y_{C_i})$ away from $g(E_0)$ is non-isolated canonical singularities.
(ii-2) $L$ is not big and $(\mathcal{E})$ splits. Let $E_\infty$ be the section of $\varphi$ disjoint from $E_0$. Then the restriction $g|_{E_0-E_\infty-Y_{C_1}}$ is an isomorphism, $g|_{Y_{C_1}}$ is as above and $g|_{E_\infty} = \Phi$.

(ii-3) $L$ is not big and $(\mathcal{E})$ does not split. Denote $L = \Phi^*L_0$ for a Cartier divisor $L_0$ on $\mathbb{P}^1$. Then the extension $\mathcal{E}$ of $\mathcal{N}$ corresponds to a non-zero section $\phi_\mathcal{E}$ of $\Gamma(\mathbb{P}^1, L_0 + K_{\mathbb{P}^1})$. Let $\phi_\mathcal{E}$ define a divisor $\sum_{k=1}^{d} m_k q_k (d \geq 0, m_k > 0)$ and $\ell_k k = 1, \ldots, b (0 \leq b \leq d)$ be smooth fibers among $\{\Phi^{-1}(q_k)\}$. A component of a singular fiber of $\Phi$ is either one of $C_i$ or $f_i$ defined above. For a general fiber $\ell \in \Phi^{-1}(q) \in \mathbb{P}^1$, $Y_\ell$ is $\mathbb{P}^1 \times \mathbb{P}^1$ and for $\ell_k, Y_{\ell_k} \simeq \mathbb{P}(\mathcal{O} \oplus \mathcal{O}(2))$. $E_0|Y_{\ell}$ is an ample section for general $\ell$, while it is the section disjoint from the negative section for $\ell = \ell_k (0 \leq k \leq b)$. Denote the negative section of $Y_{\ell_k}$ by $\tilde{\ell}_k$. Then the restriction $g|_{Y-E_0-Y_{C_i}-\bigcup_{j=1}^{r} Y_{f_j}-\bigcup_{k=1}^{b} Y_{\ell_k}}$ is isomorphic, $g|_{Y_{C_1}}$ is the second projection, $f_j$'s and $\ell_k$'s are contracted to canonical singularities in $X$.

(iii) The case that the triple $(E_0, g|_{E_0}, \Delta')$ is as follows: $E_0$ is the composite of $r$-blowing ups $E_0 \to \sigma_r \to \cdots \to \sigma_1, \mathbb{P}^1 \times \text{elliptic curve}$, where $\sigma_1$ is the blow up at a point on the fiber $C = p_i^{-1}(z)$ of a point $z \in \mathbb{P}^1$ and $\sigma_i (i > 1)$ is the blow up at the intersection of the proper transform of $C$ and the exceptional curve of $\sigma_{i-1}$. The morphism $g|_{E_0}$ is $p_1 \sigma_1 \sigma_2 \cdots \sigma_r$ and $\Delta' =$ the proper transform of $C$. 


Then $L$ is nef and big, with $L\ell_{r} > 0$ and the exact sequence $(\mathcal{E})$ splits, where $\ell_{i}$ ($i = 1, 2, \ldots, r$) are the exceptional curves of $\sigma_{i}$ respectively. Let $E_{\infty}$ be the section of $\varphi$ disjoint from $E_{0}$, and $\ell_{j} j \in J \subset \{1, 2, \ldots, r - 1\}$ be the exceptional curves with $L\ell_{j} = 0$ and $C_{i} i = 1, 2, \ldots$ be the curves on $E_{\infty}$ with $LC_{i} = 0$. Then $g|_{Y-E_{0}-Y_{\Delta}-\cup_{i\in J}Y_{\ell_{j}}-\cup_{\ell=1}^{\infty}E_{\infty}}$ is an isomorphism, $g|_{Y_{\ell_{j}}} \simeq \ell_{j} \times P^{1} \to P^{1}$ is the projection to the second factor and $g(C_{i})$ is an isolated canonical singular point on $X$ for $i = 1, 2, \ldots, s$.

(iv) The case that the triple is as (ii) of Proposition 2.

Then the exact sequence $(\mathcal{E})$ does not split. Let $C_{i}$ ($i = 1, 2, \ldots, r$) be (-2)-curves on $Z$ with $eC_{i} = LC_{i} = 0$ and $f_{j}$ ($j = 1, \ldots, s$) be (-1)-curves on $Z$ with $ef_{j} > 0$ and $Lf_{j} = 0$. Then we can take the negative section $\tilde{f}_{j}$ of $Y_{\ell_{j}}$ disjoint from $E_{0}$. Then $g|_{Y_{\Delta}-\cup_{i=1}^{s}Y_{C_{i}}-\cup_{j=1}^{r}Y_{\ell_{j}}}$ is an isomorphism, $g|_{C} : C \simeq C \times P^{1} \to P^{1}$ is the projection to the second factor for a component $C < \Delta$ and for $C = C_{i}$ ($i = 1, \ldots, r$) and $g(\tilde{f}_{j})$ is an isolated canonical singularity for $j = 1, \ldots, s$.

, where ..... is the fiber of a point in $\Sigma$
References


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