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1. INTRODUCTION AND PRELIMINARY

In the last ten years, cohomological dimension theory has striking development. A motivation of the development is surely the Edwards-Walsh theorem, [24], as follows:

1.1. Theorem. Every compact metric space $X$ of cohomological dimension $c \cdot \dim Z X \leq n$ (integer coefficient) is the image of a cell-like map $f: Z \rightarrow X$ from a compact metric space $Z$ of $\dim Z \leq n$.

Not only the result but also techniques of the proof gave an important influence to the development. After them, L. R. Rubin and P. J. Schapiro [22] showed the noncompact version of the Edwards-Walsh theorem and S. Mardešić and L. R. Rubin [17] gave the nonmetrizable version. On the other hand, A. N. Dranishnikov, [5] and [6], characterized cohomological dimension with respect to $Z_p$ by the Edwards-Walsh's way and showed the Edwards-Walsh-like theorem:

1.2. Theorem. Every compact metric space $X$ of cohomological dimension with respect to $Z_p$, $c \cdot \dim Z_p X \leq n$, is the image of a map $f: Z \rightarrow X$ from a compact metric space $Z$ of $\dim Z \leq n$ whose fibers are acyclic modulo $p$.

Motivated above results and Mardešić's characterization of $c \cdot \dim Z X \leq n$, we will show a characterization of $c \cdot \dim Z_p X \leq n$ for noncompact case. Using the characterization, we will give the existence of an acyclic resolution modulo $p$. In fact, our characterization suggests a dimension-like function, called approximable dimension, and can obtain the following more general results.

1.3. Theorem. Let $X$ be a metrizable space having approximable dimension with respect to an arbitrary coefficients $G \leq n$. Then there exists a map $f: Z \rightarrow X$ from a metrizable space $Z$ of $\dim Z \leq n$ and $w(Z) \leq w(X)$ onto $X$ such that $H^* (f^{-1}(x); G) = 0$ for all $x \in X$.

As its consequence, we have noncompact versions of Theorems 1.1 and 1.2. We may call such a mapping $f$ an acyclic resolution of $X$ (with respect to $G$), specially, in the
case of $G = \mathbb{Z}_p$, an acyclic resolution of $X$ modulo $p$. Finally we will note that there exists a compact metric space $X$ of $c$-$\dim \mathbb{Q}X = 1$ which does not admit an acyclic resolution with respect to $\mathbb{Q}$ [11,12]. Thereby we can see that approximable dimension is different from cohomological dimension and Theorem 1.3 is a good property obtained from approximable dimension.

In this paper, we mean the definition of cohomological dimension as follows: the cohomological dimension of a space $X$ with respect to a coefficient group $G$ is less than and equal to $n$, denoted by $c$-$\dim_G X \leq n$, provided that every map $f: A \to K(G,n)$ of a closed subset $A$ of $X$ into an Eilenberg-MacLane space $K(G,n)$ of type $(G,n)$ admits a continuous extension over $X$ (c.f. [10]). The dimension of a space $X$ means the covering dimension of $X$ and denotes by $\dim X$. $\mathbb{Z}$ is the additive group of all integers and for each prime number $p$, $\mathbb{Z}_p$ is the cyclic group of order $p$.

By a polyhedron we mean the space $|K|$ of a simplicial complex $K$ with the Whitehead topology. In section 5, the topology of $|K|$ may be generated by a uniformity [Appendix, 22].

If $v$ is a vertex of a simplicial complex $K$, let $\text{st}(v,K)$ be the open star of $v$ in $|K|$ and $\text{st}(v,K)$ be the closed star of $v$ in $|K|$. If $A \subseteq |K|$, then we define $\text{st}(A,K) = \bigcup \{\text{Int} \sigma : \sigma \in K, \sigma \cap A \neq \emptyset\}$ and $\text{st}(A,K) = \bigcup \{\sigma : \sigma \in K, \sigma \cap A \neq \emptyset\}$. The symbol $\text{Sd}_j K$ means the $j$-th barycentric subdivision of $K$. We define the symbols $S_i$ and $\tilde{S}_i$ for a simplicial complex $K_i$ with an index to be the cover $\{\text{st}(v,K_i) : v \in K_i^{(0)}\}$ and the cover $\{\text{st}(v,K_i) : v \in K_i^{(0)}\}$, respectively.

We use the symbol $\prec$ both to mean 'refine' for covers and 'subdivides' for subdivisions of a complex. The symbol $\prec^*$ is used for star refines.

Let $\mathcal{U}$ be an open cover of a space $X$. Then for $U \in \mathcal{U}$,

$$
\text{st}(U,\mathcal{U}) = \text{st}^1(U,\mathcal{U}) = \bigcup \{U' : U' \in \mathcal{U}, U' \cap U \neq \emptyset\},
$$

$$
\text{st}^{j+1}(U,\mathcal{U}) = \bigcup \{U' : U' \in \mathcal{U}, U' \cap \text{st}^j(U,\mathcal{U}) \neq \emptyset\}.
$$

By $\text{st}^j(\mathcal{U})$ we mean the cover $\{\text{st}^j(U,\mathcal{U}) : U \in \mathcal{U}\}$. If $f$ and $g$ are maps from a space $Z$ to a space $X$, $(f,g) \leq \mathcal{U}$ means that for each $z \in Z$, there exists $U \in \mathcal{U}$ with $f(z), g(z) \in U$. If $X$ is a metric space with a metric $d$, we write $(f,g) \leq \varepsilon$ instead of $(f,g) \leq \mathcal{U}_\varepsilon$, where $\mathcal{U}_\varepsilon$ is the cover whose consists of all $\varepsilon/2$-neighborhoods in $X$. By the symbol $\mathcal{N}(\mathcal{U})$ we mean the nerve of the cover $\mathcal{U}$. For covers $\mathcal{U}, \mathcal{V}$, the symbol $\mathcal{U} \wedge \mathcal{V}$ is used for the following cover $\{U \cap V, U, V : U \in \mathcal{U}, V \in \mathcal{V}\}$.

2. Edwards-Walsh complexes

In the latter section, we need Edwards-Walsh complexes for arbitrary simplicial complexes.

2.1. Lemma. Let $|L|$ be a simplicial complex with the Whitehead topology, $p$ be a prime number and $n$ be a natural number. Then there exists a combinatorial map (i.e.
\(\pi_{L}^{-1}(L')\) is a subcomplex of \(E\!W_{Z_{p}}(L, n)\) if \(L'\) is a subcomplex of \(L\) \(\psi_{L} : E\!W_{Z_{p}}(L, n) \to |L|\) such that

(i) for \(\sigma \in L\) with \(\dim \sigma \geq n+1\), \(\psi_{L}^{-1}(\sigma) \in K(\oplus_{1}^{r}Z_{p}, n)\), where \(r_{\sigma} = \text{rank} \, \pi_{n}(\sigma(n))\),

(ii) for \(\sigma \in L\) with \(\dim \sigma \leq n\), \(\psi_{L}^{-1}(\sigma) = \sigma\),

(iii) \(E\!W_{Z_{p}}(L, n)\) is a CW-complex,

(iv) \(\psi_{L}^{-1}(\sigma)\) is a subcomplex of \(E\!W_{Z_{p}}(L, n)\) with respect to the triangulation in (3),

(v) \(\psi_{L}^{-1}(\sigma)^{(k)}\) is a finite CW-complex for \(k \geq n\),

(vi) for any subcomplex \(L'\) of \(L\) and map \(f : |L'| \to K(Z_{p}, n)\), there exists an extension of \(f \circ \psi_{L}|_{\psi_{L}^{-1}(|L'|)}\).

**Sketch of Proof.** We give its proof by using Edwards-Walsh’s modification by Dranishnikov [6]. By the induction on \(\dim L\), \(\psi_{L}\) is constructed to satisfy the following:

1. \(\psi_{L}^{-1}(L^{(n)}) = L^{(n)}\) is a subcomplex of \(E\!W_{Z_{p}}(L, n)\) and \(\psi_{L}|_{L^{(n)}} = \text{id}_{L^{(n)}}\).

Let \(\sigma\) be a simplex of \(L\) with \(\dim \sigma = n + 1\). Let \(K(\sigma)\) be an Eilenberg-MacLane space of type \((Z_{p}, n)\) obtained from \(\partial \sigma\) by attaching an \((n+1)\)-cell by a map of degree \(p\). Hence

2. \(K(\sigma)^{(n)} = \partial \sigma\) and \(K(\sigma)^{(n+1)} = \partial \sigma \cup_{\alpha} B^{n+1}\), where \(\alpha : \partial B^{n+1} \to \partial \sigma\) is a map of degree \(p\).

If \(\dim \sigma \geq n + 2\) and \(n \geq 2\), then \(K(\sigma) = K_{1}(\sigma) \cup K_{2}(\sigma) \cup \ldots\) such that

3. \(K_{1}(\sigma) = \bigcup_{\tau \not\subset \sigma} K(\tau)\), where the union is taken over all proper faces \(\tau\) of \(\sigma\),

4. for \(i = 2, 3, \ldots\), \(K_{i}(\sigma)\) is obtained from \(K_{i-1}(\sigma)\) by attaching to \(K_{i-1}(\sigma)^{(n+i-1)}\)

a finite collection of \((n+i)\)-cells killing the \((n+i-1)\)-th homotopy group.

If \(\dim \sigma \geq n + 2\) and \(n = 1\), then \(K(\sigma) = K_{1}(\sigma) \cup K_{2}(\sigma) \cup \ldots\) such that

5. \(K_{1}(\sigma)\) is obtained from \(\bigcup_{\tau \not\subset \sigma} K(\tau)\), by attaching finite collection of 2-cells abelianizing the fundamental group,

6. for \(i = 2, 3, \ldots\), \(K_{i}(\sigma)\) is obtained from \(K_{i-1}(\sigma)\) by attaching to \(K_{i-1}(\sigma)^{(n+i-1)}\)

a finite collection of \((n+i)\)-cells killing the \((n+i-1)\)-th homotopy group.

Then we construct as

7. \(\psi_{L}^{-1}(\sigma)\) is the mapping cylinder \(M_{\sigma}\) of the embedding \(j_{\sigma} : \psi_{L}^{-1}(\partial \sigma) \hookrightarrow K(\sigma)\),

8. \(\psi_{L}|_{M_{\sigma}}\) is the cone of \(\psi_{L}|_{\psi_{L}^{-1}(\partial \sigma)}\) such that \(\psi_{L}(K(\sigma))\) is the barycentre of \(\sigma\).

Hence for each simplex \(\sigma\) of \(\dim \sigma \geq n + 1\), we have the property:

9. if \(n \geq 2\),

\[\psi_{L}^{-1}(\sigma)^{(n+1)} = \sigma^{(n)} \times [0, 1] \cup_{\alpha_{1}} B^{n+1} \cup_{\alpha_{2}} \ldots \cup_{\alpha_{r}} B^{n+1},\]

where for each \((n+1)\)-dimensional face \(\tau_{i}\) of \(\sigma\), \(\alpha_{i} : \partial B^{n+1} \to \partial \tau_{i} \times \{1\}\) is a map of degree \(p\),
(10) if $n = 1$,

$$\psi^{-1}_{L}(\sigma)^{(2)} = \sigma^{(1)} \times [0,1] \cup_{\alpha_{1}} B^{2} \cup_{\alpha_{2}} \cdots \cup_{\alpha_{r_{\sigma}}} B^{2} \cup_{\beta_{1}} B^{2} \cup_{\beta_{2}} \cdots \cup_{\beta_{k_{\sigma}}} B^{2},$$

where for each 2-dimensional face $\tau_{i}$ of $\sigma$, $\alpha_{i}: \partial B^{2} \to \partial \tau_{i} \times \{1\}$ is a map of degree $p$ and the collection $\{[\beta_{1}], \ldots, [\beta_{k_{\sigma}}]\}$ generates the commutator subgroup of $\pi_{1}(\sigma^{(1)} \times [0,1] \cup_{\alpha_{1}} B^{2} \cup_{\alpha_{2}} \cdots \cup_{\alpha_{r_{\sigma}}} B^{2})$. 

3. Characterizations for metrizable spaces

Let us establish definitions. Let $K$ be a simplicial complex and $f, g: X \to |K|$ be maps. We say that $g$ is a $K$-modifying of $f$ if for each $x \in X$ and $\sigma \in K$, $f(x) \in \sigma$ implies $g(x) \in \sigma$. Let $\mathcal{U}$ be an open cover of $X$. Then a map $b: X \to |\mathcal{N}(\mathcal{U})|$ is called $\mathcal{U}$-normal map if $b^{-1}(st(U, \mathcal{U})) = U$ for each $U \in \mathcal{U}$ and $b$ is essential on each simplex of $\mathcal{N}(\mathcal{U})$ (i.e. $b^{-1}(\sigma): b^{-1}(\sigma) \to \sigma$ is a essential map for each $\sigma \in \mathcal{N}(\mathcal{U})$). Note that if $\mathcal{U}$ is a locally finite, then $\mathcal{U}$-normal map exists.

3.1. Definition. Let $Q$, $P$ be polyhedra, $G$ be an abelian group, $\mathcal{U}$ be an open cover of $P$ and $n$ be a natural number. We say that a map $\psi: Q \to P$ is $(G, n, \mathcal{U})$-approximable if there exists a triangulation $L$ of $P$ such that for any triangulation $M$ of $Q$ there is a PL-map $\psi': |M^{(n)}| \to |L^{(n)}|$ satisfying the following conditions:

(i) $(\psi, \psi|_{M^{(n)}}) \leq \mathcal{U}$,

(ii) for any map $\alpha: |L^{(n)}| \to K(G, n)$, there exists an extension $\beta: |M^{(n+1)}| \to K(G, n)$ of $\alpha \circ \psi'$.

3.2. Definition. Let $G$ be an abelian group and $n$ be a natural number. A map $f: X \to P$ of a metrizable space $X$ to a polyhedron $P$ is called $(G, n)$-cohomological if for any open cover $\mathcal{U}$ of $P$ there exist a polyhedron $Q$ and maps $\varphi: X \to Q$, $\psi: Q \to P$ such that

(i) $(\psi \circ \varphi, f) \leq \mathcal{U}$,

(ii) $\psi$ is $(G, n, \mathcal{U})$-approximable.

3.3. Theorem. Let $X$ be a metrizable space, $p$ be a prime number and $n$ be a natural number. Then $X$ has cohomological dimension with respect to $\mathbb{Z}_{p}$ of less than and equal to $n$ if and only if every map $f$ of $X$ to a polyhedron $P$ is $(\mathbb{Z}_{p}, n)$-cohomological.

Proof of necessity. Suppose that $c\dim_{\mathbb{Z}_{p}} X \leq n$. Let $f: X \to P$ be a map of $X$ to a polyhedron $P$ and $\mathcal{U}$ be an open cover of $P$ Then take a star refinement $\mathcal{U}_{0}$ of $\mathcal{U}$.

First, we show that there exist a simplicial complex $K$ and maps $\varphi: X \to |K|$, $\psi: |K| \to P$ such that

(1) if $\sigma \in K$, there exists $U \in \mathcal{U}_{0}$ with $\psi(\sigma) \subseteq U$,
(2) for each $x \in X$ if $\varphi(x) \in \text{Int } \sigma$, $\sigma \in K$, there exists $U \in \mathcal{U}_0$ with $\psi(\sigma) \cup \{f(x)\} \subseteq U$,

(3) there exist a triangulation $L$ of $P$ and a PL-map $\psi': |K^{(n)}| \to |L^{(n)}|$ such that
   (i) $(\psi', \psi|_{|K^{(n)}|}) \leq \mathcal{U}_0$
   (ii) for any map $\alpha: |L^{(n)}| \to K(G, n)$ there is an extension $\beta: |K^{(n+1)}| \to K(G, n)$ of $\alpha \circ \psi'$.

By J. H. C. Whitehead's theorem [25], take a triangulation $L$ of $P$ such that

(4) st \{\mathcal{B}(v, L) : v \in L^{(0)}\} \prec \mathcal{U}_0.

We will construct a map $c: X \to \text{EW}_{Z_p}(L, n)$ such that

(5) $c|_{f^{-1}(\{L^{(n)}\})} = f|_{f^{-1}(\{L^{(n)}\})}$,

(6) $c(f^{-1}(\sigma)) \subseteq \psi_L^{-1}(\sigma)$ for $\sigma \in L$, where $\psi_L: \text{EW}_{Z_p}(L, n) \to L$ is the map constructed in Lemma 2.1.

We define the map $c_n \equiv f|_{f^{-1}(\{L^{(n)}\})}: f^{-1}(\{L^{(n)}\}) \to |L^{(n)}| \subseteq \text{EW}_{Z_p}(L, n)$. Inductively, suppose that for $n \leq k$ we have defined the function $c_k: f^{-1}(\{L^{(k)}\}) \to \text{EW}_{Z_p}(L, n)$ such that $c_k|_{f^{-1}(\sigma)}: f^{-1}(\sigma) \to \psi_L^{-1}(\sigma) \subseteq \text{EW}_{Z_p}(L, n)$ is continuous and $c_k|_{f^{-1}(\sigma)} = c_k|_{f^{-1}(\tau)}$ on $f^{-1}(\sigma) \cap f^{-1}(\tau)$ for $\sigma, \tau \in \{L^{(k)}\}$. Now, let $\sigma \in L$ with $\dim \sigma = k + 1$. By the construction of $c_k$ and $\text{EW}_{Z_p}(L, n)$, $c_k|_{f^{-1}(\partial \sigma)}: \partial \sigma \to \psi_L^{-1}(\sigma)$ is continuous. Hence by $c$-$\dim Z_p f^{-1}(\sigma) \leq c$-$\dim Z_p X \leq n$ and (i) in Lemma 2.1, we have an continuous extension $c_\sigma: f^{-1}(\sigma) \to \psi_L^{-1}(\sigma)$ of $c_k|_{f^{-1}(\partial \sigma)}$. Define $c_{k+1}$ to be $c_\sigma$ on $f^{-1}(\sigma)$ for $\sigma \in L$ with $\dim \sigma = k + 1$. Finally, we define $c$ to be $\bigcup_{k=n}^{\infty} c_k$. Then since $X$ is compactly generated, the function $c$ is continuous.

We define an open cover $B = \{B_\sigma : \sigma \in L\}$ in the following way:

$$B_\sigma \equiv \text{EW}_{Z_p}(L, n) \setminus \bigcup \{\psi_L^{-1}(\sigma): \sigma \cap \tau = \emptyset\}.$$ 

Then note that we have

(7) $\psi_L^{-1}(\sigma) \subseteq B_\sigma$

(8) if $x \in B_\sigma$ and $x \in \psi_L^{-1}(\tau), \sigma \cap \tau \neq \emptyset$.

Since $\text{EW}_{Z_p}(L, n)$ is LC$^n$, for a star refinement $B_1$ of $B$, there exists an open refinement $B_2$ of $B_1$ such that if $K$ is a simplicial complex of $\dim K \leq n + 1$, then every partial realization of $K$ in $\text{EW}_{Z_p}(L, n)$ relative to $B_2$ extended to a full realization relative to $B_1$ [2]. Select a star refinement $B_3$ of $B_2$.

Then by [21, Lemma 9.6], there exist an open cover $\mathcal{V}$ of $X$ refining $f^{-1}(\mathcal{U}_0) \wedge c^{-1}(B_3)$ and maps $\varphi: X \to |\mathcal{N}(\mathcal{V})|, \psi: |\mathcal{N}(\mathcal{V})| \to P$ such that

(9) $\varphi$ is $\mathcal{V}$-normal,

(10) $\psi \circ \varphi$ is $L$-modification of $f$,

(11) if $\sigma \in \mathcal{N}(\mathcal{V})$, there exists $U \in \mathcal{U}_0$ with $f(\varphi^{-1}(\sigma)) \cup \psi(\sigma) \subseteq U$. 
Then these $\mathcal{N}(\mathcal{V})$, $\varphi$ and $\psi$ satisfy the conditions (1)-(3).

It is easily seen that (11) implies (1) and (2). It remain to prove that (3) holds.

We shall construct a map $\psi_0: |\mathcal{N}(\mathcal{V})^{(n+1)}| \rightarrow \text{EW}_{Z_p}(L, n)$ in the following way: note that if $\langle U \rangle \in \mathcal{N}(\mathcal{V})^{(n+1)}$, there exists $B_U \in B_2$ with $U \subseteq c^{-1}(B_U)$. $\psi_0$ on $|\mathcal{N}(\mathcal{V})^{(0)}|$ is defined by an element $\psi_0(\langle U \rangle) \in B_U$ for each $\langle U \rangle \in \mathcal{N}(\mathcal{V})^{(0)}$. Let $\langle U_0, \ldots, U_m \rangle \in \mathcal{N}(\mathcal{V})^{(n+1)}$. Then by $\emptyset \neq U_0 \cap \cdots \cap U_m \subseteq c^{-1}(B_{U_0}) \cap \cdots \cap c^{-1}(B_{U_m})$, we have

$$\psi_0(\langle U_0, \ldots, U_m \rangle) \subseteq \text{st}(B_{U_0}, B_3) \subseteq B$$

for some $B \in B_2$. It show that $\psi_0$ is a partial realization of $\mathcal{N}(\mathcal{V})^{(n+1)}$ in $\text{EW}_{Z_p}(L, n)$ relative to $B_2$.

Therefore, by the construction of $B_2$, we may define $\psi_0$ to be a full realization relative to $B_1$. Then by the same way in [21, p245 (8)] we can show that

(12) if $t \in |\mathcal{N}(\mathcal{V})^{(n+1)}|$ with $\psi(t) \in \text{Int} \delta$ and $\psi_0(t) \in \psi_L^{-1}(\tau)$ for $\delta, \tau \in L$, then there exist $\sigma, \lambda \in L$ such that $\delta \prec \sigma$ and $\sigma \cap \lambda \neq \emptyset \neq \lambda \cap \tau$.

Now, by the property (v) in Lemma 2.1, we can choose

(13) a cellular map $\psi: |\mathcal{N}(\mathcal{V})^{(n+1)}| \rightarrow \text{EW}_{Z_p}(L, n)^{(n+1)}$ such that for each $t \in |\mathcal{N}(\mathcal{V})^{(n+1)}|$, if $\psi_0(t) \in \psi_L^{-1}(\tau)$, then $\psi_1(t) \in \psi_L^{-1}(\tau)^{(n+1)}$.

By the simplicial approximation theorem, we assume that $\psi_1$ is PL.

If $n \geq 2$, by the properties (9) and (1) in Lemma 2.1, we have

$$\text{EW}_{Z_p}(L, n)^{(n+1)} = |L^{(n)}| \cup \bigcup \{\partial \sigma \times [0, 1] \cup_{\alpha_{\sigma}} B^{n+1}_{\sigma} : \sigma \in L, \dim \sigma = n+1\},$$

where $\alpha_{\sigma}: \partial B^{n+1}_{\sigma} \rightarrow \partial \sigma$ is a map of degree $p$. For each $(n+1)$-simplex $\sigma$ of $L$, choose a point $z_{\sigma} \in B^{n+1}_{\sigma} \setminus \partial B^{n+1}_{\sigma}$, and take the retraction

$$r: \text{EW}_{Z_p}(L, n)^{(n+1)} \setminus \{z_{\sigma} \in L, \dim \sigma = n+1\} \rightarrow |L^{(n)}|$$

induced by the compositions of the radial projection of $B^{n+1}_{\sigma} \setminus \{z_{\sigma}\}$ onto $\partial \sigma \times \{1\}$ and the natural projection of $\partial \sigma \times [0, 1]$ onto $\partial \sigma \times \{0\} \subseteq |L^{(n)}|$.

If $n = 1$, for every simplex $\sigma$ of $\dim \sigma \geq 2$, $\psi^{-1}(\sigma^{(2)})$ may be represented as the form (10) in Lemma 2.1:

$$\psi^{-1}(\sigma)^{(2)} = \sigma^{(1)} \times [0, 1] \cup_{\alpha_1} B^2 \cup_{\alpha_2} \cdots \cup_{\alpha_{\tau_2}} B^2 \cup_{\beta_1} B^2 \cup_{\beta_2} \cdots \cup_{\beta_{k_\sigma}} B^2.$$

Then choose points $u^1_{\sigma}, \ldots, u^{r_\sigma}_{\sigma}, v^1_{\sigma}, \ldots, v^{s_{\sigma}}_{\sigma}$ of $\psi^{-1}(\sigma^{(2)}) \setminus \sigma^{(1)} \times [0, 1]$ for each $B^2$ and the retraction $r: \text{EW}_{Z_p}(L, n)^{(2)} \setminus \{u^1_{\sigma}, \ldots, u^{r_\sigma}_{\sigma}, v^1_{\sigma}, \ldots, v^{s_{\sigma}}_{\sigma}\} \subseteq \text{Int} \delta \times [0, 1]$ onto $\sigma^{(1)} \times [0, 1]$ and the natural projection of $\sigma^{(1)} \times [0, 1]$ onto $\sigma^{(1)} \times \{0\}$ onto $\sigma^{(1)} \times \{0\}$ onto $\text{EW}_{Z_p}(L, n)^{(1)}$. In both cases, we put

$$\psi' \equiv r \circ \psi_1|_{\mathcal{N}(\mathcal{V})^{(n)}}: |\mathcal{N}(\mathcal{V})^{(n)}| \rightarrow |L^{(n)}|.$$
Then the map $\psi'$ holds the conditions (i), (ii). First, we show the condition (i). Let $t \in |\mathcal{N}(\mathcal{V})^{(n)}|$. By (12), there exist $\sigma, \lambda, \tau \in L$ such that $\sigma \cap \lambda \neq \emptyset$ and $\lambda \cap \tau$ and $\psi(t) \in \sigma$, $\psi_0(t) \in \psi_L^{-1}(\tau)$. Then since $\psi_1(t)$ is an element of $\psi_L^{-1}(\tau)^{(n)}$, we have $\psi'(t) \in \tau$. Hence, we have $\psi(t), \psi'(t) \in \text{st}(\lambda, L) \subseteq U$ for some $U \in \mathcal{U}_0$ (see (4)). Next, we must show the condition (ii). But, it is easy to show that. Hence, we omitted it here.

Now, we shall show that $f$ is $(\mathbb{Z}_p, n)$-cohomological. By (2), we can easily see that $(\psi \circ \varphi, f) \leq \mathcal{U}$. So, we show that $\psi$ is $(\mathbb{Z}_p, n, \mathcal{U})$-approximable.

Let $M$ be a triangulation of $|K|$. Note that for a simplicial approximation $j$ of $id_{|M|} : |M| = |K| \rightarrow |K|$ with respect to $K$, we have that

$$j\left(|M^{(n+1)}|\right) \subseteq |K^{(n+1)}| \text{ and } j\left(|M^{(n)}|\right) \subseteq |K^{(n)}|.$$ 

Then by (1) and (3), we can easily see that the map

$$\psi'' \equiv \psi' \circ j : |M^{(n)}| \rightarrow |L^{(n)}|$$

holds the conditions. $\square$

The reverse implication is proved by the standard way [21]. First, we need some notations.

We may assume that the Eilenberg-MacLane space $K(\mathbb{Z}_p, n)$ is a metrizable, locally compact separable space. Then by the Kuratowski-Wojdyslawski’s theorem, we can consider that $K(\mathbb{Z}_p, n)$ is a closed subset of a convex subset $C$ of a normed linear space $E$. Note that $C$ is AR(metrizable spaces). Since $K(\mathbb{Z}_p, n)$ is ANR, there exist a closed neighborhood $F$ in $C$ and a retraction $r : F \rightarrow K(\mathbb{Z}_p, n)$. Further, we can choose an open cover $\mathcal{W}_0$ of $\text{Int}_C F$ such that

(1) for any space $Z$ and any maps $\alpha, \beta : Z \rightarrow F$ with $(\alpha, \beta) \leq \mathcal{W}_0$, the maps

$$r \circ \alpha, r \circ \beta : Z \rightarrow K(\mathbb{Z}_p, n)$$

are homotopic in $K(\mathbb{Z}_p, n)$.

Then we take an open, convex cover $\mathcal{W}$ of $C$ such that

(2) if $W \in \mathcal{W}$ with $W \cap K(\mathbb{Z}_p, n) \neq \emptyset$, there exists $U \in \mathcal{W}_0$ with $\text{st}(W, \mathcal{W}) \subseteq U$.

Select a star refinement $\mathcal{V}$ of $\mathcal{W}$.

Let $h_0 : C \rightarrow |\mathcal{N}(\mathcal{V})|$ be a Kuratowski’s map with respect to $\mathcal{V}$ and define a map $h_1 : |\mathcal{N}(\mathcal{V})| \rightarrow C$ in the following way: a map $h_1$ on $|\mathcal{N}(\mathcal{V})^{(0)}|$ is defined by an element $h_1\left(|\mathcal{N}(\mathcal{V})\right) \in C$ for each $\langle V \rangle \in |\mathcal{N}(\mathcal{V})^{(0)}|$. Next, by using the convexity of $C$, we extend $h_1$ linearly on each simplex $|\mathcal{N}(\mathcal{V})|$. Let $\sigma = \langle V_0, \ldots, V_m \rangle \in |\mathcal{N}(\mathcal{V})|$. Then by $V_0 \cap \cdots \cap V_m \neq \emptyset$,

$$h_1\left(|\mathcal{N}(\mathcal{V})|\right) \subseteq \text{st}(V_0, \mathcal{V}) \subseteq W_\sigma \text{ for some } W_\sigma \in \mathcal{W}.$$ 

Thus, by the construction of $h_1$, we have $h_1(\sigma) \subseteq W_\sigma$. 
Let $\mathcal{N}_1$ be a subcomplex $\mathcal{N}\left(\{V \in \mathcal{V} : V \cap K(Z_p, n) \neq \emptyset\}\right)$ of $\mathcal{N}(\mathcal{V})$. Let $\mathcal{N}_0$ be a simplicial neighborhood of $\mathcal{N}_1$ in $\mathcal{N}(\mathcal{V})$ such that if $(V_0) \in \mathcal{N}_0$, there exists $(V_1) \in \mathcal{N}_1$ with $V_0 \cap V_1 \neq \emptyset$. Then we can easily see the followings:

(3) for each $x \in K(Z_p, n)$, there exists $W \in \mathcal{W}$ with $x, h_1 \circ h_0(x) \in W$,
(4) $h_1(|\mathcal{N}_0|) \subseteq \text{st}(K(Z_p, n), \mathcal{W}) \subseteq F$,
(5) $h_0(K(Z_p, n)) \subseteq |\mathcal{N}_1| \subseteq |\mathcal{N}_0|.$


**Proof of sufficiency.** Let $A$ be a closed subset of $X$ and $h: A \to K(Z_p, n)$ be a map. We consider the above-mentioned nerve $\mathcal{N}(\mathcal{V})$ and maps $h_0, h_1$. We take an open cover $\mathcal{U}$ of $|\mathcal{N}(\mathcal{V})|$ such that

(6) $\text{st}^3(|\mathcal{N}_1|, \mathcal{U}) \subseteq |\mathcal{N}_0|,$
(7) $\text{st}^3(\mathcal{U}) \prec h^{-1}_1(\mathcal{W}),$

and choose a subdivision $\mathcal{N}$ of $\mathcal{N}(\mathcal{V})$ such that if $\sigma \in \mathcal{N}$ there exists $U \in \mathcal{U}$ with $\sigma \subseteq U$.

Since $C$ is AE, there is an extension $H: X \to C$ of $h$. Then by the assumption, the map $h_0 \circ H: X \to |\mathcal{N}(\mathcal{V})|$ is $(Z_p, n)$-cohomological. Hence, there exist a polyhedron $Q$ and maps $\varphi: X \to Q, \psi: Q \to |\mathcal{N}(\mathcal{V})|$ such that

(8) $(\psi \circ \varphi, h_0 \circ H) \leq \mathcal{U},$
(9) $\psi$ is $(Z_p, n, \mathcal{U})$-approximable.

By using the simplicial approximation theorem, we obtain a triangulation $M$ of $Q$ and a simplicial approximation $\psi^*: M \to \mathcal{N}$ of $\psi$. Then by (8),(9), we have

(10) $(\psi^* \circ \varphi, h_0 \circ H) \leq \text{st}\mathcal{U},$
(11) $\psi^*$ is $(Z_p, n, \text{st}\mathcal{U})$-approximable.

Now, by (11) with respect to $M$, there exist a triangulation $L$ and a PL-map $\psi': |M^{(n)}| \to |L^{(n)}|$ such that

(12) $(\psi', \psi^*|_{M^{(n)}}) \leq \text{st}\mathcal{U},$
(13) for any map $\alpha: |L^{(n)}| \to K(Z_p, n)$, there exists an extension $\beta: |M^{(n+1)}| \to K(Z_p, n)$ of $\alpha \circ \psi'$.

**Claim.** There exists a map $\xi: Q \to K(Z_p, n)$ such that $\xi|_{\psi^{-1}(|\mathcal{N}_0|)} = roh_1\circ\psi^*|_{\psi^{-1}(|\mathcal{N}_0|)}$

**Construction of $\xi$.** First, we shall see that

(14) for each $x \in D \equiv \psi^{*-1}(|\mathcal{N}_0|) \cap |M^{(n)}|$, there exists $U \in \mathcal{W}_0$ such that $h_1 \circ \psi^*(x), h_1 \circ \psi'(x) \in U$.

By (12), there exist $U_1, U_2, U_3 \in \mathcal{U}$ such that $U_1 \cap U_2 \neq \emptyset \neq U_2 \cap U_3$ and $\psi^*(x) \in U_1, \psi'(x) \in U_3$. Then by (7), we have $W \in \mathcal{W}$ with $h_1(U_1 \cup U_2 \cup U_3) \subseteq W$. Since $\psi^*(x) \in |\mathcal{N}_0|$, by (4), there exists $W' \in \mathcal{W}$ such that $h_1 \circ \psi^*(x) \in W$ and $W' \cap K(Z_p, n) \neq \emptyset$. Hence by (2), we obtain $U \in \mathcal{W}_0$ such that $h_1 \circ \psi^*(x), h_1 \circ \psi'(x) \in \text{st}(W', \mathcal{W}) \subseteq U$.

Therefore by (14) and (1), we see the followings:

(15) $h_1 \circ \psi'(D) \subseteq F,$
(16) \( r \circ h_1 \circ \psi^*|_D \simeq r \circ h_1 \circ \psi'|_D \) in \( K(Z_p, n) \).

Since \( D \) is a subpolyhedron of \( |M^{(n)}| \) and \( \psi' \) is PL, \( \psi'(D) \) is subpolyhedron of \( |L^{(n)}| \). Hence, from \( \pi_q(K(Z_p, n)) = 0 \) for \( q < n \) (if \( n = 1 \), the path-connectedness of \( K(Z_p, n) \)), there exists an extension

\[
\alpha: |L^{(n)}| \to K(Z_p, n)
\]

of \( r \circ h_1|_{\psi'(D)}: \psi'(D) \to K(Z_p, n) \).

Then by (13), we have an extension

\[
\beta: |M^{(n+1)}| \to K(Z_p, n)
\]

of \( \alpha \circ \psi' \).

Now, put

\[
R \equiv |M^{(n+1)}| \setminus \bigcup \{ \text{Int} \sigma : \sigma \in M, \dim \sigma = n + 1, \sigma \subseteq \psi^{-1}(|N_0|) \}.
\]

Then since, for each \( x \in D \subseteq R \) we have \( \beta(x) = \alpha \circ \psi'(x) = r \circ h_1 \circ \psi'(x) \),

\[
\beta|_D \simeq r \circ h_1 \circ \psi'|_D \simeq r \circ h_1 \circ \psi^*|_D \quad \text{in} \ K(Z_p, n).
\]

By the homotopy extension theorem, there exists an extension \( \xi_R: R \to K(Z_p, n) \) of \( r \circ h_1 \circ \psi^*|_D \).

Since for \( \sigma \in M \) with \( \dim \sigma = n + 1 \) and \( \sigma \subseteq \psi^{-1}(|N_0|) \), we have \( \xi_R|_{\partial \sigma} = r \circ h_1 \circ \psi^*|_{\partial \sigma} \), there exists an extension \( \xi_{n+1}: |M^{(n+1)}| \to K(Z_p, n) \) of \( \xi_R \) such that

\[
\xi_{n+1}|_{\psi^{-1}(|N_0|) \cap |M^{(n+1)}|} = r \circ h_1 \circ \psi^*|_{\psi^{-1}(|N_0|) \cap |M^{(n+1)}|}.
\]

Hence, we can define a map \( \xi': \psi^{-1}(|N_0|) \cup |M^{(n+1)}| \to K(Z_p, n) \) by the following:

\[
\xi' \equiv (r \circ h_1 \circ \psi^*|_{\psi^{-1}(|N_0|)}) \cup \xi_{n+1}.
\]

Therefore from \( \pi_q(K(Z_p, n)) = 0 \) for \( q > n \), we obtain an extension \( \xi: Q \to K(Z_p, n) \) of \( \xi' \) such that \( \xi|_{\psi^{-1}(|N_0|)} = r \circ h_1 \circ \psi^*|_{\psi^{-1}(|N_0|)} \). It completes the construction.

Now, we put

\[
h' \equiv \xi \circ \varphi: X \to K(Z_p, n).
\]

Then to complete the proof it suffices to prove

\[
h'|_A \simeq h \quad \text{in} \ K(Z_p, n).
\]

First, we shall see that

\[
\psi^* \circ \varphi(A) \subseteq |N_0|.
\]

Let \( a \in A \). By (10), there exist \( U_1, U_2, U_3 \in \mathcal{U} \) such that

\[
(19) \ U_1 \cap U_2 \neq \emptyset \neq U_2 \cap U_3 \quad \text{and} \quad \psi^* \circ \varphi(a) \in U_1, \ h_0 \circ H(a) \in U_3.
\]
Then since $h_0 \circ H(a) = h_0 \circ h(a) \in h_0(K(Z_p, n)) \subseteq |N_1|$, we have $\psi \circ \varphi(a) \in |N_0|$ by (6).

Hence, by Claim, we have for each $a \in A$ $h'(a) = \xi \circ \varphi(a) = r \circ h_0 \circ \psi \circ \varphi(a)$.

Therefore, by (1), it suffices to see that

(20) there exists $U \in \mathcal{W}_0$ such that $h_1 \circ \psi \circ \varphi(a), h(a) \in U$.

Let $U_1, U_2, U_3 \in \mathcal{U}$ with the property (19). By (7), there exists $W \in \mathcal{W}$ such that $U_1 \cup U_2 \cup U_3 \subseteq h_1^{-1}(W)$. By (3) we choose $W' \in \mathcal{W}$ such that $h(a), h_1 \circ h_0 \circ h(a) \in W'$. Therefore, since $h(a) \in K(Z_p, n)$, there exists $U \in \mathcal{W}_0$ such that

$$h_1 \circ \psi \circ \varphi(a), h(a) \in \text{st}(W', \mathcal{W}) \subseteq U.$$ 

It completes the proof. □

4. APPROXIMABLE DIMENSION

4.1. Definition. A space $X$ has approximable dimension with respect to a coefficient group $G$ of less than and equal to $n$ (abbreviated, $a$-$\dim_G X \leq n$) provided that for every polyhedron $P$, map $f: X \to P$ and open cover $\mathcal{U}$, there exist a polyhedron $Q$ and maps $\varphi: X \to Q$, $\psi: Q \to P$ such that

(i) $(\psi \circ \varphi, f) \leq \mathcal{U}$,

(ii) $\psi$ is $(G, n, \mathcal{U})$-approximable.

First, we state fundamental inequalities of $a$-$\dim_G$.

4.2. Theorem. For a metrizable space $X$ and an arbitrary abelian group $G$, we hold the following inequalities:

$$c$-$\dim_G X \leq a$-$\dim_G X \leq \dim X.$$ 

Proof. The second inequality is trivial. We can see the first inequality by the strategy similar to the proof of the sufficiency in Theorem 3.3. □

As we will show in latter sections, our approach of $a$-$\dim_G$ gives useful applications. In general, $a$-$\dim_G$ is different from $c$-$\dim_G$. However, in special cases of coefficient group $G$, $a$-$\dim_G$ coincides with $c$-$\dim_G$.

4.3. Theorem. If $G = Z$ or $Z_p$, where $p$ is a prime number, for every metrizable space $X$, we have

$$a$-$\dim_G X = c$-$\dim_G X.$$ 

Proof. From Theorem 3.3, 4.2, we see the fact. □
5. Resolutions for metrizable spaces

By a polyhedron we mean the space $|K|$ of a simplicial complex $K$ with the Whitehead topology (denoted by $|K|_w$). We may define a topology for $|K|$ by means of a uniformity in [Appendix, 22] (denoted by $|K|_u$).

5.1. Theorem. Let $X$ be a metrizable space having approximable dimension with respect to an abelian group $G$ of less than and equal to $n$. Then there exist an $n$-dimensional metrizable space $Z$ and a perfect $UV^{n-1}$ -surjection $\pi : Z \to X$ such that for $x \in X$, the set $[\pi^{-1}(x), K(G, n)]$ of homotopy classes is trivial.

Proof. The strategy is like the construction of Walsh-Rubin [24,22].

Let $d$ be a metric for $X$ and let $\{U_i : i \in \mathbb{N} \cup \{0\}\}$ be a sequence of open covers of $X$ where each $U_i$ consists of all $1/(i+1)$-neighborhoods.

First, we shall construct the followings:

open covers $\mathcal{V}_i$ of $X$ whose nerves $\mathcal{N}(\mathcal{V}_i)$ are locally finite dimensional, maps $b_i : X \to |\mathcal{N}(\mathcal{V}_i)|$ for $i \geq 0$, $f_i^* : f_i : |\mathcal{N}(\mathcal{V}_i)| \to |\mathcal{N}(\mathcal{V}_{i-1})|$ for $i \geq 1$ and sequences $\mathcal{N}_i^j, j \in \mathbb{N} \cup \{0\}$ of subdivisions of $\mathcal{N}(\mathcal{V}_i)$ for $i \geq 0$ such that

(1) $S_i^{j+1} \prec S_i^j$ for $j \geq 0$,
(2) $b_i$ is normal with respect to $b_i^{-1}(S_i^j)$ and $\mathcal{N}_i^j$ for $j \geq 0$,
(3) $f_i : \mathcal{N}_i^0 \to \mathcal{N}_i^3$ is simplicial for $i \geq 1$,
(4) $f_i \circ b_i$ is $\mathcal{N}_i^{j-1}$-modification of $b_{i-1}$, $0 \leq j \leq 3$ for $i \geq 1$,
(5) $f_i$ maps each compact set in $|\mathcal{N}_i|_w$ onto a compact set in $|\mathcal{N}_{i-1}|_w$ which is contained in a finite union of simplexes of $\mathcal{N}_{i-1}$,
(6) $S_i^0 \prec f_i^{-1}(S_i^j)$ for $i \geq 1$,
(7) $S_i^k \prec f_i^{-1}(S_i^k)$ for $k \geq 1$ and $\tilde{S}_i^k \prec f_i^{*1}(S_i^k)$ for $k \geq 4$,
(8) $\mathcal{V}_i \prec \mathcal{U}_i \wedge b_i^{-1}(S_i^0) \wedge b_i^{-1}(S_i^1) \wedge \cdots \wedge b_i^{-1}(S_i^3),$

where we regard $|\mathcal{N}_i|_w$ as the uniform space with the uniform topology induced by the uniform base $\{S_i^j\}_{j=0}^{\infty}$.

Further, we shall construct continuous (w.r.t. the Whitehead topology), uniformly continuous (w.r.t. the uniform topology) PL-maps $g_i : |(\mathcal{N}_i^3)^{(n)}| \to |(\mathcal{N}_{i-1}^3)^{(n)}|$ such that

(9) for each $t \in |(\mathcal{N}_i^3)^{(n)}|$, there exist $\sigma, \tau \in \mathcal{N}_{i-1}$ such that $f_i(t) \in \sigma$, $g_i(t) \in \tau$ and $\sigma \cap \tau \neq \emptyset$,
(10) for any map $\alpha : |(\mathcal{N}_{i-1}^3)^{(n)}|_w \to K(G, n)$, there exists an extension $\beta : |(\mathcal{N}_i^3)^{(n+1)}|_w \to K(G, n)$ of $\alpha \circ g_i : |(\mathcal{N}_i^3)^{(n)}|_w \to |(\mathcal{N}_{i-1}^3)^{(n)}|_w \to K(G, n),$
(11) for each $x \in |\mathcal{N}_i|$, $g_i \left(\text{st}(x, \tilde{S}_i^2) \cap |(\mathcal{N}_i^3)^{(n)}|\right)$ is a Whitehead (i.e. finite) compact polyhedral subset of $|\mathcal{N}_{i-1}|$.

Let us start the construction. We take an open refinement $\mathcal{V}_0$ of $\mathcal{U}_0$ in $X$ whose nerve $\mathcal{N}(\mathcal{V}_0)$ is locally finite dimensional and $\mathcal{V}_0$-normal map $b_0 : X \to |\mathcal{N}(\mathcal{V}_0)|$. We
define $\mathcal{N}^{j}_{0}$ to be a subdivision of $Sd_{2j}\mathcal{N}(\mathcal{V}_{0})$ for $j = 0, 1, 2$ with $S_{0}^{j} \prec S_{0}^{j-1}$. By using \cite[Proposition A.3]{22}, for the cover $\mathcal{E}_{0} \equiv \{ \text{st}(x, S_{0}^{3}) : x \in |\mathcal{N}(\mathcal{V}_{0})| \}$, we obtain an open cover $B_{0}$ of $|\mathcal{N}(\mathcal{V}_{0})|$ and a PL, $\mathcal{N}^{3}_{0}$-modification $r_{0} : |\mathcal{N}^{3}_{0}| \to |\mathcal{N}^{3}_{0}|$ of the identity such that

$$(12)_{0} \quad r_{0}(\text{Cl } B) \text{ is compact for } B \in B_{0},$$

$$(13)_{0} \quad \text{Cl } B \cup r_{0}(\text{Cl } B) \subseteq E \text{ for some } E \in \mathcal{E}_{0}. $$

Since $b_{0}$ is $(G, n)$-cohomological, from the similar argument to the proof of the necessity in Theorem 3.3 we can take the followings:

subdivision $\mathcal{N}^{3}_{0}$ of $Sd_{2}\mathcal{N}^{3}_{0}$, locally finite open cover $\mathcal{V}_{1}$ of $X$ and maps $b_{1} : X \to |\mathcal{N}(\mathcal{V}_{1})|$, $f_{1}^{*} : |\mathcal{N}(\mathcal{V}_{1})| \to |\mathcal{N}^{3}_{0}|$ such that

$$(14)_{1} \quad \overline{S}_{0}^{3} \prec^{*} S_{0}^{3} \wedge B_{0},$$

$$(15)_{1} \quad \mathcal{V}_{1} \prec^{*} \mathcal{U}_{1} \wedge b_{0}^{-1}(S_{0}^{3}),$$

$$(16)_{1} \quad b_{1} \text{ is } \mathcal{V}_{1}\text{-normal},$$

$$(17)_{1} \quad f_{1}^{*} \circ b_{1} \text{ is } \mathcal{N}^{3}_{0}\text{-modification of } b_{0},$$

$$(18)_{1} \quad \text{for each } \sigma \in \mathcal{N}(\mathcal{V}_{1}), \text{ there exists } U \in \text{st } S_{0}^{3} \text{ such that } b_{0} \left(b_{0}^{-1}(\sigma) \cup f_{1}^{*}(\sigma) \subset U \right),$$

$$(19)_{1} \quad \text{for any triangulation } M \text{ of } |\mathcal{N}(\mathcal{V}_{1})|, \text{ there exists a PL-map } p' : |M^{(n)}| \to |\mathcal{N}^{3}_{0}(n)| \text{ such that}$$

$$(i) \quad (p', f_{1}^{*}|_{M^{(n)}}) \leq \{ \text{st}(\lambda, \mathcal{N}^{3}_{0}) : \lambda \in \mathcal{N}^{3}_{0} \},$$

$$(ii) \quad \text{for any map } \alpha : |\mathcal{N}^{3}_{0}(n)| \to K(G, n), \text{ there exists an extension } \beta : |M^{(n+1)}| \to K(G, n) \text{ of } \alpha \circ p'.$$

Let $\mathcal{N}^{j+1}_{0}$ denote a subdivision of $Sd_{2}\mathcal{N}^{j}_{0}$ with $\overline{S}_{0}^{j+1} \prec^{*} S_{0}^{j}$ for $j \geq 3$.

Now, let $|\mathcal{N}^{3}_{0}|_{m}$ denote $|\mathcal{N}^{3}_{0}|$ with the metric topology \cite[p301]{19}. Then there is a $\mathcal{N}^{3}_{0}$-modification $j_{0} : |\mathcal{N}^{3}_{0}|_{m} \to |\mathcal{N}^{3}_{0}|_{w}$ of the identity function \cite[p302]{19}. By the simplicial approximation theorem, we obtain a subdivision $\mathcal{N}_{1}$ of $\mathcal{N}(\mathcal{V}_{1})$ and a simplicial approximation $f_{1} : \mathcal{N}_{1} \to \mathcal{N}^{3}_{0}$ of $j_{0} \circ f_{1}^{*}$. Let $\mathcal{N}_{0}^{1}$ denote $\mathcal{N}_{1}$. Then by the simpliciality of $f_{1}$ and (17)_{1}, we have

$$(20) \quad S_{0}^{3} \prec f_{1}^{-1}(S_{0}^{3}),$$

$$(21) \quad f_{1} \circ b_{1} \text{ is } \mathcal{N}_{0}^{3}\text{-modification of } b_{0}. $$

We take a subdivisions $\mathcal{N}^{j+1}_{1}$ of $\mathcal{N}^{j}_{0}$ for $j = 0, 1$ such that

$$(22) \quad \overline{S}_{0}^{j+1} \prec^{*} S_{0}^{j} \text{ for } j = 0, 1,$$

$$(23) \quad \overline{S}_{0}^{j} \prec f_{1}^{-1}(S_{0}^{j+3}) \text{ for } j = 1, 2,$$

$$(24) \quad \mathcal{N}^{j}_{1} \prec Sd_{2j}\mathcal{N}^{0}_{1} \text{ for } j = 1, 2.$$

By using Lemma \cite[Proposition A.3]{22}, for the cover $\mathcal{E}_{1} \equiv \{ \text{st}(x, S_{1}^{2}) : x \in |\mathcal{N}_{1}| \}$, we obtain an open cover $B_{1}$ of $|\mathcal{N}(\mathcal{V}_{0})|$ and a PL, $\mathcal{N}^{3}_{1}$-modification $r_{1} : |\mathcal{N}^{3}_{1}| \to |\mathcal{N}^{3}_{1}|$ of the identity map such that

$$(12)_{1} \quad r_{1}(\text{Cl } B) \text{ is compact for } B \in B_{1},$$

$$(13)_{1} \quad \text{Cl } B \cup r_{1}(\text{Cl } B) \subseteq E \text{ for some } E \in \mathcal{E}_{1}. $$
Since $b_1$ is $(G, n)$-cohomological, from the similar argument to the proof of the necessity in Theorem 3.3 we can take the followings:

subdivision $\mathcal{N}_1^3$ of Sd$_2\mathcal{N}_0^2$, locally finite open cover $\mathcal{V}_2$ of $X$ and maps $b_2 : X \to |\mathcal{N}(\mathcal{V}_2)|, f_2^* : |\mathcal{N}(\mathcal{V}_2)| \to |\mathcal{N}_1^3|$ such that

(14)$_2 \bar{S}_1^3 \prec^* S_1^2 \cup B_1 \cup f_1^{-1}(S_0^6),$
(15)$_2 \mathcal{V}_2 \prec^* \mathcal{U}_2 \cup b_1^{-1}(S_1^2) \cup b_0^{-1}(S_0^6),$
(16)$_2 b_2$ is $\mathcal{V}_2$-normal,
(17)$_2 f_2^* \circ b_2$ is $\mathcal{N}_1^3$-modification of $b_1$, 
(18)$_2$ for each $\sigma \in \mathcal{N}(\mathcal{V}_2)$, there exists $U \in \text{st} S_1^3$ such that $b_1(b_2^{-1}(\sigma)) \cup f_2^*(\sigma) \subseteq U$, 
(19)$_2$ for any triangulation $M$ of $|\mathcal{N}(\mathcal{V}_2)|$, there exists a PL-map $p' : |M^{(n)}| \to |(\mathcal{N}_1^3)^{(n)}|$ such that

(i) $|p', f_2^*|_{|M^{(n)}|} \leq \{\tilde{\text{st}}(\lambda, \mathcal{N}_1^3) : \lambda \in \mathcal{N}_0^3\}$,
(ii) for any map $\alpha : |(\mathcal{N}_1^3)^{(n)}| \to K(G, n)$, there exists an extension $\beta : |(\mathcal{N}_1^3)^{(n+1)}| \to K(G, n)$ of $\alpha \circ p'$.

Now, by using (19)$_1$ about the triangulation $\mathcal{N}_1^3$ of $|\mathcal{N}(\mathcal{V}_1)|$, we obtain a PL-map $g_1^* : |(\mathcal{N}_1^3)^{(n)}| \to |(\mathcal{N}_0^3)^{(n)}|$ such that

(25)$_1 (g_1^*, f_1^*|_{|(\mathcal{N}_1^3)^{(n)}|}) \leq \{\tilde{\text{st}}(\lambda, \mathcal{N}_0^3) : \lambda \in \mathcal{N}_0^3\}$,
(26)$_1$ for any map $\alpha : |(\mathcal{N}_0^3)^{(n)}| \to K(G, n)$, there exists an extension $\beta : |(\mathcal{N}_1^3)^{(n+1)}| \to K(G, n)$ of $\alpha \circ g_1^*$.

Consider the inclusion map $i_0 : |(\mathcal{N}_0^3)^{(n)}| \hookrightarrow |\mathcal{N}_0^3|$ and the composition

$r_0 \circ i_0 \circ g_1^* : |(\mathcal{N}_1^3)^{(n)}| \to |(\mathcal{N}_0^3)^{(n)}| \hookrightarrow |\mathcal{N}_0^3| = |\mathcal{N}(\mathcal{V}_0)| \to |\mathcal{N}(\mathcal{V}_0)|.$

The image $A$ of the PL-map $r_0 \circ i_0 \circ g_1^*$ has dimension $\leq n$. Then we can take a $\mathcal{N}_0^3$-modification $s_0 : A \to |(\mathcal{N}_0^3)^{(n)}|$ of the inclusion map $A \hookrightarrow |\mathcal{N}_0^3|$. Let $g_1 : |(\mathcal{N}_1^3)^{(n)}| \to |(\mathcal{N}_0^3)^{(n)}|$ denote the composition map $s_0 \circ r_0 \circ i_0 \circ g_1^*$.

Then this has the following properties:

Claim 1.

(9)$_1$ for each $t \in |(\mathcal{N}_1^3)^{(n)}|$, there exist $\sigma, \tau \in \mathcal{N}_0^3$ such that $f_1(t) \in \sigma, g_1(t) \in \tau$ and $\sigma \cap \tau \neq \emptyset$, 
(10)$_1$ for any map $\alpha : |(\mathcal{N}_0^3)^{(n)}| \to K(G, n)$, there exist an extension $\beta : |(\mathcal{N}_1^3)^{(n+1)}| \to K(G, n)$ of $\alpha \circ g_1$, 
(11)$_1$ for each $x \in |\mathcal{N}_1^3|$, $g_1 \left(\text{st}(x, \bar{S}_1^2) \cap |(\mathcal{N}_1^3)^{(n)}|\right)$ is a Whitehead (i.e. finite) compact polyhedral subset of $|\mathcal{N}_0^3|.$

Proof of Claim 1. We show the property (9)$_1$. Let $t \in |(\mathcal{N}_1^3)^{(n)}|$. By (25)$_1$, there exist $\sigma, \lambda, \tau \in \mathcal{N}_0^3$ such that $f_1^*(t) \in \sigma, g_1^*(t) \in \tau$ and $\sigma \cap \lambda \neq \emptyset \neq \lambda \cap \tau$. We may assume that $\lambda = |v_0, v_1|, v_0 \in \sigma$ and $v_1 \in \tau$. 


Since $j_0$ is $N_0^3$-modification of the identity function, we have $j_0 \circ f_1^*(t) \in \sigma$. Since $f_1$ is simplicial approximation of $j_0 \circ f_1^*$, we have $f_1(t) \in \sigma$.

Select $\hat{\sigma} \in N_0^3$ with $\sigma \subseteq \hat{\sigma}$. Since $r_0$ is $N_0^3$-modification of the identity map, we have $r_0 \circ i_0 \circ g_i^*(t) \in \hat{\sigma}$. Further since $s_0$ is $N_0^3$-modification of $A \hookrightarrow |N_0^3|$ and $N_0^3 \prec N_0^2$, we have $g_1(t) = s_0 \circ r_0 \circ i_0 \circ g_i^*(t) \in \hat{\sigma}$.

**Case 1.** $v_1 \in (N_0^3)^{(0)}$ (i.e. $v_1 \in \hat{\sigma}^{(0)}$).

By $N_0^3 \prec Sd_2 N_0^2$, we have $v_0 \notin (N_0^2)^{(0)}$. Hence, there exists $\gamma \in N_0^2$ such that $|v_0, v_1| \subseteq \gamma$ and $v_0 \in \operatorname{Int} \gamma$. Then if $\bar{\sigma} \in N_0^2$ with $\sigma \subseteq \bar{\sigma}$, we have $\gamma \prec \bar{\sigma}$. Therefore we have $\bar{\sigma} \cap \hat{\sigma} \neq \emptyset$, $f_1(t) \in \bar{\sigma}$ and $g_1(t) \in \hat{\sigma}$.

**Case 2.** $v_1 \notin (N_0^3)^{(0)}$.

If $v_0 \in (N_0^2)^{(0)}$, the proof is similar to Case 1. Let $v_0 \notin (N_0^2)^{(0)}$. By $N_0^3 \prec Sd_2 N_0^2$, there exist $\gamma_0, \gamma_1 \in N_0^2$ such that $v_0 \in \operatorname{Int} \gamma_0$, $v_1 \in \operatorname{Int} \gamma_1$ and $\gamma_0 \prec \gamma_1$ or $\gamma_1 \prec \gamma_0$. Then if $\bar{\sigma} \in N_0^2$ with $\sigma \subseteq \bar{\sigma}$, we have $\gamma_0 \prec \bar{\sigma}$. Similarly, we have $\gamma_1 \prec \hat{\sigma}$. Therefore we have $\bar{\sigma} \cap \hat{\sigma} \neq \emptyset$, $f_1(t) \in \bar{\sigma}$ and $g_1(t) \in \hat{\sigma}$.

By $g_1^* \simeq g_1$, we can see the property (10)$_1$ by the homotopy extension theorem and (26)$_1$.

We show the property (11)$_1$. First, we shall see that

\begin{equation}
(27) \quad g_1^* \left(\operatorname{st}(x, \bar{S}_1^2) \cap (N_1^3)^{(n)}\right) \subseteq B \text{ for some } B \in B_0.
\end{equation}

Let $\operatorname{st}(x, \bar{S}_1^2)$ be represented by $\bigcup \{\operatorname{st}(v_{\alpha}, N_1^3) : \alpha \in A\}$. There exists $\sigma_x \in N_1^2$ with $x \in \operatorname{Int} \sigma_x$.

For each $\alpha \in A$, we choose $\sigma_{x, \alpha} \in N_1^2$ such that $\sigma_x \not\prec \sigma_{x, \alpha}$ and $v_{\alpha} \in \sigma_{x, \alpha}$. Further we select minimum and maximal dimensional simplexes $\tau_x, \tau_{\alpha} \in N_1^2$ with $\tau_x \not\prec \tau_{\alpha}$ respectively such that $\sigma_x \subseteq \tau_x$ and $\sigma_{x, \alpha} \subseteq \tau_{\alpha}$.

If $\sigma_x \subseteq \operatorname{Int} \tau_x$, we have $\operatorname{st}(v_{\alpha}, N_1^2) \subseteq \tau_{\alpha}$ from $v_{\alpha} \in \operatorname{Int} \tau_{\alpha}$. Then there exists a vertex $v \in N_1^2$ such that $\bigcup_{\alpha} \tau_{\alpha} \subseteq \operatorname{st}(v, N_0^0)$. Since $f_1$ is the simplicial map from $N_1^0$ to $N_1^3$, we have $f_1(\bigcup_{\alpha} \tau_{\alpha}) \subseteq f_1(\operatorname{st}(v, N_0^0)) \subseteq \operatorname{st}(f_1(v), N_0^3)$. By the nearness between $f_1$ and $g_1^*$ (see proof of (9)$_1$) and (14)$_1$, we obtain

\begin{equation}
(28) \quad g_1^* \left(\operatorname{st}(x, \bar{S}_1^2) \cap (N_1^3)^{(n)}\right) \subseteq \operatorname{st}(f_1(v), N_0^3) \subseteq B \text{ for some } B \in B_0.
\end{equation}

If $\sigma_x \not\prec \tau_x$ and $\sigma_x \not\subset \tau_x$, we choose a face $\bar{\tau}_x$ with $\bar{\tau}_x \not\subseteq \tau_x$ such that $\sigma_x \not\subset \partial \tau_x \subseteq \bar{\tau}_x$. Then there exists a vertex $v \in \bar{\tau}_x$ such that $\bigcup_{\alpha} \operatorname{st}(v_{\alpha}, N_1^2) \subseteq \operatorname{st}(v, N_1^2)$. Hence we have (28) in the same way.

Since $\operatorname{st}(x, \bar{S}_1^2) \cap (N_1^3)^{(n)}$ is a subpolyhedron of $|N_1|$ and $g_1^*$ is a PL-map, we see that $g_1^* \left(\operatorname{st}(x, \bar{S}_1^2) \cap (N_1^3)^{(n)}\right)$ is a subpolyhedron of $|N_0|$. Then by (27) and (12)$_0$, $r_0 \circ i_0 \circ g_i^* \left(\operatorname{st}(x, \bar{S}_1^2) \cap (N_1^3)^{(n)}\right)$ is a subpolyhedron of $|N_0|$ and a compact set of $|N_0|_w$. Since $s_0$ is a PL-map, we have see the property (11)$_1$.  

Now, we shall take a base for a uniformity for $|\mathcal{N}_1|$. We choose a subdivisions $\mathcal{N}^j_1$ for $j \geq 4$ of $\mathcal{N}_1$ such that

\begin{align*}
(29) & \quad \mathcal{N}^j_1+1 \prec \text{Sd}_2 \mathcal{N}^j_1 \quad \text{for} \quad j \geq 3, \\
(30) & \quad \mathcal{S}^j_1 \prec \mathcal{S}^j_1 \quad \text{for} \quad j \geq 3, \\
(31) & \quad \mathcal{S}^j_1 \prec f_1^{-1}(\mathcal{S}^{j+1}_0) \wedge f_1^{-1}(\mathcal{S}^{j+4}_0) \wedge \mathcal{F}^{j+4}_1 \quad \text{for} \quad j \geq 3, \\
\end{align*}

where $\mathcal{F}^{j+4}_1$ is defined as follows. $g_1^{-1}\left(\mathcal{S}^{j+4}_0 \cap |(\mathcal{N}^3_1(n)|\right)$ is the open cover of $|(\mathcal{N}^3_1(n)|_w$. Extend it to an open cover $\mathcal{F}^{j+4}$ of $|\mathcal{N}_1|$. Then clearly the uniformity make $f_1$, $f_1^*$ and $g_1$ uniformly continuous.

We shall show that $f_1$ holds the property (5). First, note that the composition

$$j_0 \circ \text{id} \circ f_1^*: |\mathcal{N}_1| \rightarrow |\mathcal{N}_0|_u \rightarrow |\mathcal{N}_0|_m \rightarrow |\mathcal{N}_0|_w,$$

where $\text{id}: |\mathcal{N}_0|_u \rightarrow |\mathcal{N}_0|_m$ is the identity map, is continuous.

Let $K$ be a compact set of $|\mathcal{N}_1|_w$. There exist $\sigma_1, \ldots, \sigma_l \in \mathcal{N}_0$ such that $j_0 \circ f_1^*(K) = j_0 \circ \text{id} \circ f_1^*(K) \subseteq \sigma_1 \cup \cdots \cup \sigma_l$. Since $f_1$ is a simplicial approximation of $j_0 \circ f_1^*$, we have $f_1(K) \subseteq \sigma_1 \cup \cdots \cup \sigma_l$. By the continuity of $f_1$, $f_1(K)$ is a compact set of $|\mathcal{N}_0|_u$.

As we proceed in this work, we have $\mathcal{V}_i$, $f_i^*$, $f_i$, $\mathcal{N}^j_i$ and $g_i$ with the properties (1)-(11).

From now on, we consider $X$ to be the uniform space with the uniformity generated by the sequence $\{\mathcal{V}_i\}_{i=0}^\infty$ of open covers of $X$ and $|\mathcal{N}_i|$ to be the uniform space with the uniformity generated by the sequence $\{S^j_i\}_{j=0}^\infty$. Then by the construction, the topology induced by $\{\mathcal{V}_i\}_{i=0}^\infty$ and the original metric topology are identical.

We shall construct the resolution of $X$. The construction essentially depends on Rubin’s way [22]. Hence, the detail is omitted here.

For $j \geq 0$, let $f_{j,j}$ denote the identity on $\mathcal{N}_j$ and let $f_{i,j}$ denote the composition $f_{j+1} \circ \cdots \circ f_{i}: |\mathcal{N}_i| \rightarrow |\mathcal{N}_j|$ for $i > j$.

The functions

$$b_i: (X, \{\mathcal{V}_i\}_{i=0}^\infty) \rightarrow (|\mathcal{N}_i|, \{S^j_i\}_{j=0}^\infty)$$

and

$$f_{i+1,i}: (|\mathcal{N}_{i+1}|, \{S^j_{i+1}\}_{j=0}^\infty) \rightarrow (|\mathcal{N}_i|, \{S^j_i\}_{j=0}^\infty)$$

are uniformly continuous for $i \geq 0$. Then since the sequence $\{f_{i,j} \circ b_i\}_{i,j=0}^\infty$ is Cauchy in the uniform space $C(X, |\mathcal{N}_j|_w)$ with the uniformity of uniform convergence, we have a uniformly continuous, limit map

$$f_{\infty,i} \equiv \lim_{q \rightarrow \infty} f_{q,i} \circ b_q: (X, \{\mathcal{V}_i\}_{i=0}^\infty) \rightarrow (|\mathcal{N}_j|, \{S^j_i\}_{j=0}^\infty),$$

such that

\begin{align*}
(32) & \quad f_{\infty,j} \in \mathcal{N}^j_j \text{-modification of } b_j, \\
(33) & \quad (f_{\infty,j}, b_j) \leq S^j_j, \\
(34) & \quad f_{\infty,j} \text{ is a topological irreducible (i.e. surjective) map relative to } \mathcal{N}^j_j, \\
(35) & \quad f_{i+1,i} \circ f_{\infty,i+1} = f_{\infty,i} \text{ for } i \geq 0.
\end{align*}
We consider $\prod_{i=0}^{\infty} |\mathcal{N}_i|_u$ to be the uniform space by the product uniformity. Note that $\lim \{ |\mathcal{N}_j|_u, f_{i+1,i} \}$ is a non-empty subspace by the property (34).

Then by (35), there exist a uniformly continuous map $f_\omega: X \to \prod_{i=0}^{\infty} |\mathcal{N}_i|_u$ with $f_{\infty,i} = pr_i \circ f_\omega$, and especially the map $f_\omega$ is a uniformly embedding onto a dense subset $f_\omega(X)$ in $\prod_{i=0}^{\infty} |\mathcal{N}_i|_u$, where $pr_i: \prod_{j=0}^{\infty} |\mathcal{N}_j|_u \to |\mathcal{N}_i|_u$ is the natural projection.

Let $Z$ denote the limit of the inverse sequence $\{(\mathcal{N}_i^{3})^{(n)}|_u, g_{i+1,i} \}$. Then we consider $Z$ to be the sub-uniform space of the uniform space $\prod_{i=0}^{\infty} |\mathcal{N}_i|_u$. Note that $Z$ has dimension $\leq n$.

We begin with a description of the map $\pi$. For $j \geq 0$, a uniformly continuous map $\pi_j: Z \to \prod_{i=0}^{\infty} |\mathcal{N}_i|_u$ is defined by

$$\pi_j(z) \equiv (f_{j,0}(z_j), f_{j,1}(z_j), \ldots, f_{j,j-1}(z_j), z_j, z_{j+1}, \ldots)$$

for $z = (z_j) \in Z$ and let $\pi_0$ be the inclusion map. Then since the sequence $\{\pi_j\}_{j=0}^{\infty}$ is Cauchy in $C(Z, \prod_{i=0}^{\infty} |\mathcal{N}_i|_u)$, there is a uniformly continuous, limit map $\pi: Z \to \prod_{i=0}^{\infty} |\mathcal{N}_i|_u$. Then the map $\pi$ is proper from $Z$ onto $\lim \{ |\mathcal{N}_i|_u, f_{i+1,i} \}$. We must show that $\pi^{-1}(x)$ is a UV$^{n-1}$-set and the set $[\pi^{-1}(x), K(G,n)]$ is trivial for $x \in \lim \{ |\mathcal{N}_i|_u, f_{i+1,i} \}$.

For $x = (x_i) \in \lim \{ |\mathcal{N}_i|_u, f_{i+1,i} \}$, let $\delta N(x_i)$ and $\epsilon N(x_i)$ denote $st(x_i, S_i^0)$ and $st(x_i, S_i^2)$, respectively. Then we have the following properties [22]: for $x = (x_i) \in \lim \{ |\mathcal{N}_i|_u, f_{i+1,i} \},$

$$\lim_{\omega} \{ \delta N(x_i) \cap |(\mathcal{N}_i^3)^{(n)}|, g_{i+1,i} \} \subseteq \epsilon N(x_i) \subseteq \delta N(x_i) \subseteq \delta N(x_i) \cap |(\mathcal{N}_i^3)^{(n)}|, g_{i+1,i} \}$$

By $S_i^2 \prec S_i^1$, there exists $F_i \in S_i^1$ such that $st(x_i, S_i^2) \subseteq F_i$. Further, by $S_i^2 \prec S_i^0$, there is a $S \in S_i^0$ such that $F_i \subseteq S$. Hence we have the contractible set $F_i$ such that

$$\lim_{\omega} \{ \delta N(x_i) \subseteq F_i \subseteq \delta N(x_i) \} \subseteq \delta N(x_i).$$

**Claim 2.** $\pi^{-1}(x)$ is a UV$^{n-1}$-set for for $x = (x_i) \in \lim \{ |\mathcal{N}_i|_u, f_{i+1,i} \}$.

*Proof of Claim 2.* It suffices to show that the map

$$g_{i+1,i} \lim_{\omega} \{ \delta N(x_{i+1}) \cap |(\mathcal{N}_i^3)^{(n)}|, g_{i+1,i} \} \subseteq F_i \cap |(\mathcal{N}_i^3)^{(n)}| \subseteq \delta N(x_i) \cap |(\mathcal{N}_i^3)^{(n)}|$$

induces a zero homomorphism of homotopy group of dimension less than $n$. By (36) and (38), we have

$$g_{i+1,i} \left( \delta N(x_{i+1}) \cap |(\mathcal{N}_i^3)^{(n)}| \right) \subseteq F_i \cap |(\mathcal{N}_i^3)^{(n)}| \subseteq \delta N(x_i) \cap |(\mathcal{N}_i^3)^{(n)}|.$$}

Since $F_i$ is contractible, we have

$$\pi_k \left( F_i \cap |(\mathcal{N}_i^3)^{(n)}| \right) = 0 \quad \text{for } k < n.$$}

Therefore $g_{i+1,i} \lim_{\omega}$ induces a zero homomorphism of homotopy group of dimension less than $n$. 

Claim 3. \([\pi^{-1}(x), K(G, n)] \approx \check{H}^n(\pi^{-1}(x); G)\) is trivial for \(x \in \lim\{N_i|\}_{u, f_{i+1,i}}\).

**Proof of Claim 3.** By (11),(36),(37) and the continuity of Čech cohomology, we have

\[ \check{H}^n(\pi^{-1}(x); G) \approx \lim \left\{ H^n(g_{i,i-1}(\varepsilon N(x_i) \cap |(N^3_i)^{(n)}|_u); G), g_{i,i-1}|_{\star} \right\}. \]

Hence it suffices to show that

\[ g_{i,i-1}|_{\star} : H^n \left( g_{i,i-1}(\varepsilon N(x_i) \cap |(N^3_i)^{(n)}|); G \right) \to H^n \left( g_{i+1,i}(\varepsilon N(x_{i+1}) \cap |(N^3_{i+1})^{(n)}|); G \right) \]

is the zero homomorphism.

Let \(G_{i,i-1}\) denotes \(g_{i,i-1}(\varepsilon N(x_i) \cap |(N^3_i)^{(n)}|_u)\). Then by (11) the subspace \(G_{i,i-1}\) of \(|(N^3_{i-1})^{(n)}|_u\) and the subspace \(G_{i,i-1}\) of \(|(N^3_{i-1})^{(n)}|_w\) is identical. Hence from now on, we may consider that \(G_{i,i-1}\) is the subspace of \(|(N^3_{i-1})^{(n)}|_w\).

Let \(\alpha \in [G_{i,i-1}, K(G, n)]\). Then from \(\pi_q(K(G, n)) = 0\) for \(q < n\), there exists an extension \(\tilde{\alpha} : |(N^3_{i-1})^{(n)}|_w \to K(G, n)\) of \(\alpha\). By (10), we have an extension \(\beta : |(N^3_{i})^{(n+1)}|_w \to K(G, n)\) of \(\tilde{\alpha} \circ g_{i,i-1}|_{G_{i+1,i}}\).

Since \(F_i\) is the contractible set, \(F_i \cap |(N^3_i)^{(n+1)}|_w\) is contractible in \(|(N^3_i)^{(n+1)}|_w\). Hence, there exists a homotopy \(H : (F_i \cap |(N^3_i)^{(n)}|_w) \times I \to F_i \cap |(N^3_i)^{(n+1)}|_w\) such that \(H_0\) is the inclusion map and \(H_1\) is a constant map. Since \(G_{i+1,i} \subseteq \varepsilon N(x_i) \cap |(N^3_i)^{(n)}|_w \subseteq F_i \cap |(N^3_i)^{(n)}|_w\), we can define the following compositions:

\[ \tilde{H} \equiv \beta \circ i_2 \circ H \circ i_1 : G_{i+1,i} \times I \leftrightarrow \left( F_i \cap |(N^3_i)^{(n)}|_w \right) \times I \to F_i \cap |(N^3_i)^{(n+1)}|_w \]

\[ \leftrightarrow |(N^3_i)^{(n+1)}|_w \to K(G, n), \]

where \(i_1\) and \(i_2\) are the inclusion maps.

Then we have \(\tilde{H}_0 = \beta|_{G_{i+1,i}} = \alpha \circ g_{i,i-1}|_{G_{i+1,i}}\) and \(\tilde{H}_1 = \) a constant . It completes the proof of Claim 3. Then the map

\[ \pi_X \equiv \pi|_{\pi^{-1}(X)} : \pi^{-1}(X) \to X \]

is a desired one for Theorem. \(\square\)

5.2. **Corollary.** Let \(X\) be a metrizable space having cohomological dimension with respect to \(Z_p\) of less than and equal to \(n\). Then there exist an \(n\)-dimensional metrizable space \(Z\) and a perfect \(UV^{n-1}\)-surjection \(\pi : Z \to X\) such that for \(x \in X\), the set \([\pi^{-1}(x), K(Z_p, n)]\) of homotopy classes is trivial.
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