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# Tolerance Analysis of Linear Resistive Network by Interval Mathematics

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## 1. Introduction

In reference [2], the application of the interval Gaussian algorithm to the tolerance analysis, particularly, the worst case analysis of the linear resistive network has been presented. The interval hybrid equation is proposed to formulate the network equation. Hansen's method is applied to the interval hybrid equation to perform the interval Gaussian algorithm. In order to have a good estimation of the interval solutions, we proposed to use the maximally distant tree pair for the formulation of the network equation and take the intersection of the solutions of the hybrid equation for each tree. The maximally distant tree pair is introduced under assumption that the tree pair covers all the resistive branches.

In this report, we try to deal with the network of a little larger scale where the maximally distant tree pair can not cover the network. The interval Gaussian algorithm is compared with the iteration method. Finally, in order to have a better estimated solution, we describe a method for solving the interval hybrid equations with the subdivided interval parameters and taking the union of the interval solutions.

## 2. Interval Hybrid Equation

The network is assumed to have a single resistive element between nodes. We suppose that every branch of the network has connected across it an ideal current source and inserted into it an ideal voltage source. The hybrid equation of the linear resistive time-invariant network becomes

$$\mathbf{H}\mathbf{x} = \mathbf{a} \tag{1}$$

where

$$\mathbf{H} = \begin{bmatrix} \mathbf{G} & \mathbf{Q}_l \\ \mathbf{B}_t & \mathbf{R} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} \mathbf{V}_t \\ \mathbf{I}_l \end{bmatrix}, \mathbf{a} = \begin{bmatrix} \mathbf{J}_s \\ \mathbf{E}_s \end{bmatrix}, \mathbf{J}_s = -\mathbf{j}_{st} + \mathbf{G}\mathbf{e}_{st}, \mathbf{E}_s = -\mathbf{e}_{sl} + \mathbf{R}\mathbf{j}_{sl} \tag{2}$$

The matrix $\mathbf{G}$ is the diagonal branch conductance matrix of dimension $\rho$ and $\mathbf{R}$ is the diagonal branch resistance matrix of dimension $\mu$, where $\rho$ and $\mu$ are the rank and nullity of the simple graph associated with the network with the voltage sources short and current sources open. The submatrices $\mathbf{Q}_l$ and $\mathbf{B}_t$ are the principal parts of the cutset and tieset matrices, respectively. The vectors $\mathbf{V}_t$ and $\mathbf{I}_l$ are the tree branch voltage and link current vectors, respectively. The vectors $\mathbf{J}_s$ and $\mathbf{E}_s$ are the cutset current source and loop voltage source vectors, respectively. The vectors $\mathbf{j}_t$ and $\mathbf{j}_l$ are the current source vectors associated with the tree and the link, respectively. The vectors $\mathbf{e}_t$ and $\mathbf{e}_l$ are the voltage source vectors associated with the tree and the link, respectively.

### 3. Selection of Trees

We pick up a tree $T_1$ which contains as many conductors as possible and select the maximally distant tree $T_2$ from $T_1$. At this stage, we check whether the pair of the trees $T_1, T_2$ covers all the resistive branches of the network. If not, we choose another tree $T_3$ and check whether the tree set $T_1, T_2, T_3$ all the resistive branches. We repeat this procedure untill the set of the trees $T_1, T_2, ..., T_m$ covers all the tree branches where the integer $m$ is the minimum number of the trees.

For the tree $T_i$ $(i = 1, 2, ..., m)$ we formulate the hybrid equation

$$\mathbf{H}_i \mathbf{x}_i = \mathbf{a}_i \; i = 1, 2, ..., m. \tag{3}$$

We assume that the matrix $\mathbf{H}_i \, (i = 1, 2, ..., m)$ is nonsingular. Using Hansen's method, we transform Eq.(3) into

$$\tilde{\mathbf{H}}_i \mathbf{x}_i = \tilde{\mathbf{a}}_i \; i = 1, 2, ..., m \tag{4}$$

where

$$\tilde{\mathbf{H}}_i = m(\mathbf{H}_i)^{-1}\mathbf{H}_i, \; \tilde{\mathbf{a}}_i = m(\mathbf{H}_i)^{-1}\mathbf{a}_i \; i = 1, 2, ..., m. \tag{5}$$

The interval Gaussian algorithm or the iteration method is applied to Eq.(5) for each $i$ to have the interval branch voltage solution $\mathbf{V}^{(i)} \, (i = 1, 2, ..., m)$ and the branch current solution $\mathbf{I}^{(i)} \, (i = 1, 2, ..., m)$. We represent the true branch voltage and current solutions as $\mathbf{V}_{true}$ and $\mathbf{I}_{true}$, respectively. Hence the relations

$$\mathbf{V}_{true} \subseteq \mathbf{V}^{(i)}, \; \mathbf{I}_{true} \subseteq \mathbf{I}^{(i)} \; i = 1, 2, ..., m \tag{6}$$

Clearly we have

$$\mathbf{V}_{true} \subseteq (\cap_{i=1}^{m} \mathbf{V}^{(i)}) \subseteq \mathbf{V}^{(i)} \; i = 1, 2, ..., m \tag{7}$$

$$\mathbf{I}_{true} \subseteq (\cap_{i=1}^{m} \mathbf{I}^{(i)}) \subseteq \mathbf{I}^{(i)} \; i = 1, 2, ..., m \tag{8}$$

Hence we take $\mathbf{V} = \cap_{i=1}^{m} \mathbf{V}^{(i)}$ and $\mathbf{I} = \cap_{i=1}^{m} \mathbf{I}^{(i)}$ as the nearest interval branch voltage and current solutions to the true solutions.

### 4. Partitioning Interval Parameters

If the tolerance of the resistive parameter is larger, patitioning the interval parameters is effective to have a good estimation of the interval solutions. Let the network have $b = \rho + \mu$ resistive parameters. Partitioning i-th resistive parameters into $b_i$ $(i = 1, 2, ..., b)$ subintervals provides us with $\Pi_{i=1}^{b} b_i$ hybrid equations for each tree. Associated with the tree $T_i$, let $\mathbf{V}_k^{(i)}$ and $\mathbf{I}_k^{(i)}$ be the interval branch voltage and current solutions of the hybrid equation for the $k$-th combination of each partitioned parameter. For the tree $T_i$, we have

$$\mathbf{V}^{(i)} = \cup_{k=1}^{K} \mathbf{V}_k^{(i)}, \; \mathbf{I}^{(i)} = \cup_{k=1}^{K} \mathbf{I}_k^{(i)}, K = \Pi_{i=1}^{b} b_i \tag{9}$$

Solving $K$ hybrid equations for the tree $T_i$, we have the interval solutions which are of the form

$$\mathbf{V} = \cap_{i=1}^{m} \mathbf{V}^{(i)} = \cap_{i=1}^{m} (\cup_{k=1}^{K} \mathbf{V}_k^{(i)}) \tag{10}$$

$$\mathbf{I} = \cap_{i=1}^{m} \mathbf{I}^{(i)} = \cap_{i=1}^{m} (\cup_{k=1}^{K} \mathbf{I}_k^{(i)}). \tag{11}$$

If $b$ interval parameters take part in partitioning, $Km$ combinations of the intervals involves the solution of the hybrid equations. For example, by each halving of the interval parameters, we have to solve $2^b m$ hybrid equations.

### 5. Numerical Examples

This section demonstrates the worst-case analysis of a linear resistive network with rather large number of branches. Hybrid equation (3) is solved by the interval Gaussian algorithm(abbreviated as IGA) as well as symmetric single step Method(abbreviated as SS method)[1]. The latter is one of the iteration methods in which the hybrid matrix $\tilde{\mathbf{H}}$ is decomposed into a diagonal, a strictly lower and a strictly upper triangular matrix. Both methods are compared. Further, the validity of partitioning of the interval parameters is shown.

### 5.1 Comparison of IGA with SS Method

Fig.1 shows a linear resistive network the branch voltages of which are analysed. Each resistor has its own tolerance. The resistors and voltage sources assign respectively the center values $g_i$ $(i = 1, 2, ..., 26)$ as conductances and numerical values $E_i\,(i = 1, 2)$ in table 1. We give 10 percents tolerance of the center values for each resistor. Table 2 shows the branch numbers of the trees chosen. The distance between $T_1$ and $T_2$ is maximal. The set of the trees $T_1, T_2, T_3$ covers all the resistive branches.

Table 3 shows the branch voltages computed by IGA and SS method as well as Monte Carlo method(abbreviated as MC method). The name of each column indicates the method as its name shows. The interval numbers in MONTE CARLO are computed by MC method. The numbers in GAUSS and SS are the ratios of the widths of the voltages by IGA and SS method to the widths by MC method. The branch voltages are well estimated except $V_9, V_{12}, V_{13}, V_{19}$ and $V_{24}$. The four times of iterations are enough to converge the procedures of SS method. The starting interval are the point interval numbers which are determined by the solutions of the network equation without the tolerances of resistances.

### 5.2 Validity of Partitioning

We deal with the ladder network which was used as an example in Ref.[2]. The resistive parameters are the same as given in Ref.[2]. As a typical case, the interval of each resistive parameter is halved($K = 2^9$). The maximally distant tree pair covers all the resistive branches($m = 2$). Table 4 shows the results computed by IGA and SS method as well as MC method. The numbers in each column have the same meaning as in Table 3. Clearly the interval branch voltages by both methods are best estimated in comparison with MC method. However, IGA and SS methods require longer CPU time than MC method. Hence the number of interval parameters subject to partitioning should not be excessive in practice.

### 6. Conclusion

We have presented methods for better estimation of the interval solutions of linear resistive network. In order to reduce the computational time, chosing the minimum number of trees to cover all the resistive branches is of importance. Partitioning of the interval parameters leads us to having a good estimation of the solutions although the number of hybrid equations to be solved increases excessively. The methods described here will be effective when a parallel machine will be more common.

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### References

[1] G. Alefeld and J. Herzberger;"Introduction to Interval Computations",translation by J. Rokne, Academoic Press, New York, New York, 1983.

[2] K. Okumura;"Several Applications of Interval Mathematics to Electrical Network Analysis", submitted to RIMS Kokyuroku of Kyoto University, 1993.

Table 1 Center values of conductances.

| | |
|---|---|
| $g_1$ | 0.400 |
| $g_2$ | 0.013 |
| $g_3$ | 0.800 |
| $g_4$ | 0.038 |
| $g_5$ | 0.320 |
| $g_6$ | 0.928 |
| $g_7$ | 0.019 |
| $g_8$ | 0.031 |
| $g_9$ | 0.019 |
| $g_{10}$ | 0.013 |
| $g_{11}$ | 0.800 |
| $g_{12}$ | 0.063 |
| $g_{13}$ | 0.448 |
| $g_{14}$ | 0.928 |
| $g_{15}$ | 0.063 |
| $g_{16}$ | 0.031 |
| $g_{17}$ | 0.013 |
| $g_{18}$ | 0.031 |
| $g_{19}$ | 0.013 |
| $g_{20}$ | 0.063 |
| $g_{21}$ | 1.280 |
| $g_{22}$ | 0.031 |
| $g_{23}$ | 0.800 |
| $g_{24}$ | 0.038 |
| $g_{25}$ | 0.768 |
| $g_{26}$ | 1.440 |
| $E_1$ | 0.500 |
| $E_2$ | 1.050 |

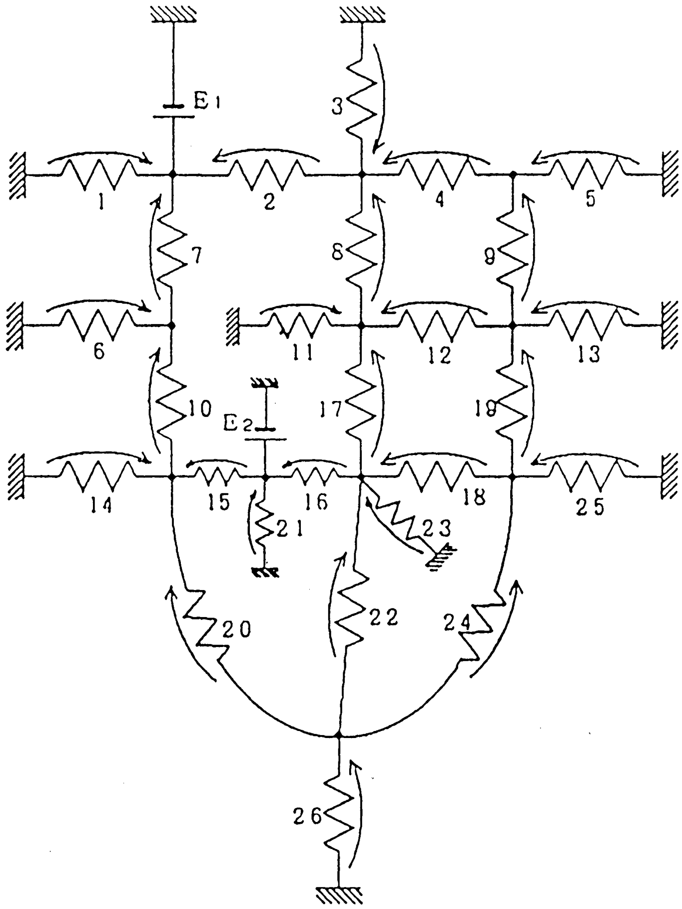


Fig. 1 Resistive network. Arrow shows direction of branch voltage.

Table 2 Trees and their branch numbers.

| | Branch number |
|---|---|
| $T_1$ | 1, 3, 5, 6, 11, 13, 14, 21, 23, 25, 26 |
| $T_2$ | 1, 2, 4, 7, 9, 10, 12, 16, 18, 19, 22 |
| $T_3$ | 2, 3, 5, 7, 8, 13, 15, 17, 20, 21, 24 |

Table 3 Interval solutions of branch voltages by MC method, IGA and SS method.

| | MONTE CARLO | | GAUSS | SS |
|---|---|---|---|---|
| V1 | [ 0.500000 | 0.500000 ] | 1.0000 | 1.0000 |
| V2 | [ 0.490969 | 0.493896 ] | 1.0755 | 1.0763 |
| V3 | [ 0.006104 | 0.009031 ] | 1.0258 | 1.0666 |
| V4 | [ 0.005377 | 0.008249 ] | 1.0939 | 1.1080 |
| V5 | [ 0.000524 | 0.001084 ] | 1.1392 | 1.1520 |
| V6 | [ 0.008742 | 0.013217 ] | 1.0393 | 1.0411 |
| V7 | [ 0.486783 | 0.491258 ] | 1.0812 | 1.0824 |
| V8 | [ 0.005119 | 0.008351 ] | 1.1505 | 1.1585 |
| V9 | [ 0.000331 | 0.000933 ] | 1.3198 | 1.3306 |
| V10 | [ -0.065901 | -0.038925 ] | 1.1266 | 1.1295 |
| V11 | [ 0.000543 | 0.001122 ] | 1.1903 | 1.2002 |
| V12 | [ 0.000412 | 0.000938 ] | 1.3521 | 1.3751 |
| V13 | [ 0.000098 | 0.000240 ] | 1.4369 | 1.4569 |
| V14 | [ 0.051612 | 0.075056 ] | 1.0697 | 1.0878 |
| V15 | [ -0.998388 | -0.974944 ] | 1.0741 | 1.0775 |
| V16 | [ 1.006290 | 1.020232 ] | 1.0792 | 1.0811 |
| V17 | [ -0.042951 | -0.028955 ] | 1.1441 | 1.1473 |
| V18 | [ 0.028348 | 0.042195 ] | 1.0876 | 1.0949 |
| V19 | [ -0.001947 | -0.000848 ] | 1.3302 | 1.3353 |
| V20 | [ 0.048242 | 0.071966 ] | 1.0879 | 1.0978 |
| V21 | [ 1.050000 | 1.050000 ] | 1.0000 | 1.0000 |
| V22 | [ 0.025721 | 0.041037 ] | 1.1178 | 1.1237 |
| V23 | [ 0.029768 | 0.043709 ] | 1.0741 | 1.0834 |
| V24 | [ -0.003205 | -0.000524 ] | 1.2613 | 1.2698 |
| V25 | [ 0.001010 | 0.002110 ] | 1.1673 | 1.1758 |
| V26 | [ 0.002273 | 0.004589 ] | 1.1572 | 1.1642 |

Table 4 Interval solution of ladder network[2].

| | MONTE CARLO | | GAUSS | SS |
|---|---|---|---|---|
| V1 | [ 6.4248 | 7.8960 ] | 1.0441 | 1.0503 |
| V2 | [ 3.6234 | 4.8511 ] | 1.0640 | 1.0723 |
| V3 | [ 4.9306 | 6.0860 ] | 1.0755 | 1.0821 |
| V4 | [ 1.8081 | 2.6369 ] | 1.0812 | 1.0903 |
| V5 | [ 0.8304 | 1.4132 ] | 1.0709 | 1.0827 |
| V6 | [ 2.3123 | 3.5820 ] | 1.0546 | 1.0663 |
| V7 | [ -1.8596 | -0.7401 ] | 1.0746 | 1.0878 |
| V8 | [ 2.7787 | 3.8542 ] | 1.0636 | 1.0732 |
| V9 | [ 0.8399 | 1.4170 ] | 1.0871 | 1.0991 |