結び目解消操作

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　この報告では、以前に作成した最小交叉数が 9 以下の素な結び目の結び目解消数の表について、訂正及び修正をまずします。その後、結び目解消操作及び一般化された結び目解消操作について参考論文のリストを挙げることにします。結び目解消数 u(K) の評価に用いたのは、次の定理です。

　定理 **1** ([Ms]) 結び目 K について、

$$0 \leq |\sigma(K)|/2 \leq g^*(K) \leq u(K)$$

が成り立つ。ただし、$\sigma(K)$ は K の signature を、$g^*(K)$ は K の4次元種数を表す。

　定理 **2** ([N1]) 結び目 K について、

$$0 \leq m(K) \leq u(K)$$

が成り立つ。ただし、m(K) は K の Alexander 行列の最小次数を表す。

定理 **3** ([L1], [KaM]) ２橋結び目 K については、u(K) = 1 か u(K) ≧ 2 であるかは判定できる。

これらと [N2] で掲げた次の予想に基づいて表が作られています。

予想　結び目の最小交叉の射影図には、結び目解消操作を施せば、結び目解消数が少なくなるような交叉が必ず存在する。

なお、以前の表で、u (9_{29}) = 1 としましたが、以上のどれかに引っかかるわけではなく、1 または 2 に訂正します。

表において、N_n は Alexander-Briggs notation で、N は最小交叉数を、n は交叉数 N の中での順番を表す。σ' は signature の絶対値の半値を、

g* は4次元種数を、m は Alexander 行列の最小次数を表す。また、未確定の数値については、A で 1 または 2 を、X で 2 または 3 を表す。

Table of Unknotting Number

k	σ'	g^*	u	m	k	σ'	g^*	u	m	k	σ'	g^*	u	m	k	σ'	g^*	u	m
3_1	1	1	1	1	8_8	0	0	2	1	9_8	1	1	2	1	9_{29}	1	1	A	1
4_1	0	1	1	1	8_9	0	0	1	1	9_9	3	3	3	1	9_{30}	0	1	1	1
5_1	2	2	2	1	8_{10}	1	1	A	1	9_{10}	2	2	X	1	9_{31}	1	A	2	1
5_2	1	1	1	1	8_{11}	1	1	1	1	9_{11}	2	2	2	1	9_{32}	1	A	A	1
6_1	0	0	1	1	8_{12}	0	1	2	1	9_{12}	1	1	1	1	9_{33}	0	1	1	1
6_2	1	1	1	1	8_{13}	0	1	1	1	9_{13}	2	2	X	1	9_{34}	0	1	1	1
6_3	0	1	1	1	8_{14}	1	1	1	1	9_{14}	0	1	1	1	9_{35}	1	1	X	2
7_1	3	3	3	1	8_{15}	2	2	2	1	9_{15}	1	1	2	1	9_{36}	2	2	2	1
7_2	1	1	1	1	8_{16}	1	A	A	1	9_{16}	3	3	3	1	9_{37}	0	1	2	2
7_3	2	2	2	1	8_{17}	0	1	1	1	9_{17}	1	A	A	1	9_{38}	2	2	X	A
7_4	1	1	2	1	8_{18}	0	A	2	2	9_{18}	2	2	2	1	9_{39}	1	1	1	1
7_5	2	2	2	1	8_{19}	3	3	3	1	9_{19}	0	1	1	1	9_{40}	1	A	2	2
7_6	1	1	1	1	8_{20}	0	0	1	1	9_{20}	2	2	2	1	9_{41}	0	0	2	2
7_7	0	1	1	1	8_{21}	1	1	1	1	9_{21}	1	1	1	1	9_{42}	1	1	1	1
8_1	0	1	1	1	9_1	4	4	4	1	9_{22}	1	1	1	1	9_{43}	2	2	2	1
8_2	2	2	2	1	9_2	1	1	1	1	9_{23}	2	2	2	1	9_{44}	0	1	1	1
8_3	0	1	2	1	9_3	3	3	3	1	9_{24}	0	1	1	1	9_{45}	1	1	1	1
8_4	1	1	2	1	9_4	2	2	2	1	9_{25}	1	1	A	1	9_{46}	0	0	2	2
8_5	2	2	2	1	9_5	1	1	2	1	9_{26}	1	1	1	1	9_{47}	1	A	2	2
8_6	1	1	2	1	9_6	3	3	3	1	9_{27}	0	0	1	1	9_{48}	1	A	2	2
8_7	1	1	1	1	9_7	2	2	2	1	9_{28}	1	1	1	1	9_{49}	2	2	X	2

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