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Kyoto University
Continuation of Real Analytic Solutions of Partial Differential Equations up to Convex Conical Singularities

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In his talk at the RIMS Seminar in December 1985, Kaneko gave the following conjecture (cf. [Kn3]):

Kaneko’s Conjecture. Let $P = D_t^2 - \Delta$ be the wave operator on the Euclidean $n$ space $\mathbb{R}^n$. Let $\Gamma$ be a closed convex proper cone of $\mathbb{R}^n$ with vertex at the origin, sharp enough in a certain direction; i.e., $\Gamma$ is contained in $\{x_1 \geq C|x_2|\}$ for a Euclidean coordinate $(x_1, \cdots, x_n)$ of $\mathbb{R}^n$, for a large $C > 0$. Let $R > 0$ and set $K = \{(x, t) \in \mathbb{R}^n \times \mathbb{R} | x \in \Gamma, |t| \leq R|x|\}$. Then any real analytic solution to the wave equation $Pu = 0$ defined outside $K$ can be analytically continued up to the origin $(0, 0)$ of $\mathbb{R}^n \times \mathbb{R}$.

We give an answer to this conjecture in a general context.

Definition. Let $K$ be a closed subset of a real analytic manifold $M$ of dimension $n$. $K$ is said to be $C^\alpha$-convex at $x \in M$ $(1 \leq \alpha \leq \omega)$ if there exist a neighborhood $U$ of $x$ and an open $C^\alpha$-immersion $\phi : U \to \mathbb{R}^n$ such that $\phi(U \cap K)$ is convex in $\mathbb{R}^n$. $K$ is said to have a conical singularity at $x$ if $x \in K$ and the tangent cone $C_x(K)$ is a closed proper cone of $T_xM$.

Theorem 0.1. Let $K$ be a $C^1$-convex closed subset of a real analytic manifold $M$, having a conical singularity at $x$. Let $P = P(x, D)$ be a second order differential operator with analytic coefficients defined in a neighborhood of $x$. Assume that $P$ is of real principal type and is not elliptic. Then any real analytic solution to the equation $Pu = 0$ defined outside $K$ is analytically continued up to $x$.

In order to state a similar result for overdetermined systems of differential equations, we first recall the notion of a virtual bicharacteristic manifold of a system $\mathcal{M}$ of differential equations.

Let $V = \text{Char}(\mathcal{M})$; $V^c$ denotes the complex conjugate of $V$ with respect to $T_M^*X$. Let $p \in V \cap (T_M^*X \setminus M)$. Assume the following:

(b.1) $V$ is nonsingular at $p$.
(b.2) $V$ and $V^c$ intersect cleanly at $p$; i.e., $V \cap V^c$ is a smooth manifold and

$$T_pV \cap T_pV^c = T_p(V \cap V^c).$$

(b.3) $V \cap V^c$ is regular; i.e., $\omega|_{V \cap V^c} \neq 0$, with $\omega$ being the fundamental 1-form on $T^*X$.
(b.4) The generalized Levi form of $V$ has constant rank in a neighborhood of $p$. 
Then one can define the virtual bicharacteristic manifold $\Lambda_p$ of $\mathcal{M}$ passing through $p$ (cf. [SKK, Ch.III, Sect.2.4]). We assume

(b.5) $d\pi(T_p\Lambda_p) \neq \{0\}$.

**Theorem 0.2.** Let $(K, x)$ be as in Theorem 0.1. Let $\mathcal{M}$ be a system of differential equations defined in a neighborhood of $x$. Assume that $\text{Char}(\mathcal{M}) \cap \pi^{-1}(x)$ has codimension $\geq 2$ in $\pi^{-1}(x)$ and that $V = \text{Char}(\mathcal{M})$ satisfies conditions (b.1)–(b.5) at each point $p$ of $V \cap (T^*_M X \setminus \mathcal{M}) \cap \pi^{-1}(x)$. Then any real analytic solution to $\mathcal{M}$ defined outside $K$ is analytically continued up to $x$.

**Corollary.** Let $(K, x)$ be as in Theorem 0.2. Let $\mathcal{M}$ be an elliptic system of differential equations and assume that $\text{Char}(\mathcal{M}) \cap \pi^{-1}(x)$ has codimension $\geq 2$ in $\pi^{-1}(x)$. Then any solution $u$ of $\mathcal{M}$ defined outside $K$ can be analytically continued up to $x$.

**Remark.** Cf. [Kw], theorems 4 and 5, for general results on analytic continuation of the solutions of overdetermined systems of differential equations.

The following theorem is a generalization of Theorem 0.1 to higher order differential equations for $K = \{x_0\}$. Cf. Theorem 17 and Corollary 22 of [Kn2].

**Theorem 0.3.** Let $P = P(x, D)$ be a differential operator of real principal type. Assume that the polynomial $f(x_0; \zeta)$ in $\zeta$ has no elliptic factors. Then any real analytic solution to the equation $Pu = 0$ defined in a neighborhood of $x_0$ except $x_0$ can be analytically continued on the whole of a neighborhood of $x_0$.


