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On the Hausdorff dimension of the attractor for the heat convection equation

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§1. Introduction.

In our previous study [3], we considered the heat convection equation (HC) in a time-dependent domain $\Omega(t) \subset R^2$ and showed the existence of the absorbing set for (HC).

On the other hand, Foias-Manley-Temam [1] showed the existence of the attractor for the Bénard problem and obtained the estimates of the Hausdorff and fractal dimensions of the attractor.

In this paper we consider (HC) in a wider class of fixed bounded domains of R^2 with inhomogeneous boundary conditions and we estimate the Hausdorff and fractal dimensions of the attractors for (HC).

§2. Equations and assumptions.

Let Ω be a bounded domain in R^2 included in an open ball $B = B(0,d)$. The boundary $\partial\Omega$ consists of N connected components, namely, $\partial\Omega = \Gamma_1 + \cdots + \Gamma_N$, where Γ_i are smooth (say, of class C^2) and they does not intersect each other.

We consider the following heat convection equation :

$$(1)\quad \begin{cases} u_t + (u\cdot\nabla)u = -\nabla p/\rho + \{1 - \alpha(\theta-T_0)\}g + \nu\Delta u \ , \\ \qquad \operatorname{div} u = 0 \ , \\ \theta_t + (u\cdot\nabla)\theta = \kappa\Delta\theta \ , \quad \text{in } \Omega \ , \end{cases}$$

$$(2)\quad u|_{\partial\Omega} = \beta(x) \ , \quad \theta|_{\partial\Omega} = T(x) \geqq 0 \ ,$$

(3) $u|_{t=o} = a(x)$, $u|_{t=o} = h(x)$, $x \in \Omega$,

where u, p and θ denote the velocity of the fluid, the pressure and the temperature, respectively ; $g(x)$ means the gravitational vector and ν, κ, α, β are physical constants.

Now we make an assumption on the boundary function β.

Assumption. β is smooth and satisfies the condition

(4) $\int_{\Gamma_K} \beta\cdot n \, ds = 0$ $(k = 1, \cdots, N)$,

where n is the outer normal vector to Γ_k.

Then, the next lemma is known :

Lemma 1. Let $\beta \in H^{3/2}(\partial\Omega)$, then for any $\varepsilon > 0$, there exists $b \in H^2(\Omega)$ such that $b = \beta$ on $\partial\Omega$, $\mathrm{div}\, b = 0$ and

(5) $|((v\cdot\nabla)b, v)| \leqq \varepsilon\|\nabla v\|^2$ for any $v \in H^1_\sigma(\Omega)$.

Remark 1. We assume the function $T(x)$ is continuous on $\partial\Omega$. Then we can have a function $\overline{\theta}(x)$ such that $\Delta\overline{\theta} = 0$ in Ω and $\overline{\theta}(\partial\Omega) = T(x)$.

Now we make changes of variables : $u = \hat{u} + b$, $\theta = \hat{\theta} + \overline{\theta}$; $(x,y) = d(x^*, y^*)$, $t = (d^2/\nu)\cdot t^*$, $\hat{u} = (\nu/d)u^*$, $\hat{\theta} = (\nu T_0/\kappa)\theta^*$ and $p = (\rho\nu^2/d^2)p^*$, where $T_0 = \max_x T(x)$. Abbreviating asterisks $*$ and using the same letters u, θ, p, x, y, t, the heat convection equation (1) $\sim$ (3) are rewritten as follows :

$$
(6) \quad \begin{cases} u_t + (u\cdot\nabla)u = -\nabla p + \Delta u - (u\cdot\nabla)b - (b\cdot\nabla)u - R\theta - (b\cdot\nabla)b + \Delta b \\ \qquad\qquad + d^3 g/\nu^2 - R(\overline{\theta} - 1/P) \quad , \\ \mathrm{div}\, u = 0 \quad , \end{cases}
$$

$$\theta_t + (u\cdot\nabla)\theta = (1/P)\Delta\theta - (u\cdot\nabla)\overline{\theta} - (b\cdot\nabla)\theta - (b\cdot\nabla)\overline{\theta} \quad ,$$

(7) $u|_{\partial\Omega} = 0 \quad , \quad \theta|_{\partial\Omega} = 0 \quad ,$

(8) $u|_{t=0} = a-b \quad , \quad \theta|_{t=0} = h-\overline{\theta} \quad ,$

where $R = \alpha g T_0 d^3/k\nu$ and $P = \nu/\kappa$.

We introduce the following abstract heat convection equation (AHC) :

(AHC) $\frac{dU}{dt} + AU(t) + FU(t) + MU(t) = P(\Omega)f,$

here $U = {}^t(u,\theta)$, $AU(t) = {}^t(-P_\sigma(\Omega)(\Delta u), -(1/P)\Delta\theta)$, $FU(t) = {}^t(P_\sigma(\Omega)(u\cdot\nabla)u, (u\cdot\nabla)\theta)$, $MU(t) = {}^t(P_\sigma(\Omega)((u\cdot\nabla)b + (b\cdot\nabla)u + R\theta), (u\cdot\nabla)\overline{\theta} + (b\cdot\nabla)\theta)$, $f = {}^t(-(b\cdot\nabla)b + \Delta b + d^3g/\nu^2 - R(\overline{\theta} - 1/P), -(b\cdot\nabla)\overline{\theta})$, $P(\Omega) = {}^t(P_\sigma(\Omega), 1_\Omega)$ and $P_\sigma(\Omega)$ is the projection $L^2(\Omega) \longrightarrow H_\sigma(\Omega)$.

§3, Results.

To explain our results, we give some preliminaries.

Definition 1. Let $U : [0,T] \longrightarrow H_\sigma(\Omega) \times L^2(\Omega)$, $T \in (0,\infty)$. Then U is called a strong solution of (AHC) on [0,T] if it satisfies the following properties (i) and (ii).

(i) $U \in C([0,T] ; H_\sigma(\Omega) \times L^2(\Omega))$ and $U(t)$ is absolutely continuous on (0,T].

(ii) $U(t) \in D(A) = (H^2(\Omega) \cap H^1_\sigma(\Omega)) \times (H^2(\Omega) \cap H^1_0(\Omega))$ for a.e. $t \in [0,T]$ and U satisfies (AHC) for a.e. $t \in [0,T]$.

Definition 2. If a strong solution U of (AHC) satisfies

(9) $U(0) = U_0 = {}^t(a-b,\ h-\overline{\theta})$ in $H_\sigma(\Omega) \times L^2(\Omega)$,

then it is called a strong solution of the initial value problem for (AHC).

Here we put $H = H_\sigma(\Omega) \times L^2(\Omega)$ and $V = H^1_\sigma(\Omega) \times H^1_0(\Omega)$.
Then we have the following existence theorem ([4]).

Theorem 0. Suppose the assumptions hold. Then for anu $U_0 \in H$ there exists a unique strong solution U with $U(0) = U_0$ such that $U \in C([0,T]\ ;\ H) \cap L^2(0,T\ ;\ V)$ and $dU/dt \in L^2(\delta,T\ ;\ H)$ where δ is an arbitrary number in (0,T). In particular, if $U_0 \in V$, then $U \in C([0,T]\ ;\ V) \cap L^2(0,T\ ;\ D(A))$ and $dU/dt \in L^2(0,T\ ;\ H)$.

Put $S(t) : U_0 \longrightarrow U(t)$, $U(t)$ being a solution, then we have

Theorem 1. There exists a V-bounded absorbing set $\mathcal{A}$ in V for (AHC) in the following sense : For every bounded set $\mathcal{B} \subset V$, there exists $t = t(\mathcal{B}) > 0$ such that $S(t)\mathcal{B} \subset \mathcal{A}$ for all $t \geqq t(\mathcal{B})$. Furthermore, for any bounded set $\mathcal{B}' \subset H$ we can take $t(\mathcal{B}') > 0$ satisfying $S(t)\mathcal{B}' \subset \mathcal{A}$ for all $t \geqq t(\mathcal{B}')$.

Next we state the definition of an attractor.

Definition 3. Let $\{S(t)\}_{t\geqq 0}$ be a semigroup of continuous operators in a Hilbert space H. Then a functional invariant set for S(t) is a set $X \subset H$ such that $S(t)X = X$ for any $t > 0$.

Definition 4. Let X be a functional invariant set for S(t). Then X is said to be an attractor in H if it possesses a neighbourhood $\mathcal{O}$ of X in H such that for any $\varphi_0 \in \mathcal{O}$ $\mathrm{dist}(S(t)\varphi_0, X) \longrightarrow 0$ as $t \longrightarrow \infty$.

Then we have

Theorem 2. Let $\mathcal{A}$ be the absorbing set obtained in Theorem 1. Putting $X = \bigcap_{s \geqq 0} \bigcup_{t \geqq s} \overline{S(t)\mathcal{A}}^{H}$, then X is an attractor for (AHC).

Here we introduce the Hausdorff dimension of X.

Definition 5. Let E be a metric space and X be a subset of E. The number $\mu_H(X,d) \in [0,\infty]$ defined by

$$\mu_H(X,d) = \lim_{\varepsilon \to 0} \mu_H(X,d,\varepsilon) = \sup_{\varepsilon > 0} \mu_H(X,d,\varepsilon)$$

is the d-dimensional Hausdorff measure of X, where

$$\mu_H(X,d,\varepsilon) = \inf \sum_i r_i^d$$

and the infimum is for all covering of X by a family (B_i) of balls of E of radii $r_i \leqq \varepsilon$.

Definition 6. Let E be a metric space and X be a subset of E. The number $d_H(X) \in [0,\infty]$ is called the Hausdorff dimension of X if it satisfies

$$\mu_H(X,d) = \begin{cases} 0 , & d > d_H(X) \\ +\infty , & d < d_H(X) \end{cases} ,$$

where $\mu_H(X,d)$ is the d-dimensional Hausdorff measure of X.

Now we will give our main theorem.

Theorem 3. Let X be the attractor in Theorem 2. Then the Hausdorff dimension $d_H(X)$ is finite and the following estimate holds :

(10) $\quad d_H(X) \leqq 1 + 2(\gamma_2/\gamma_1 + \sqrt{\gamma_3/\gamma_1})$,

where $\gamma_1 = C_2(\lambda_1 + \lambda_1')/2(1+P)$, $\gamma_2 = (2/P + |R|/2)$, $\gamma_3 = (C_1+4)\|\nabla b\|^2 + 4C_\Omega^2\{\|b\|\cdot\|\nabla b\|^3 + (3|R|/P + d^3\|g\|_\infty^2/\nu^2)^2\cdot|\Omega|\}$, $|R| = \alpha\|g\|_\infty T_0 d^3/\kappa\nu$.

$\|g\|_\infty^2 = \|g_1\|_{L^\infty}^2 + \|g_2\|_{L^\infty}^2$; λ_1 and λ_1' are the smallest eigenvalues of the Stokes operator and $-\Delta$ with the homogeneous Dirichlet condition, respectively.

Remark 2. The following estimate is known ([4],p118) : The function b given in Lemma 1 (satisfying (5)) also satisfies an estimate of the form

$$(11)\quad \|b\|_{L^2(\Omega)} \leqq \|b\|_{H^1(\Omega)} \leqq C\varepsilon \exp(4/\varepsilon)\|\beta\|_{H^{1/2}(\partial\Omega)},$$

where C depends on the domain Ω and physical constants.

Remark 3. We will denote the fractral dimension of X by $d_F(X)$. Then we can obtain the estimate

$$d_F(X) \leqq 2 + 4(\gamma_2/\gamma_1 + \sqrt{\gamma_3/\gamma_1}) .$$

§4. Some lemmas.

To prove the theorems, we prepare some lemmas.

Lemma 2. Let X be a subset of a Hilbert space H and $\{S(t)\}_{t\geqq 0}$ be a semigroup in H. Suppose that $S(t)X = X$ for any $t > 0$, $S(t)$ is differentiable on X with the differential $L(t,u)$ and $\sup_{u\in X}\|L(t_0,u)\|_{L(H)} < +\infty$ for some $t_0 > 0$. Denote the Lyapunov exponents for X by $\mu_j (j\geqq 1)$. If for some $n \geqq 1$, $\mu_1 + \cdots + \mu_{n+1} < 0$, then $\mu_{n+1} < 0$, $(\mu_1 + \cdots + \mu_n)/|\mu_{n+1}| < 1$ and the Hausdorff dimension $d_H(X)$ is bounded as

$$(12)\quad d_H(X) \leqq n + \frac{(\mu_1+\cdots+\mu_n)_+}{|\mu_{n+1}|} < n+1 .$$

The next elementary lemma is also useful ([4], p303).

Lemma 3. We assume that the sequence $\{\mu_j\}_{j\geqq 1}$ satisfies the following inequalities

(13) $\mu_1 + \cdots + \mu_j \leqq -\alpha j^{\theta} + \beta$ for any $j \geqq 1$,

where α, β, $\theta > 0$. Let $m \in N$ be defined by

(14) $m - 1 < (2\beta/\alpha)^{1/\theta} \leqq m$.

Then $\mu_1+\cdots+\mu_m < 0$ and $(\mu_1+\cdots+\mu_j)_+/|\mu_1+\cdots+\mu_m| < 1$ for $j = 1,\cdots,m$.

To state the next lemma, we prepare a framework as follows.

Let $\{S(t)\}_{t\geqq 0}$ be a semigroup in a Hilbert sapce H generated by a nonlinear evolution equation

(15) $\frac{du}{dt} = F(u(t))$ for $t > 0$, $u(0) = u_0 \in H$.

We assume (15) has a linearized equation

(16) $\frac{dU}{dt} = A_F(S(t)u_0)U(t)$, $U(0) = \xi$,

and moreover we assume (16) is well-posed for any u_0 and $\xi \in H$. Finally we assume $S(t)$ is differentiable in H with the differential $L(t,u_0)$ defined by

(17) $L(t,u_0)\xi = U(t)$ for any $\xi \in H$,

where $U(t)$ is a solution of (16).

Under these assumptions, we have ([4]) :

Lemma 4. If X is a functional invariant set of $S(t)$ and μ_j ($j \geqq 1$) are Lyapunov exponents for X, then

(18) $\mu_1 + \cdots + \mu_m \leqq q_m$,

where q_m is defined by

(19) $q_m \equiv \limsup_{t\to\infty} q_m(t)$

$$= \lim_{t\to\infty}\sup \Big(\sup_{u_0\in X} \sup_{\substack{\xi_i\in H\\ \|\xi_i\|\leq 1}} \frac{1}{t}\int_0^t T_r(A_F(u)\circ Q_m(\tau))d\tau\Big)$$

and $Q_m(t,u_o,\xi_1,\cdots,\xi_m)$ is the projector from H onto the space spanned by $U_1(t)$, $\cdots$, $U_m(t)$; $U_i(t)$ being solutions of (16) with $U_i(0) = \xi_i$.

We use later the known facts as below.

Lemma 5. Let $\{\lambda_j\}$ and $\{\lambda'_j\}$ be eigenvalues of the Stokes operator and $-\Delta$ with the homogeneous Dirichlet condition on Ω, respectively. If $\Omega \subset R^2$, then

(20) $\lambda_j \sim c\lambda_1 j$ as $j \longrightarrow \infty$ (by Metivier),

(21) $\lambda'_j \sim c\lambda'_1 j$ as $j \longrightarrow \infty$ (by Courant-Hilbert).

Lemma 6. ([4].) Let A be a linear positive self-adjoint operator in a Hilbert space H. Suppose A^{-1} is compact. Let $\{\lambda_j\}$ be eigenvalues of A. Then, for any family of elements φ_1, $\cdots$, φ_m of $V = D(A^{1/2})$ which is orthonormal in H,

(22) $\sum_{j=1}^{m}(A\varphi_j,\varphi_j) \geqq \lambda_1 + \cdots + \lambda_m$.

If, furthermore, $\lambda_j \sim c\lambda_1 j^{\alpha}(\alpha>0)$ as $j \longrightarrow \infty$, c depending on A, then

(23) $\sum_{j=1}^{m}(A\varphi_j,\varphi_j) \geqq \lambda_1 + \cdots + \lambda_m \geqq c'\lambda_1 m^{\alpha+1}$,

with another constant c' depending A and α.

§5. Proofs of the results.

We will only give the proof of Theorem 3 which is the main theorem of our work. First, we introduce the linearized equation of (AHC). Let $\varphi = {}^t(u,\theta)$ be a solution of (AHC) with $\varphi(0) = \varphi_o$

$= {}^t(u_0, \theta_0)$. For $\Phi = {}^t(U,\theta) \in (H^2(\Omega) \cap H^1_\sigma(\Omega)) \times (H^2(\Omega) \cap H^1_0(\Omega))$, we define an operator

$$(24) \quad A_F(\varphi)\Phi = \begin{pmatrix} P_\sigma(\Omega)\{\Delta U - ((u+b)\cdot\nabla)U - (U\cdot\nabla)(u+b) - R\theta\} \\ (1/P)\Delta\theta - ((u+b)\cdot\nabla)\theta - (U\cdot\nabla)(\theta+\overline{\theta}) \end{pmatrix}$$

Then the linearized equation (LAHC) of (AHC) is given by

$$(\text{LAHC}) \quad \frac{d\Phi}{dt} = A_F(\varphi)\Phi \quad ,\Phi(0) = {}^t(\xi,\eta) \ .$$

Remark 4. (LAHC) is well-posed for any ${}^t(\xi,\eta) \in H = H_\sigma(\Omega) \times L^2(\Omega)$. On the other hand, we can show S(t) is defferentiable in H and its differential $L(t,\varphi_0)$ is written for every ${}^t(\xi,\eta) \in H$ as $L(t,\varphi_0)\,{}^t(\xi,\eta) = {}^t(U(t),\theta(t))$ where ${}^t(U(t),\theta(t))$ is a solution of (LAHC) with ${}^t(U(0),\theta(0)) = {}^t(\xi,\eta)$. Therefore, we can apply Lemma 4 to (AHC). Moreover, we see that for some $t_0 > 0$, $\sup_{\varphi_0 \in X} \|L(t_0,\varphi_0)\| < +\infty$ where X is the attractor in Theorem 2, whence Lemma 2 is applicable to (AHC). We omit these verification.

Now, let X be the attractor for (AHC), we define q_m by

$$(25) \quad q_m = \limsup_{t\to\infty} \sup_{\binom{u_0}{\theta_0}\in X} \sup_{\substack{{}^t(\xi_i,\eta_i)\in H \\ \|{}^t(\xi_i,\eta_i)\|\leqq 1}} \frac{1}{t}\int_0^t T_r(A_F(\varphi(\tau))\circ Q_m(\tau))d\tau ,$$

where $\varphi = {}^t(u,\theta)$ is a solution of (AHC) with $\varphi(0) = {}^t(u_0,\theta_0)$ and Q_mH is spanned by ${}^t(U_1,\theta_1), \cdots, {}^t(U_m,\theta_m)$; ${}^t(U_i,\theta_i)$ are solutions of (LAHC) with ${}^t(U_i(0),\theta_i(0)) = {}^t(\xi_i,\eta_i)$. Then we present the following lemma by which we can prove Theorem 3.

Lemma 7. Consider (AHC) equation. Then we have

$$(26) \quad q_m \leqq -\gamma_1 m^2 + \gamma_2 m + \gamma_3 \leqq -\frac{\gamma_1}{2}m^2 + \frac{\gamma_2^2}{2\gamma_1} + \gamma_3 \ ,$$

where γ_i are defined in Theorem 3.

Remark 5. If Lemma 7 is proved, then from (18) of Lemma 4, we get an inequality like a type of (13) of Lemma 3, from which we can find m such that $\mu_1 + \cdots + \mu_m < 0$ and, using Lemma 2, we conclude that (10) of Theorem 3 holds.

Proof of Lemma 7.

We recall that X is the attractor for (AHC), q_m is defined by (25) and Q_m is the projector. To estimate q_m, let $\Psi_j(s) = {}^t(w_j(s), \theta_j(s))$ be an orthonormal basis of H, $\Psi_j \in V$ and $\Psi_1, \cdots, \Psi_m$ span $Q_m H$. Now we calculate

$$(27)\quad T_r(A_F(\varphi)\circ Q_m) = \sum_{j=1}^{m}(A_F(\varphi)\Psi_j,\ \Psi_j)$$

$$= \sum_{j=1}^{m}\{(\Delta w_j, w_j) - (((u+b)\cdot\nabla)w_j, w_j) - ((w_j\cdot\nabla)(u+b), w_j) - (R\theta_j, w_j)\}$$

$$+ \sum_{j=1}^{m}\{(\tfrac{1}{P}\Delta\theta_j, \theta_j) - (((u+b)\cdot\nabla)\theta_j, \theta_j) - ((w_j\cdot\nabla)(\theta+\overline{\theta}), \theta_j))\}.$$

Here we notice :

$$(28)\quad \sum_{j=1}^{m}|((w_j\cdot\nabla)(u+b), w_j)| \leqq \int_\Omega(\sum_{j=1}^{m}|w_j|^2)\cdot|\nabla(u+b)|dx$$

$$\leqq \|\rho\|\cdot\|\nabla(u+b)\| \quad (\text{where } \rho(x) \equiv \sum_{j=1}^{m}|w_j(x)|^2)$$

$$\leqq C_1(\|\nabla u\|^2 + \|\nabla b\|^2) + \frac{1}{2}\sum_{j=1}^{m}\|\nabla w_j\|^2\ ,$$

here we used $\|\rho\|^2 \leqq C_1\sum_{j=1}^{m}\|\nabla w_j\|^2$, C_1 depending on Ω.

$$(29)\quad \sum_{j=1}^{m}|(R\theta_j, w_j)| \leqq \frac{|R|}{2}m\ ,$$

since $\|w_j\|^2 + \|\theta_j\|^2 = 1$ (normalized).

$$(30)\quad \sum_{j=1}^{m}|((w_j\cdot\nabla)(\theta+\overline{\theta}), \theta_j)| = \sum_{j=1}^{m}|((w_j\cdot\nabla)\theta_j, (\theta+\overline{\theta}))|$$

$$\leqq \frac{2}{P}\sum_{j=1}^{m}\|w_j\|\cdot\|\theta_j\| \leqq \frac{1}{2P}\sum_{j=1}^{m}\|\nabla\theta_j\|^2 + \frac{2}{P}m\ ,$$

where we employed ${}^t(w_j, \theta_j) \in V$, $\|w_j\| \leqq 1$ together with $|\theta(\cdot,t)|$

$\leqq 1/P$ for $\theta \in x$ and $|\overline{\theta}| \leqq 1/P$ (maximal principle).

Using (28), (29), (30) and noticing

(31) $\|\nabla w_j\|^2 + P^{-1}\|\nabla\theta_j\|^2 \geqq (1+P)^{-1}(\|\nabla w_j\|^2 + \|\nabla\theta_j\|^2)$,

and with the aid of Lemma 5 and Lemma 6, then we have

(32) $T_r(A_F(\varphi)\circ Q_m)$

$$\leqq -\frac{1}{2(1+P)}\sum_{j=1}^{m}(\|\nabla w_j\|^2+\|\nabla\theta_j\|^2)+(\frac{2}{P}+\frac{|R|}{2})m+C_1(\|\nabla u\|^2+\|\nabla b\|^2)$$

$$\leqq -\frac{C_2}{2(1+P)}(\lambda_1+\lambda_1')m^2 + (\frac{2}{P}+\frac{|R|}{2})m+C_1(\|\nabla u\|^2+\|\nabla b\|^2),$$

where C_2 depends on Ω.

Next, we estimate $\|\nabla u(t)\|^2$. To do this, recall that ${}^t(u,\theta)$ is a solution of (AHC), then we get

(33) $\frac{1}{2}\frac{d}{dt}\|u(t)\|^2 + \|\nabla u(t)\|^2$

$$= -((u\cdot\nabla)b,u) - ((b\cdot\nabla)b,u) -(\Delta b,u) - (R\theta,u)$$

$$+ d^3\nu^{-2}(g,u) - (R\overline{\theta},u) + P^{-1}(R,u)$$

$$\leqq 4\times\frac{1}{8}\|\nabla u\|^2+2C_\Omega^2(\|b\|\cdot\|\nabla b\|^3+(3|R|P^{-1}+d^3\nu^{-2}\|g\|_\infty)^2|\Omega|)+2\|\nabla b\|^2,$$

where we used Lemma 1 with $\varepsilon = 1/8$.

Thus we obtain

(34) $\limsup_{t\to\infty}\frac{1}{t}\int_0^t\|\nabla u(\tau)\|^2 d\tau$

$$\leqq 4\|\nabla b\|^2 + 4C_\Omega^2(\|b\|\cdot\|\nabla b\|^3+(3|R|P^{-1}+d^3\nu^{-2}\|g\|_\infty)^2|\Omega|) .$$

Hence, finally we have

(35) $q_m = \limsup_{t\to\infty}\frac{1}{t}\int_0^t T_r(A_F(\varphi)\circ Q_m)d\tau$

$$\leqq -C_2 2^{-1}(1+P)^{-1}(\lambda_1+\lambda_1')m^2+(2P^{-1}+|R|\cdot 2^{-1})m$$

$$+ (C_1+4)\|\nabla b\|^2+4C_\Omega^2(\|b\|\cdot\|\nabla b\|^3+(3|R|P^{-1}+d^3\nu^{-2}\|g\|_\infty)^2|\Omega|).$$

$$\equiv -\gamma_1 m^2 + \gamma_2 m + \gamma_3 \quad .$$

Recalling Remark 4 and 5, we have proved Theorem 3.

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