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NONLINEAR UNSTEADY KELVIN-HELMHOLTZ FLOWS

M. NAGATA

School of Mathematics and Statistics, The University of Birmingham, Edgbaston, Birmingham, B15 2TT U.K.
Tel: (021) 414-6596
Fax: (021) 414-3389
E-mail:nagatam@vax1.bham.ac.uk.

ABSTRACT

We develop a nonlinear analysis of interfacial waves between two inviscid fluids with different dencities which flow parallel to each other in an oscillatory fashon. It is known from the linear analysis by Kelly(1965) that besides the ordinary Kelvin-Helmholtz instability which results from the difference in the mean uniform velocities across the interface exceeding some critical value, parametric resonances occur at the interface producing a pair of travelling waves which propagate in opposite directions with subharmonic frequencies.

We are concerned with the evolution of the pair of the trav-

elling waves in a long time scale T. It is described by the following coupled nonlinear equations for the two complex-valued amplitudes A^+ and A^- with the plus and the minus signs indicating the directions of propagation:

$$i\frac{\mathrm{d}A^+}{\mathrm{d}T} = aA^- + f_1A^+ + p \mid A^+ \mid^2 A^+ + q \mid A^- \mid^2 A^+$$

$$i\frac{\mathrm{d}A^-}{\mathrm{d}T} = -aA^+ + f_2A^- - q \mid A^+ \mid^2 A^- + s \mid A^- \mid^2 A^-,$$

where a measures the modulation amplitude of the velocity fields, f_1 and f_2 are linear functions of the difference in the mean velocities, and p, q, and r are real constants. This system is a four-dimensional real-valued Hamiltonian system with SO_2 symmetry.

The mean velocity difference is varied for a fixed a. The present analysis demonstrates the following findings. Firstly, after exhibiting a Hamiltonian Hopf bifurcation, one of the two pairs of complex conjugate eigenvalues on the imaginary axis collides at the origin, when in-phase or out-of-phase *steady* solutions bifurcate from the flat interface.

Secondly, our system turns out to be integrable with $\mid A^+ \mid^2 - \mid A^- \mid^2$ being an additional conserved quantity. The analysis of the resulting two-dimensional system enables us to clarify the behaviour of the original system at the Hamiltonian Hopf bifurcation with the result that in-phase or out-of-phase *periodic* solutions are shed from the quiescent interface. These two types of the periodic solutions are positioned in the phase spase in such a way that one of the types is inside a heteroclinic loop connecting two solutions of the other type.

References

Kelly, R. E. 1965 The stability of an unsteady Kelvin-Helmholtz flow. *J. Fluid Mech.* **22**, 547-560.