数理解析研究所講究録
第 808 巻 1992 年 39-48

Some D-modules on the moduli spaces of curves associated to CFT

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§ Introduction

The aim of this exposition is two-fold. The first is to present a short review of some recent works on the geometric construction of the space of conformal blocks in the theory of conformal field theory (abbreviated as CFT) in terms of certain D-modules on the moduli spaces of curves.

The second is to sketch our recent study [SU] of "abelian" CFT jointly with Prof.Kenji Ueno in the same principle as the above, cf.[S].

The construction of the spaces of conformal blocks is due to [BF] in the case of minimal series representations of the Virasoro algebra and to [TUY] in the case of integrable representations of affine Lie algebras. Both works realize the space of conformal blocks as fibers of certain D-modules on the dressed moduli spaces of curves by the method of localization.

Projective connections on those modules are neatly explained in [BK] by a kind of "heat equation" which reformulates Hitchin's approach [H].

The contents are as follows. In §1 we briely review the method of localization. Then we treat the case of Virasoro algebra in §2 and the case of abelian CFT in §3. Finally in §4 we comment on the factorization property.

§0 Notations

T_Y denotes the tangent sheaf of a smooth scheme Y.

Vir denotes the Virasoro algebra :

$$Vir := \mathbb{C}((z))\frac{d}{dz} \oplus \mathbb{C}\cdot c \quad \mathbb{C}((z)) = \mathbb{C}[[z]][z^{-1}]$$

with the commutation relation

$$[f(z)\frac{d}{dz}, g(z)\frac{d}{dz}] := (fg' - f'g)\frac{d}{dz} + \frac{1}{12}\mathrm{Res}_{z=0}(f'''gdz)\cdot c$$

for $f(z), g(z) \in \mathbb{C}((z))$ and c is a central element. For more details on Vir, cf.[KR].

$\hat{u}(1)$ denotes the (completed) affinization of the one-dimensional Lie algebra $u(1) := \mathbb{C}$:

$$\hat{u}(1) := \mathbb{C}((z)) \oplus \mathbb{C} \cdot K,$$

where K is a central element and its Lie bracket is defined to be

$$[f(z), g(z)] = \mathrm{Res}_{z=0}(f' g dz) \cdot K.$$

(The oscillator algebra in [KR])

We introduce "N-point variants" of the above algebras :

$$Vir_N := \bigoplus_{i=1}^{N} \mathbb{C}((z_i))\frac{d}{dz_i} \oplus \mathbb{C} \cdot c$$
$$\hat{u}_N(1) := \bigoplus_{i=1}^{N} \mathbb{C}((z_i)) \oplus \mathbb{C} \cdot K.$$

Here c and K are again central elements and the Lie brackets are given by

$$\begin{aligned}[(f_i(z_i)\frac{d}{dz_i}), (g_i(z_i)\frac{d}{dz_i})] &= \sum_{i=1}^{N}(f(z_i)g'(z_i) - f'(z_i)g(z_i))\frac{d}{dz_i} \\ &+ \frac{1}{12}\sum_{i=1}^{N} \mathrm{Res}_{z_i=0}(f_i'''(z_i)g_i(z_i)dz_i) \cdot c \\ [(f_i(z_i)), (g_i(z_i))] &= \sum_{i=1}^{N} \mathrm{Res}_{z_i=0}(f_i'(z_i)g_i(z_i)dz_i) \cdot K.\end{aligned}$$

Finally let us recall the Fock space representations which have two complex parameters λ, w :

$$\begin{aligned} F_{\lambda,w} &:= U(\hat{u}(1))/I(\lambda, w) \\ U(\hat{u}(1)) &:= \text{the universal enveloping algebra of } \hat{u}(1) \\ I(\lambda, w) &:= \text{the left ideal of } U(\hat{u}(1)) \text{ generated by } \mathbb{C}[[z]],\ z^0 - w \text{ and } K - \lambda. \end{aligned}$$

These are $\hat{u}(1)$-modules and becomes Vir_N-modules by the so-called Sugawara construction cf.[KR].

Put $F_0 := F_{0,0}$ for later use and put

$$F_{\lambda,\vec{w}} := \otimes_{i=1}^{N} F_{\lambda,w_i}$$

for $\lambda, w_1, \cdots, w_N \in \mathbb{C}$ and $\vec{w} = (w_1, \ldots, w_N)$. These are naturally $\hat{u}_N(1)$-modules.

§1 Localization

1.1 First we recall the definition of a ring of twisted differential operators (abbreviated as a tdo),cf.[B],[K].

Let X be a smooth scheme.

A tdo on X is a filtered ring (=sheaf of rings) (D,F), which satisfies the following conditions :

(1) $\bigcup_i F_i D = D, \quad F_{-1}D = 0.$

(2) $F_i D/F_{i-1}D \simeq S^i(T_X)$ compatibly with the multiplications on the both sides.

We will sometimes write $F_i D = D^{\leq i}$.

If $\mathcal{L}$ is a line bundle on X, then the sheaf $D_{\mathcal{L}}$ of differential operators acting on the (local) sections of $\mathcal{L}$ is a basic example of tdo.

1.2 Let Y be a smooth scheme, D a tdo on Y. By the action of a Lie algebra $\mathfrak{g}$ on (Y,D), we mean a Lie algebra homomorphism $\alpha : \mathfrak{g} \to D^{\leq 1}(Y)$, where we put $\mathcal{F}(Y) = \Gamma(Y,\mathcal{F})$ for a sheaf $\mathcal{F}$.

If we have an action of $\mathfrak{g}$ on (Y,D), then we have an algebra homomorphism $\alpha : U(\mathfrak{g}) \to D(Y)$ and also a $\mathfrak{g}$-action on Y, i.e., $\mathfrak{g} \to D^{\leq 1}(Y) \to T_Y(Y)$.

Definition (Localization functor)

Assume that we are given an action of $\mathfrak{g}$ on (Y,D). Then the following correspondence

$$M \longmapsto D \otimes_{U(\mathfrak{g})} M$$

defines a functor

$$\Delta : (\mathfrak{g}\text{-modules}) \longmapsto (D\text{-modules}).$$

This is a right-exact functor.

Lemma. *Let $\mathfrak{g}_p$ be the stabilizer at a point $p \in Y$ ($= Ker(\mathfrak{g} \to D^{\leq 1}(Y) \to D_p^{\leq 1})$). Then we have*

$$\Delta(M) \otimes \mathcal{O}_Y/\mathfrak{m}_p \simeq M/\mathfrak{g}_p M.$$

The right hand side is the space of coinvariants.

§2 The case of Virasoro algebra

2.1 We fix non-negative integers g, N with $3g-3+N \geq 0$. A scheme (or a stack) means a $\mathbb{C}$-scheme (or a $\mathbb{C}$-stack) in what follows.

Let $\mathcal{M}_{g,N}$ be the moduli space of N-pointed smooth projetive algebraic curves over $\mathbb{C}$ of genus g, and $\overline{\mathcal{M}}_{g,N}$ its natural compactification, i.e., the moduli space of N-pointed stable curves of genus g. These are smooth stacks of dimension $3g-3+N$ and $\overline{\mathcal{M}}_{g,N}$ is also proper over $\mathbb{C}$, cf.[DM],[Kn].

Consider the morphism

$$\pi : \overline{\mathcal{C}} = \overline{\mathcal{M}}_{g,N+1} \to \overline{\mathcal{M}} = \overline{\mathcal{M}}_{g,N}$$

which forgets the $(N+1)$-th point and is the "univesal" curve. By restriction this gives rise to the universal curve

$$\pi : \mathcal{C} = \mathcal{C}_{g,N} \to \mathcal{M} = \mathcal{M}_{g,N}.$$

In general, we will denote the determinant line bundle $detR\pi_*(\mathcal{F})$ by $d(\mathcal{F})$ for a family of curves $\pi : X \to S$ and a coherent sheaf $\mathcal{F}$ on X, cf.[KM].

Returning to our situation, we put

$$\lambda_j = detR\pi_*(\omega_{\mathcal{C}/\mathcal{M}}^{\otimes j}).$$

Then $\lambda_j = \lambda_{1-j}$ (Serre duality) and $\lambda_0 = \lambda_1 =: \lambda_H$ is the (so-called) Hodge line bundle.

We are going to apply the localization procedure not for $\mathcal{M}_{g,N}$ but for the following dressed moduli space $Y = \mathcal{M}_{g,N}^{(\infty)}$.

$\mathcal{M}_{g,N}^{(\infty)}$ is the moduli space of dressed N-pointed curves, which is introduced by Beilinson and Kontsevich, cf.[KNTY]. A dressed N-pointed curve $(C; x_1, \cdots, x_N; t_1, \cdots, t_N)$ consists of a N-pointed $(C; x_1, \cdots, x_N)$ and isomorphisms of $\mathbb{C}$-algebras $t_i : \widehat{\mathcal{O}}_{C,x_i} \simeq \mathbb{C}[[z]]$ $(1 \le i \le N)$ (formal local parametrizations).

The obvious projection $\mathcal{M}_{g,N}^{(\infty)} \to \mathcal{M}_{g,N}$ makes $\mathcal{M}_{g,N}^{(\infty)}$ into a $\mathrm{Aut}_{\mathbb{C}-alg}(\mathbb{C}[[z]])$-torsor over $\mathcal{M}_{g,N}$. The pull-back by this projection gives rise to a family of curves over $\mathcal{M}_{g,N}^{(\infty)}$ and the determinant line bundle λ_j.

We have similar notions for the stable moduli.

2.2 PROPOSITION ([BS §4]). *There is an action of the Lie algebra Vir_N on $(\mathcal{M}_{g,N}^{(\infty)}, D_{\lambda_H})$ (with central charge 1).*

The proof of this fact uses a construction of the Virasoro algebra associated to a pointed curve and a theorem on the integration of trace algebras : $D_{\lambda_H}^{\le 1} = R^0\pi_*({}^{tr}\mathcal{A}_{\mathcal{O}})$ where ${}^{tr}\mathcal{A}_{\mathcal{O}}$ is a certain complex of Lie algebras cf.[BS,§1,2].

It is easy to produce the associated action of Vir_N on $\mathcal{M}_{g,N}^{(\infty)}$: $\mathcal{O}_{\mathcal{M}_{g,N}^{(\infty)}} \hat{\otimes} Vir_N/\mathbb{C}c \simeq \dot{\pi}_*(T_\pi)$.

REMARK There is an operation of multiplying a tdo by a complex number $c \in \mathbb{C}$. For example, $cD_L = D_{cL} = D_{L^{\otimes c}}$ for a line bundle L and $c \in \mathbb{Z}$.

Thus we dispose the following functor :

$$\Delta : (Vir_N\text{-modules with central charge c}) \to (D_{c\lambda_H}\text{-modules}).$$

2.3 As an illustration of the localization technique, we recall the beautiful results by Beilinson-Feigin [BS,§4], [BFM,§§7,8].

Let M be a finitely generated Vir_N-module which satisfies the condition : $(\otimes_{i=1}^N \mathbb{C}[[z_i]]z_i\partial_{z_i})m$ is finite-dimensional for any $m \in M$. (Then we say M is integrable with respect to $(\otimes_{i=1}^N \mathbb{C}[[z_i]]z_i\partial_{z_i})$.)

For such an M with central charge c, $\Delta(M)$ is a coherent $D_{c\lambda_H}$-module and descends to $\mathcal{M}_{g,N}^{(1)}$, where (1) means that we consider local coordinates up to the first order instead of formal local parameters for the case of (∞).

Among such representations are the Verma module $M_{c,h}$ and its irreducible quotient $L_{c,h}$. (c and h are eigenvalues of the operators c and L_0 on the vacuum vector.)

2.4 THEOREM. *The following are equivalent.*

(1) $L_{c,h}$ *is lisse, i.e., the characteristic variety of* $L_{c,h}$ $SS(L_{c,h})$ *is included in the orthogonal complement of* $\{L_{-1}, L_{-2}, \cdots\}$ *in the dual space* Vir^*.

(2) $N_{c,h} = \mathrm{Ker}(M_{c,h} \to L_{c,h})$ *is generated by two singular vectors and* $N_{c,h} \subset M_{c,h'}$ *implies* $(c,h) = (c,h')$.

(3) $L_{c,h}$ *is in the minimal series, i.e.,*

$$\begin{cases} c = 1 - \dfrac{6(p-q)^2}{pq} \\ h = \dfrac{(pm-qn)^2-(p-q)^2}{4pq} \end{cases} \qquad (1<p<q, \quad 1 \le m < q, \quad 1 \le n < p)$$

The meaning of lisse-ness is clear from the following :

2.5 THEOREM. *If* M *is a lisse* Vir_N*-module, then* $\Delta(M)$ *is a vector bundle with a (twisted) integrable connection.*

§3 The case of $U(1)$-current algebra

We would like to treat the case of $U(1)$-current algebra using the localization technique as in §2. Unlike the minimal series representations for Virasoro algebra, Fock space representations (cf.§0) produce non-coherent D-modules on the moduli spaces of curves.

We consider "dressed" invertible sheaves on a curve and "dress" the relative Picard scheme over $\mathcal{M}_{g,N}^{(\infty)}$. Then, by a theorem on tdo's by Beilinson-Kazhdan, we can produce the modules of conformal blocks for our abelian CFT. This generalizes the earlier work [KNTY].

3.1 Recall the situation in 2.1. We have the universal family of curves $\pi : \mathcal{C} \to \mathcal{M}$ and $\pi : \overline{\mathcal{C}} \to \overline{\mathcal{M}}$.

The relative Picard group (of degree d) $Pic^d_{\mathcal{C}/\mathcal{M}}$ is also a smooth algebraic stack, which is projective over $\mathcal{M}$ and is moreover a (relative) abelian scheme of dimension

g over $\mathcal{M}$. Put $P = Pic^{g-1}_{\mathcal{C}/\mathcal{M}} \xrightarrow{p} \mathcal{M}$. We have the (universal) Poincaré bundle $\mathcal{P}$ on $\mathcal{C} \times_{\mathcal{M}} P$ and dispose the determinant line bundle $\mathcal{L} = d(\mathcal{P}) = \det R\pi_*(\mathcal{P})$ on $\mathcal{P}$, where π is the projection $\mathcal{C} \times_{\mathcal{M}} P \to P$. cf.[Sz]. It is known that $\mathcal{L} = \mathcal{O}(-\Theta)$ holds where Θ is the theta divisor on P.

We also dispose the relative Picard group $\overline{P} = Pic^{g-1}_{\overline{\mathcal{C}}/\overline{\mathcal{M}}}$, which is a (relative) semi-abelian scheme over $\overline{\mathcal{M}}$. The Poincaré bundle extends to $\overline{\mathcal{P}}$ over $\overline{\mathcal{C}} \times_{\overline{\mathcal{M}}} \overline{P}$ and we have the determinant line bundle $\overline{\mathcal{L}}$.

Consider the following fiber product :

$$P^{(\infty)} = P \times_{\mathcal{M}_{g,N}} \mathcal{M}^{(\infty)}_{g,N} \xrightarrow{p} \mathcal{M}^{(\infty)}_{g,N}.$$

In order to localize representations of the Lie algebra $\hat{u}_N(1)$, we have to consider the "dressing" of invertible sheaves on a curve. A dressed invertible sheaf on a dressed N-pointed curve $(C; x_1, \cdots, x_N; t_1, \cdots, t_N)$ is an invertible sheaf L (or a line bundle) equipped with t_i-linear isomorphisms $v_i : \widehat{L}_{x_i} \simeq \mathbb{C}[[z]] \quad (1 \leq i \leq N)$.

Denote by $P^{(\#)}$ the moduli space of dressed invertible sheaves over dressed N-pointed curves. Thus $P^{(\#)}$ is a $\mathbb{G}_m(\mathbb{C}[[z]])^N$-torsor over $P^{(\infty)}$.

3.2 Proposition. *There exists an action of* $\hat{u}_N(1)$ *(with central charge 1) on* $(P^{(\#)}, D_{r^*\mathcal{L}})$. *Namely there is a Lie algebra homomorphism :* $\theta : \hat{u}_N(1) \to D^{\leq 1}_{r^*\mathcal{L}}$.

The homomorphism θ factors through $D_{r^*\mathcal{L}/\mathcal{M}^{(\infty)}}$ by the construction, for which we do the same thing as for 2.2. Moreover we know that $\mathrm{Ker}\theta = \pi_*\mathcal{O}_{\mathcal{C}}(*\sum s_i)$.

Thanks to this fact, we dispose the following functor :

$$\Delta : (\hat{u}_N(1)\text{-modules }) \to (D_{r^*\mathcal{L}}\text{-modules})$$

3.3 We define the module of conformal blocks in the following way. Put (cf.§0)

$$\mathfrak{M} = \Delta(F_0^{\otimes N}).$$
$$\mathfrak{N} = (r_*\mathfrak{M})^{\prod_{i=1}^N \mathbb{G}_m(\widehat{\mathcal{O}}_{s_i})}$$

$\mathcal{O}_{P^{(\infty)}}$-module $\mathfrak{N}$ has a structure of $D_{\mathcal{L}}$-module inherited from that of $D_{r^*\mathcal{L}}$-module on $\mathfrak{M}$.

We define the module of conformal blocks to be the direct image $p_*\mathfrak{N}$, which has a priori a structure of $p_*D_{\mathcal{L}}$-module.

Before explaining how a structure of twisted D-module on $p_*\mathfrak{N}$ is deduced, we define the "dual" of $p_*\mathfrak{N}$.

Recall that the completion $F^{\dagger}_{\lambda,w}$ of the Fock space $F_{\lambda,w}$ with respect to the degree equals the topological dual of $F_{-\lambda,w}$ considered as a left $\hat{u}(1)$-module via the anti-automorphism of $U(\hat{u}(1))$ which is $-id$ on $\hat{u}(1)$. We use the notation $F^{*}_{\lambda,w}$ for the same space considered as a right module.

There is a natural pairing called "expectation value" :

$$F^*_{\lambda,w} \times F_{\lambda,w} \xrightarrow{<|>} \mathbf{C},$$

or

$$F^\dagger_{-\lambda,w} \times F_{\lambda,w} \longrightarrow \mathbf{C}.$$

This gives rise to :

$$\mathcal{O}_{P(\#)} \otimes F^\dagger_{-\lambda,w} \times \mathcal{O}_{P(\#)} \otimes F_{\lambda,w} \longrightarrow \mathcal{O}_{P(\#)}$$

We define the dual of $\mathfrak{M}$ to be the orthogonal space to it :

$$\mathfrak{M}^\dagger := \{u \in \mathcal{O}_{P(\#)} \otimes (F^{\dagger}_{0,0}{}^{\widehat{\otimes} N}); f \cdot u = 0 \quad \text{for all} \quad f \in \mathrm{Ker}\theta\}$$

This has a structure of $D_{r^*\mathcal{L}^{-1}}$-module.

We set

$$\mathfrak{N}^\dagger := (r_* \mathfrak{M}^\dagger)^{\prod_{i=1}^N \mathfrak{G}_m(\widehat{\mathcal{O}}_{s_i})},$$

which is naturally a $D_{\mathcal{L}^{-1}}$-module.

Then $p_* \mathfrak{N}^\dagger$ is called as the module of conformal (co-)blocks.

So far $p_* \mathfrak{N}$, $p_* \mathfrak{N}^\dagger$ have only the module structure over $D_\mathcal{L}$,$D_{\mathcal{L}^{-1}}$ respectively. The following result shows that they have twisted D-module structure.

3.4 PROPOSITION [BK]. *$p_* D_\mathcal{L}$ (resp. $p_* D_{\mathcal{L}^{-1}}$) is a tdo. Moreover we have : $p_* D_{\mathcal{L}^{-1}} = D_{\frac{1}{2}\lambda_H}$, where λ_H is the Hodge line bundle, cf.2.1.*

The filtration for tdo structure comes from the spectral sequence with respect to the filtration as to the order of operators along the base space of p.

We refrain from giving the exact statement of the following :

3.5 SCHOLIE. *"$p_* D_{\mathcal{L}^{-1}} = D_{\frac{1}{2}\lambda_H}$" is compatible with the Sugawara construction and embodies the heat equation.*

This implies that $p_* \mathfrak{N}^\dagger$ satisfies the gauge condition and the equation of motion in the sense of [KNTY,§7], where the case of $N = 1, g \geq 2$ is considered. But it doesn't obey the modular transformation property and doesn't characterize the τ-function (or θ-function), cf.3.6.

3.6 "Plücker embedding"

The determinant line bundle $\mathcal{L}$ is known to equal $\mathcal{O}(-\Theta)$ for the theta divisor Θ on $Pic^{g-1}_{\mathcal{C}/\mathcal{M}}$, cf.[Sz]. We relate $\mathcal{L}$ or its inverse to the structure of the modules of conformal blocks.

Let us calculate a fiber of the determinant line bundle $\mathcal{L}^{-1}$. Let

$$\mathcal{X} = (C; x_1, \dots, x_N; t_1, \dots, t_N; L; v_1, \dots, v_N)$$

be a point of $P^{(\#)}$. Then the fiber at $\mathcal{X}$ of $r^*\mathcal{L}^{-1}$ is isomorphic to $d(\omega_C \otimes L^{-1}) := \det R\Gamma(C, \omega_C \otimes L^{-1})$ where ω_C denotes the dualizing sheaf of the curve C. From the exact sequence

$$0 \to \omega_C \otimes L^{-1} \to \omega_C \otimes L^{-1}(m \sum_i x_i) \to \oplus_{i=1}^N \oplus_{k=-1}^{-m} \mathbb{C} z_i^k dz_i \to 0$$

we have

$$d(\omega_C \otimes L^{-1}) = d(\omega_C \otimes L^{-1}(m \sum_i x_i)) \cdot \wedge^{max}(\oplus_{i=1}^N \oplus_{k=-1}^{-m} \mathbb{C} z_i^k dz_i)^{-1}.$$

Note that the data t_i's and v_i's are necessary here.

Recall that $F_0^\dagger$ is realized as a semi-infinite form module, which is obtained from the vector space $\mathbb{C}((z))$ by semi-infinite wedge product, cf.[KNTY,§1].

In our situation, the isomorphisms v_i 's induce an embedding :

$$H^0(C, L^{-1} \otimes \omega_C(* \sum x_i)) \hookrightarrow \bigoplus_{i=1}^N \mathbb{C}((z_i)).$$

Then, it defines a line in $(F_0^\dagger)^{\widehat{\otimes} N}$ by the semi-infinite wedge product. This leads to the following natural embedding

$$r^*\mathcal{L}^{-1} \hookrightarrow \mathfrak{M}^*$$

of $D_{r^*\mathcal{L}^{-1}}$-modules as well as the natural embedding

$$\mathcal{L}^{-1} \hookrightarrow \mathfrak{N}.$$

Hence we have

$$p_*\mathcal{L}^{-1} \hookrightarrow p_*\mathfrak{N}.$$

We have similar construction for the "duals".

The basic problem is to understand the structure of $p_*\mathfrak{N}$ or $p_*\mathfrak{N}^*$. This can be done by the above embedding of $p_*\mathcal{L}$ or $p_*\mathcal{L}^{-1}$ and the consideration of theta structures, cf.[SU]. As to the chracterization of the θ-function mentioned in 3.5, we may say that the action of the theta group replaces that of the modular group.

§4 Comments on factorization property

4.1 "Line bundles on moduli"

To formulate the factorization property for our D-modules of conformal blocks, we need to know the boundary behaviour of the basic line bundle $\mathcal{L}$.

Consider the diagram :

$$
\begin{tikzcd}
\overline{P}_g \arrow[r, phantom, "\supset"] \arrow[d, "p"'] & P^b \arrow[d] & & \sigma^* P^b \arrow[r, "\varpi"] \arrow[d] & P_{g-1} \arrow[dl] \\
\overline{\mathcal{M}}_{g,N} \arrow[r, phantom, "\supset"] & D_0 \simeq \overline{\mathcal{M}}_{g-1,N+2}/\mathfrak{S}_2 & & \overline{\mathcal{M}}_{g-1,N+2}/\mathfrak{S}_2 \arrow[ll, "\sigma"'] &
\end{tikzcd}
$$

Here D_0 is the open dense subset corresponding to smooth curves of the irreducible divisor of $\overline{\mathcal{M}}_{g,N}$ whose general point represents an irreducible curve with only one node. The left square is cartesian and ϖ has a structure of $\mathbb{C}^*$-bundle.

Then we have

4.2 PROPOSITION.

$$\sigma^* \mathcal{L}_g|_{D_0} \simeq \varpi^* \mathcal{L}_{g-1}.$$

This results is analogous to the theorem of Beilinson and Manin [BM] which states that the restriction of the Hodge line bundle to D_0 is again the Hodge line bundle of the genus less by one.

4.3 It is implicit (or semi-explicit) in [TUY] that The so-called factorization property of the conformal blocks (in the non-abelian CFT) can be formulated in terms of nearby-cycle functor. Thus the above result may be viewed as a preliminary to formulate the factorization property of the conformal blocks along the boundary D_0.

In [SU], we develop necessary techniques for this purpose such as the nearby cycle functor for twisted D-modules, correspondence with monodromic D-modules on the total sspace of line bundles, etc.

We have to care about the compactification of our Picard schemes and D-modules on (singular) algebraic stacks,cf.[OS].

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