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Convergence of Numerical Interface Curves to Nonlinear Diffusion Equation with Absorption

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1. Introduction.

We are concerned with difference approximations to the following initial value problem for the nonlinear diffusion equation described by

$$v_t = (v^m)_{xx} - cv^p, \qquad x \in \mathbf{R}^1, \quad t > 0 \tag{1.1}$$

with an initial condition

$$v(0,x) = v^0(x), \qquad x \in \mathbf{R}^1, \tag{1.2}$$

where $m(>1)$, $p(>0)$ and $c(\geq 0)$ are constants, and $v^0(x)$ has compact support. The equation of the form (1.1) is known as a simple mathematical model for several physical phenomena.

The first model with $c = 0$ describes the flow of an ideal gas through a homogeneous porous medium, where v represents a density of the gas. Physically, v^{m-1} is the pressure of the gas and $(v^{m-1})_x$ is the velocity.

The second model with $c > 0$ describes the transport of the thermal energy in plasma. Here v means the temperature. The term $-cv^p$ is understood as volumetric absorption caused by radiation.

In both models with $c = 0$ and with $c > 0$ the most interesting phenomenon is the occurrence of finite propagation of the initial support. It is already shown that there are three cases of the behavior of supp $v(t,\cdot)$.

Case 1. Positivity. For $c = 0$ and $m > 1$, or $c > 0$ and $p \geq m > 1$ supp $v(t,\cdot)$ expands as t increases and supp $v(\infty,\cdot) = \mathbf{R}^1$ ([1],[3],[4],[8],[9],[15]).

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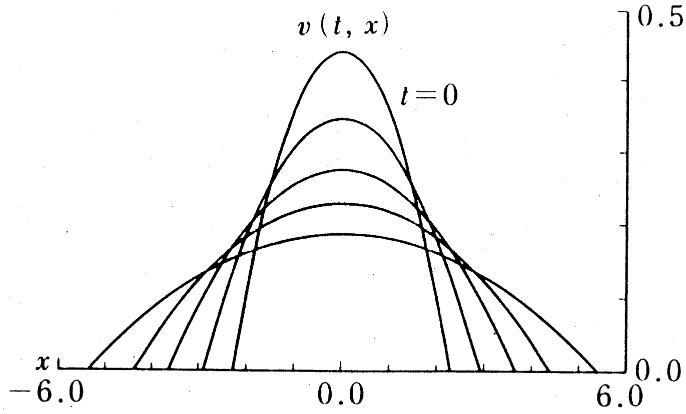
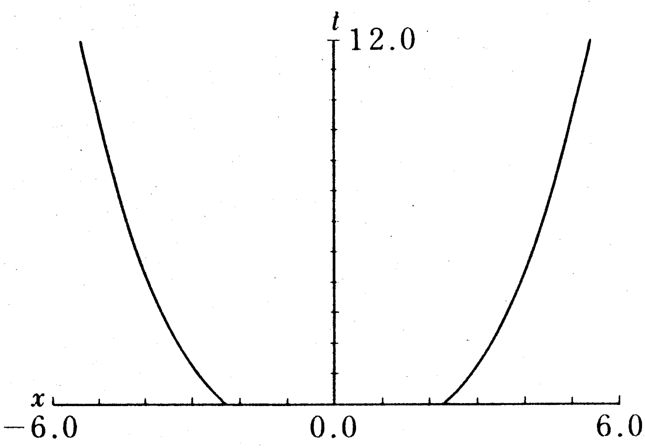
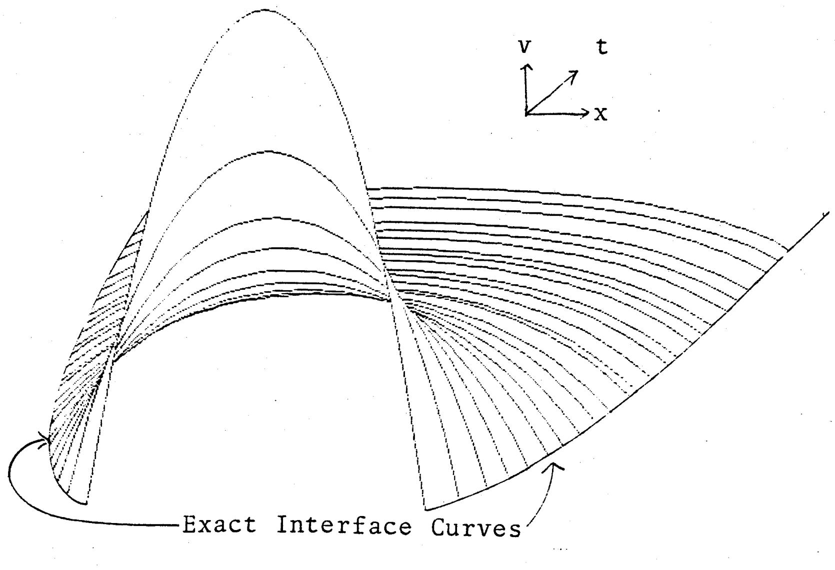




$$v_t = (v^2)_{xx}$$

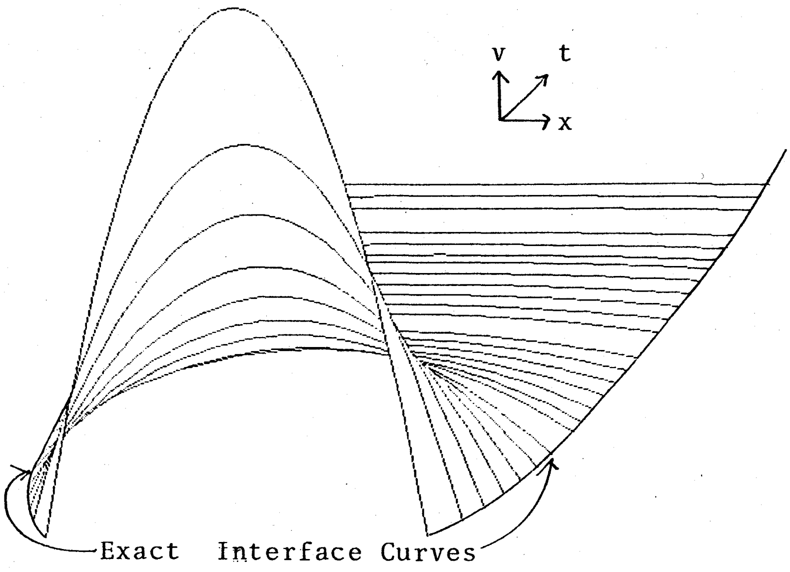
Barenblatt and Pattle's exact solution and interface curve .



$$V_t=(V^2)_{xx}- V^2$$

Bertsch, Kersner and Peletier's solution.

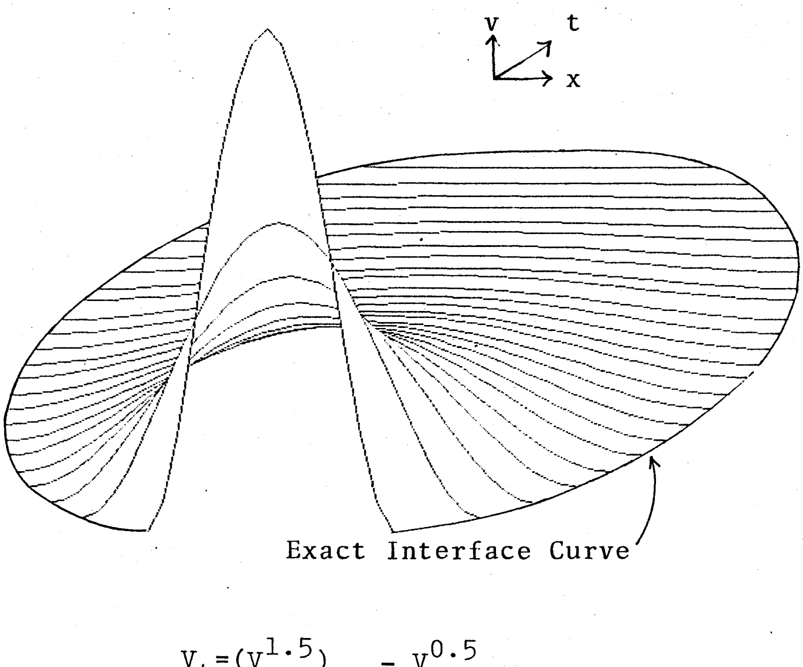
Case 2. Localization. For $c > 0$ and $m > p \geq 1$ supp $v(t, \cdot)$ expands as t increases and is uniformly bounded with respect to t ([7],[9],[11]).



$V_t = (V^2)_{xx} - 0.5V$

Exact solution (Gurtin and MacCamy).

Case 3. Total Extinction. For $c > 0$, $m > 1$ and $0 < p < 1$ supp v is compact in $[0, \infty) \times \mathbf{R}^1$ and $v(t, x)$ extincts in finite time ([9],[10],[11]).



$V_t = (V^{1.5})_{xx} - V^{0.5}$

Kersner's solution

From a numerical point of view, it is interesting to determine the behavior of supp $v(t,\cdot)$, that is, *interface curves* appearing between $v>0$ and $v=0$.

Several difference schemes to (1.1)–(1.2) with $c=0$ have been investigated. In [6] Graveleau and Jamet proved the finite propagation of the support by using their difference scheme. However, their scheme does not give good approximations to the exact interface curves. DiBenedetto and Hoff's scheme [5] and Mimura, Nakaki and Tomoeda's scheme [13] give the convergence of numerical interface curves. It is observed that the numerical interface curves by the latter scheme are more accurate rather than the former's .

Numerical computations to (1.1)–(1.2) with $c>0$ are investigated by Rosenau and Kamin [16], Mimura, Nakaki and Tomoeda [13] and Nakaki[14]. In [13] the convergence of numerical solutions is proved for Cases 1 and 2,and the convergence of numerical interface curves is also proved for Case 1 and Case 2 with $p=1$. In Case 3 Rosenau and Kamin numerically examined the problem of the pulse splitting into several sub-pulses, but the theoretical results of the numerical scheme are not discussed. In Case 3 with $m+p=2$ it is shown in [14] that not only numerical solutions but also interface curves converge to exact ones under

Condition A: $(v^0(x))^{m-1}$ is concave downward on its support.

In this paper we show the convergence of numerical interface curves without Condition A for Case 3 with $m+p=2$. However, instead of Condition A we have to impose the following condition on numerical results.

Condition B: There exist positive numbers M, T^* and h^* such that

$$\dot{\ell}_h(t) < M \quad \text{and} \quad \dot{r}_h(t) > -M \quad \left(\dot{} = \frac{d}{dt}\right) \tag{1.3}$$

for almost all $t\in[0,T^*]$ and for all $h\in(0,h^*)$. Here $\ell_h(t)$ and $r_h(t)$ denote left and right numerical interface curves, respectively, and h is a space mesh width.

We now state the existence and uniqueness of weak solution of (1.1)–(1.2) in Section 2. In Section 3 we present Mimura, Nakaki and Tomoeda's scheme for Cases 1 and 2, and demonstrate some numerical solutions and interface curves. In Section 4 we introduce the modified Mimura, Nakaki and Tomoeda's scheme for Case 3 with $m+p=2$ and $c>0$, and show the convergence of numerical interface curves under the Condition B.

2. Existence and Uniqueness.

To show the convergence of the difference approximation to the exact solution, we prepare the existence and uniqueness of the weak solution of (1.1)–(1.2) with $m+p\geq 2$ and $c\geq 0$.

Definition. (Herrero and Vázquez[8]). A function $v(t,x)$ defined on $S=[0,\infty)\times\mathbf{R}^1$ is said to be a weak solution of (1.1)–(1.2), if

(i) $v \in C^0(S) \cap L^\infty(S)$ and $v \geq 0$ on S ;

(ii) for any $x \in \mathbf{R}^1$, $v(0,x) = v^0(x)$;

(iii) for any function $\phi(x) \in C^{1,2}(S)$ with compact support in S, the following integral relation holds:

$$\iint_S (v^m \phi_{xx} + v\phi_t - cv^p\phi)dxdt + \int_{\mathbf{R}^1} v(0,x)\phi(0,x)dx = 0. \tag{2.1}$$

To show the stability of our difference scheme, we set $u = v^{m-1}$ and rewrite (1.1)–(1.2) as

$$u_t = muu_{xx} + a(u_x)^2 - (m-1)cu^q, \quad a = \frac{m}{m-1}, \quad q = \frac{m+p-2}{m-1}, \tag{2.2}$$

$$u(0,x) = u^0(x) \equiv (v^0(x))^{m-1}. \tag{2.3}$$

Definition . A function $u(t,x)$ defined on S is said to be a weak solution of (2.2)–(2.3), if

(i) $u \in C^0(S) \cap L^\infty(S)$, $u_x \in L^\infty(S)$ and $u \geq 0$ on S ;

(ii) for any $x \in \mathbf{R}^1$, $u(0,x) = u^0(x)$;

(iii) for any function $\phi(x) \in C^{1,2}(S)$ with compact support in S, the following integral relation holds:

$$\iint_S (u\phi_t - muu_x\phi_x - (m-a)(u_x)^2\phi - (m-1)cu^q\phi)dxdt + \int_{\mathbf{R}^1} u(0,x)\phi(0,x)dx = 0. \tag{2.4}$$

Theorem 2.1 (Herrero and Vázquez [8]). *Let $m > 1$, $m+p \geq 2$, $p > 0$ and let v^0 be a continuous, nonnegative and bounded real function. Then there exists a unique weak solution v of (1.1)–(1.2) and v is smooth in the set $\{(t,x) : v(t,x) > 0\}$.*

Remark. Let D be the space of all continuous functions with compact support in S, and D' be its dual. If a weak solution u of (2.2)–(2.3) with $m > 1$ satisfies

$$u_{xx}, \quad u_t \in D',$$

then $v = u^{1/(m-1)}$ is the unique weak solution of (1.1)–(1.2) (see [6]).

3. Difference Schemes to (1.1)–(1.2) with $m > 1$ and $p \geq 1$.

Our difference scheme approximates the problem (2.2)–(2.3) instead of (1.1)–(1.2). The difference scheme is constructed based on splitting the equation (2.2) into three parts:

$$(3.1) \qquad u_t = Pu = muu_{xx},$$

$$(3.2) \qquad u_t = Hu = a(u_x)^2, \qquad a = m/(m-1),$$

$$(3.3) \qquad u_t = Du = -(m-1)cu^q, \qquad q = (m+p-2)/(m-1).$$

We denote by V_h the set of the nonnegative continuous functions u_h with the following properties:

(i) u_h has compact support $[\ell(u_h), r(u_h)]$;

(ii) u_h is linear on each interval $[x_i, x_{i+1}]$ $(i \in \mathbf{Z})$, where

$$x_i = ih \quad \text{for all} \quad ih \in (\ell(u_h), r(u_h)) \quad (i = L, L+1, \cdots, R-1, R),$$
$$x_{L-1} \equiv \ell(u_h), \quad x_{R+1} \equiv r(u_h).$$

Let $h_i = x_{i+1} - x_i$ and $u_i = u(x_i)$. Then our difference scheme [13] is described as follows:

Find the sequence $\{u_h^n\}_{n=0,1,2,\cdots} \subset V_h$ such that

$$(3.4) \qquad u_h^{n+1} = S_{h,k}u_h^n \equiv (D_{h,k/\nu})^\nu \cdot (P_{h,k/\mu})^\mu \cdot H_{h,k}u_h^n \quad \text{for } n = 0, 1, 2, \cdots.$$

Here $k \equiv k_{n+1} \equiv t_{n+1} - t_n$ is a variable time step , $\mu \equiv \mu_{n+1}$ and $\nu \equiv \nu_{n+1}$ are integers depending on k_{n+1}.

Difference operator $\boldsymbol{H}_{h,k}$

We define the operator $H_{h,k}$ mapping from V_h to V_h by

$$(3.5) \qquad u_h' = (H_{h,k}u_h)(x_i) = \text{exact solution } u(k, x_i) \text{ of } u_t = Hu$$
$$\text{with the initial value } u(0,x) = u_h(x).$$

Let $\{L', L'+1, \cdots, R'-1, R'\}$ be the set of integers such that

$$x_i = ih \quad \text{for all} \quad ih \in (\ell(u_h'), r(u_h')) \quad (i = L', L'+1, \cdots, R'-1, R').$$

Then

$$(3.6)\qquad u_h' = \begin{cases} u_i + a(\delta u_i)^2 k & \text{if} \quad i \in S^+ = S_S^+ \cup S_R^+, \\ u_i + a(\delta u_{i-1})^2 k & \text{if} \quad i \in S^- = S_S^- \cup S_R^-, \\ u_i & \text{if} \quad i \in S^0, \\ (L'h - \ell')\delta u_{L-1} & \text{if} \quad i = L' = L-1, \\ (R'h - r')\delta u_R & \text{if} \quad i = R' = R+1, \\ 0 & \text{if} \quad i \in \mathbf{Z} \setminus \{L', \cdots, R'\}, \end{cases}$$

where

$$\begin{aligned} S_S^+ &= \Big\{ i \in \{L, \cdots, R\} : \delta u_{i-1} < \delta u_i \quad \text{and} \quad \delta u_{i-1} > -\delta u_i \Big\}, \\ S_S^- &= \Big\{ i \in \{L, \cdots, R\} : \delta u_{i-1} < \delta u_i \quad \text{and} \quad \delta u_{i-1} \le -\delta u_i \Big\}, \\ S_R^+ &= \Big\{ i \in \{L, \cdots, R\} : \delta u_{i-1} \ge \delta u_i > 0 \Big\}, \\ S_R^- &= \Big\{ i \in \{L, \cdots, R\} : 0 > \delta u_{i-1} \ge \delta u_i \Big\}, \\ S^0 &= \Big\{ i \in \{L, \cdots, R\} : \delta u_{i-1} \ge 0 \ge \delta u_i \Big\}. \end{aligned}$$

Stability Condition: $k \equiv k_{n+1}$ is the largest number satisfying the following inequality:

$$(3.7)\qquad a\|(u_h)_x\|_\infty k \le \min\{h/4,\ Lh - \ell(u_h),\ r(u_h) - Rh\}.$$

$$(3.8)\qquad k \le Ch^s \quad \text{(For simplicity we put } C = 1,\ s = 1/2).$$

Difference operator $P_{h,k/\mu}$

$$(3.9)\qquad (P_{h,k/\mu} u_h)(x_i) = u_i + \frac{k}{\mu} m u_i \delta^2 u_i \quad \text{for all } i \in \mathbf{Z},$$

$$\delta u_i = (u_{i+1} - u_i)/h_i, \qquad \delta^2 u_i = 2(\delta u_i - \delta u_{i-1})/(h_i + h_{i-1}),$$

Stability Condition: μ satisfies the following inequalities:

$$(3.10)\qquad m\|u_h\|_\infty (k/\mu)[1/h^2 + 2/\{h(h+h_j)\}] \le 1 \quad \text{for } j = L-1, R,$$

$$(3.11)\qquad 4m\|(u_h)_x\|_\infty (k/\mu)/(h+h_j) \le 1 \quad \text{for } j = L-1, R.$$

Difference operator $D_{h,k/\nu}$

$$(3.12)\qquad (D_{h,k/\nu} u_h)(x_i) = \{[u_h - (m-1)c(u_h)^q (k/\nu)]^+\}(x_i), \quad [f]^+ = \max\{f, 0\}.$$

Stability Condition: ν satisfies the following inequality:

$$(k/\nu)(m-1)cq\|u_h\|_\infty^{q-1} < 1. \tag{3.13}$$

We note that the interfaces of $D_{h,k/\nu}u_h$ and $P_{h,k/\mu}u_h$ are same ones of u_h. From the property of the operator $H_{h,k}$ the numerical interfaces of solutions given by (3.4) can be expressed as follows:

$$\ell_{n+1} = \ell_n - a\delta u_{L-1}^n k, \tag{3.14}$$

$$r_{n+1} = r_n - a\delta u_R^n k, \tag{3.15}$$

where

$$\ell_n \equiv \ell(u_h^n), \quad r_n \equiv r(u_h^n) \quad \text{for } n = 0, 1, 2, \cdots. \tag{3.16}$$

To start the scheme (3.4), we take

$$t_0 = 0, \quad \ell_0 = \ell(u_h^0), \quad r_0 = r(u_h^0), \quad u_h^0(x_i) = u^0(x_i), \tag{3.17}$$

and impose the following assumptions on the initial value $u^0(x)$.

Assumption I. $u \in C^0(\mathbf{R}^1) \cap BV(\mathbf{R}^1)$ has compact support $[\ell(u^0), r(u^0)]$ and satisfies $u_x^0 \in L^\infty(\mathbf{R}^1) \cap BV(\mathbf{R}^1)$.

Assumption II. u^0 satisfies

$$u_{xx}^0(x) > -C, \quad \text{where } C \text{ is some positive constant.} \tag{3.18}$$

Theorem 3.1 (Stability [13], [18]). *Under Assumption I let $m > 1$, $p \geq 1$ and $c > 0$. Then*

$$0 \leq u_h^n \leq \|u^0\|_\infty \quad \textit{for all} \quad n \geq 0, \tag{3.19}$$

$$\lim_{n\to\infty} t_n = \infty. \tag{3.20}$$

Moreover, for each $T > 0$, the following estimates

$$(3.21)\qquad \begin{cases} \|(u_h^n)_x\|_\infty, \quad \|(u_h^n)_x\|_{L^1(\mathbf{R}^1)}, \quad V((u_h^n)_x), \\ \|(u_h^{n+1} - u_h^n)/k_{n+1}\|_{L^1(\mathbf{R}^1)} \quad \leq C_1(T) \qquad \text{for all } t_{n+1} \leq T, \end{cases}$$

hold for all $n \geq 0$ satisfying $t_{n+1} \in [0, T]$, where $C_1(T)$ is a constant depending on T, but independent of h.

Remark. The conclusion of this theorem is also valid for $c = 0$.

To state the convergence, we define a function $u_h(t, x)$ by

$$(3.22)\qquad u_h(t, x) = u_h^n(x) \quad \text{on} \quad [t_n, t_{n+1}) \qquad \text{for all } n \geq 0.$$

Theorem 3.2 (Convergence [13],[18]). *Under the same assumptions as stated in Theorem 3.1, $(u_h)^{1/(m-1)}$ converges uniformly in any bounded domain of $[0, \infty) \times \mathbf{R}^1$ to the unique weak solution of (1.1)-(1.2).*

By piecewise-linearly interpolating (t_n, ℓ_n) (resp.(t_n, r_n)) $(n \geq 0)$ we define the left (resp. right) numerical interface curve $\ell_h(t)$ (resp. $r_h(t)$).

Theorem 3.3 (Numerical Interface Curves [13],[18]). *Under Assumptions I and II let*

$$(m, p) \in \{(m, p) : p \geq m > 1\} \cup \{(m, 1) : m > 1\}.$$

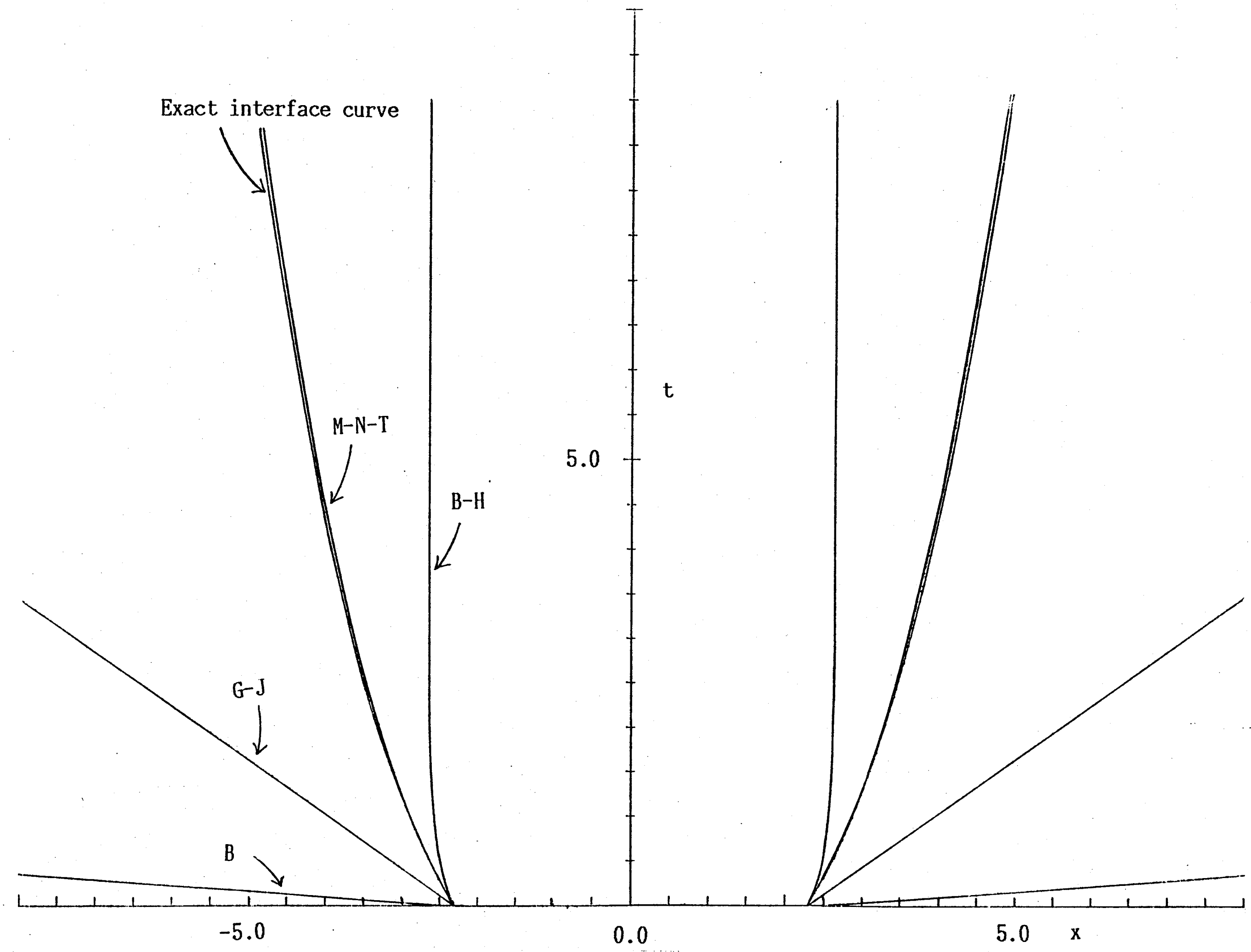
Then, for any $T > 0$,

$$(3.23)\qquad \|\ell_h - \ell^*\|_{L^\infty([0,T])} \longrightarrow 0 \quad as \quad h \to 0,$$

$$(3.24)\qquad \|r_h - r^*\|_{L^\infty([0,T])} \longrightarrow 0 \quad as \quad h \to 0,$$

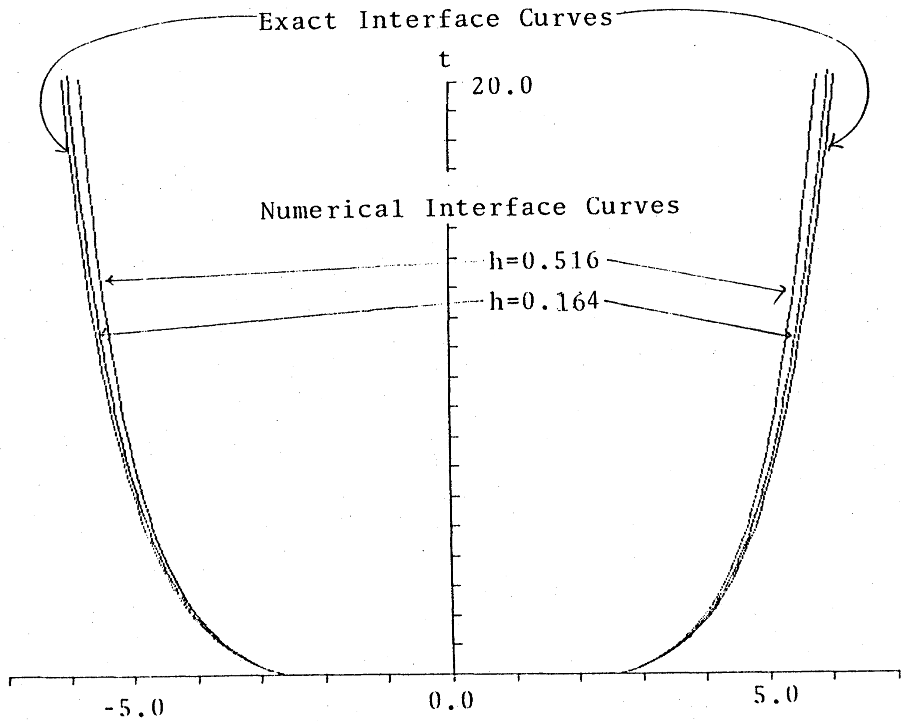
where ℓ^* and r^* are the exact interface curves.

Remark. (3.23) and (3.24) also hold for $c = 0$.



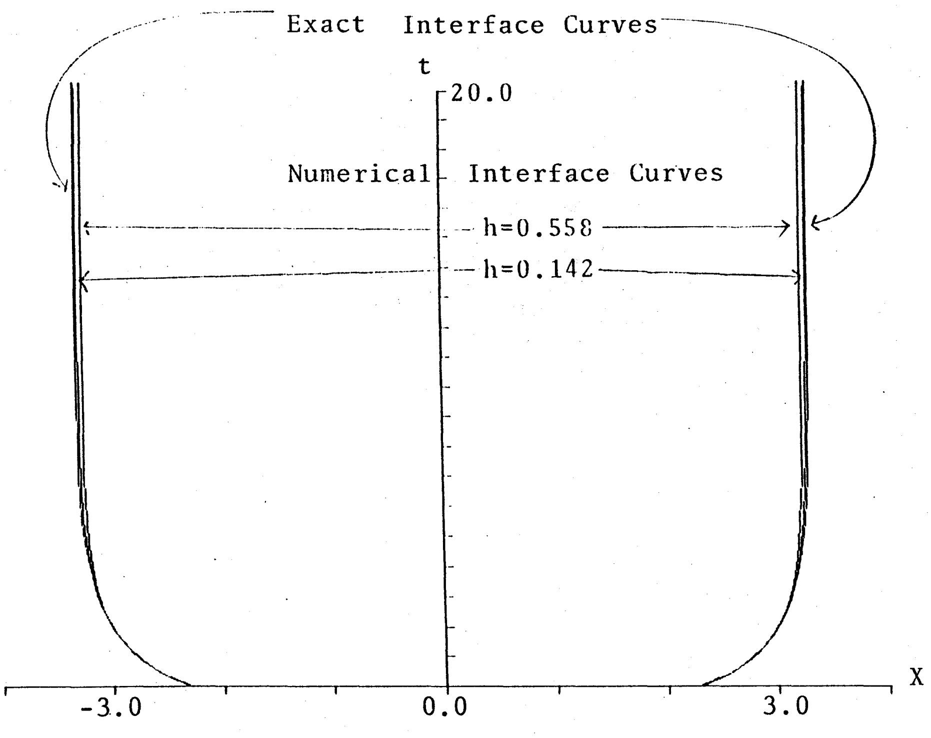
Numerical interface curves to
Barenblatt and Pattle's solution.

B: Baklanovskaya's scheme [2] h=0.057
G-J: Graveleau and Jamet's scheme [6] h=0.114
B-H: DiBenedetto and Hoff's scheme [5] h=0.125
M-N-T: Mimura, Nakaki and Tomoeda' scheme [17, 18] h=0.125



$V_t=(V^2)_{xx} - V^2$

The initial value is the same one as Bertsch, Kersner and Peletier's solution at t=0.



$V_t=(V^2)_{xx} - 0.5V$

The initial value is the same one as the exact solution at t=0 (Gurtin and MacCamy).

4. Difference Schemes to (1.1)–(1.2) with $m+p=2$.

In this section we assume that $m+p=2$, $m>1$ and $p>0$. The solution (1.1)–(1.2) extincts in finite time. The front of support may expand and/or shrink. Taking this property into consideration, we approximate (2.2) in the following way [14]:

Find the sequence $\{u_h^n\}_{n=0,1,2,\cdots} \subset V_h$ such that

$$u_h^{n+1} = S_{h,k}u_h^n \equiv (P_{h,k/\mu})^{\mu} \cdot H_{h,k} \cdot D_{h,k}u_h^n \quad \text{for } n=0,1,2,\cdots \tag{4.1}$$

The difference operators $H_{h,k}$ and $P_{h,k/\mu}$ are given by (3.5) and (3.9), respectively. Since $q=0$, the difference operator $D_{h,k}$ is written as

$$(D_{h,k}u_h)(x_i) = \{[u_h - (m-1)ck]^+\}(x_i), \tag{4.2}$$

Stability Condition:

1) $k = k_{n+1}$ is the largest number satisfying the inequality (3.7)–(3.8) with $u_h = D_{h,k}u_h^n$;

2) Every connected component of the set [supp u_h] $\setminus$ [supp $D_{h,k}u_h$] has at most one point x such that x/h is an integer;

3) μ satisfies the stability conditions (3.10)–(3.11).

From this stability condition the numerical interface curves can be expressed as follows:

$$\ell_{n+1} = \ell_n + \frac{(m-1)ck}{\delta u_{L-1}^n} - a\delta u_{L-1}^n k \qquad \text{if } x_{L-1} < \ell(D_{h,k}u_h^n) < x_L, \tag{4.3}$$

$$\ell_{n+1} = \ell_n + \frac{u_L^n}{\delta u_{L-1}^n} + \frac{(m-1)ck - u_L^n}{\delta u_L^n} - a\delta u_L^n k \quad \text{if } x_L \leq \ell(D_{h,k}u_h^n) < x_{L+1}, \tag{4.4}$$

$$r_{n+1} = r_n + \frac{(m-1)ck}{\delta u_R^n} - a\delta u_R^n k \qquad \text{if } x_R < r(D_{h,k}u_h^n) < x_{R+1}, \tag{4.5}$$

$$(4.6)\ r_{n+1} \;=\; r_n + \frac{u_R^n}{\delta u_R^n} + \frac{(m-1)ck - u_R^n}{\delta u_{R-1}^n} - a\delta u_{R-1}^n k \quad \text{if } x_{R-1} < r(D_{h,k}u_h^n) \leq x_R,$$

where ℓ_n and r_n are given by (3.16) .

Theorem 4.1. *Under Assumption I let $m + p = 2$, $m > 1$ and $p > 0$. Then (3.19) and (3.21) hold for all $n \geq 0$, where the constant C_1 is independent of T and h.*

Theorem 4.2. *Under Assumption II and the same assumptions as stated in Theorem 4.1 let Condition B be satisfied. Then there exist Lipschitz continuous functions $\ell^*(t)$ and $r^*(t)$ on $[0, T^*]$ such that*

$$(4.7) \qquad ||v_h - v||_{L^\infty(H)} \longrightarrow 0 \quad \text{as} \quad h \to 0,$$

$$(4.8) \qquad ||\ell_h - \ell^*||_{L^\infty([0,T^*])} \longrightarrow 0 \quad \text{as} \quad h \to 0,$$

$$(4.9) \qquad ||r_h - r^*||_{L^\infty([0,T^*])} \longrightarrow 0 \quad \text{as} \quad h \to 0,$$

where $H = [0, T^*] \times \mathbf{R}^1$, $v_h \equiv (u_h)^{1/(m-1)}$ and $v(t,x)$ is the unique weak solution of (1.1)-(1.2). Moreover, ℓ^* and r^* become the left and right interface curves, respectively.

Numerical interface curves and numerical extinction time with m=1.5, p=0.5 and c=1. The initial value is the same one as Kersner's solution:

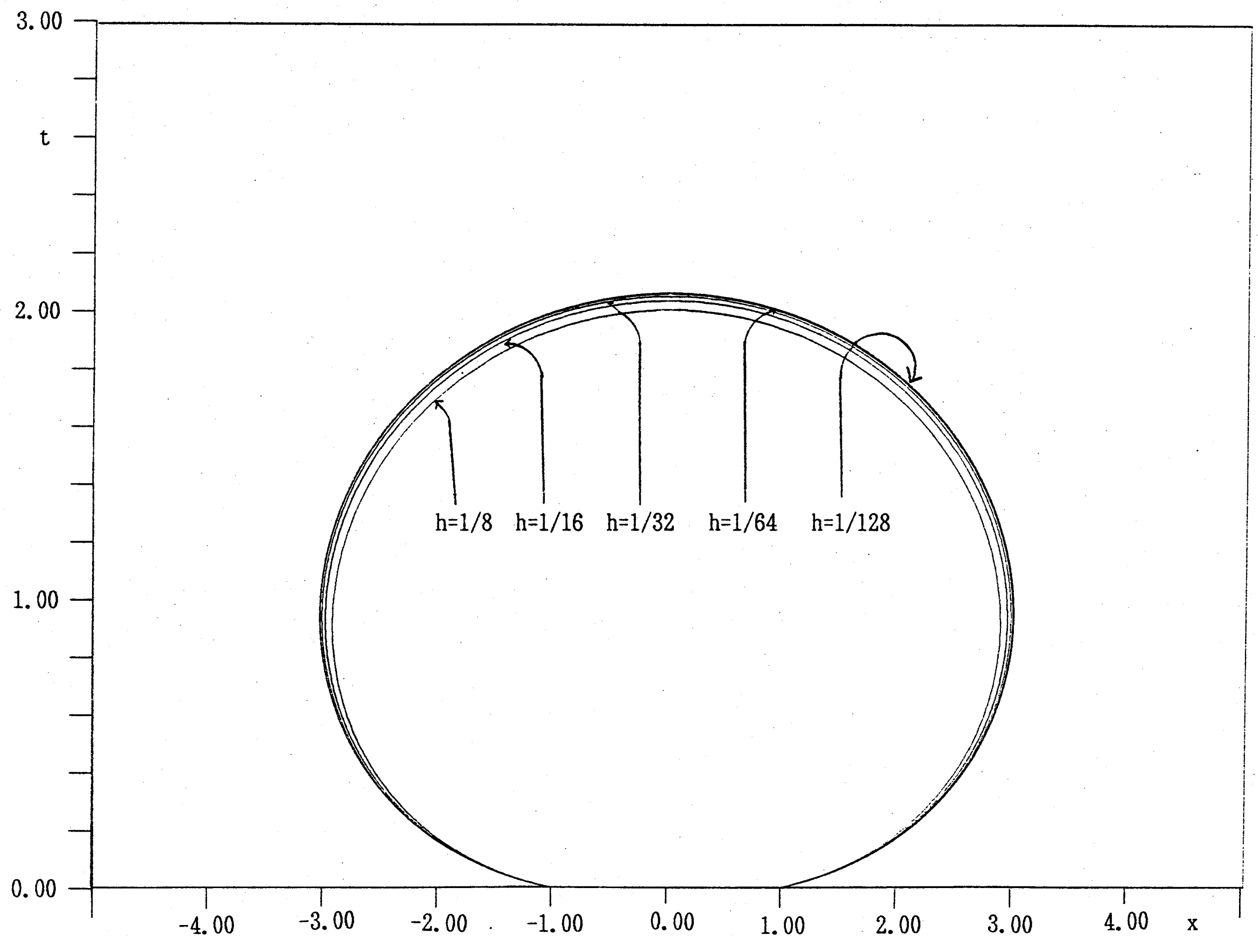
$$v(t,x)=(b_1 t + b_2)^{-1/(m-1)}[\lambda^2(t) - x^2]^{1/(m-1)} \quad \text{for} \quad |x| \leqq \lambda(t),$$

$$\lambda(t)=\pm[a_1(b_1 t + b_2)^{2/(m+1)} - a_2(b_1 t + b_2)^2]^{1/2},$$

$$a_1 = \frac{c(m-1)^4 + 4m^2}{4m^2(m-1)^{2/(m+1)}}, \quad a_2 = \frac{c(m-1)^2}{4m^2},$$

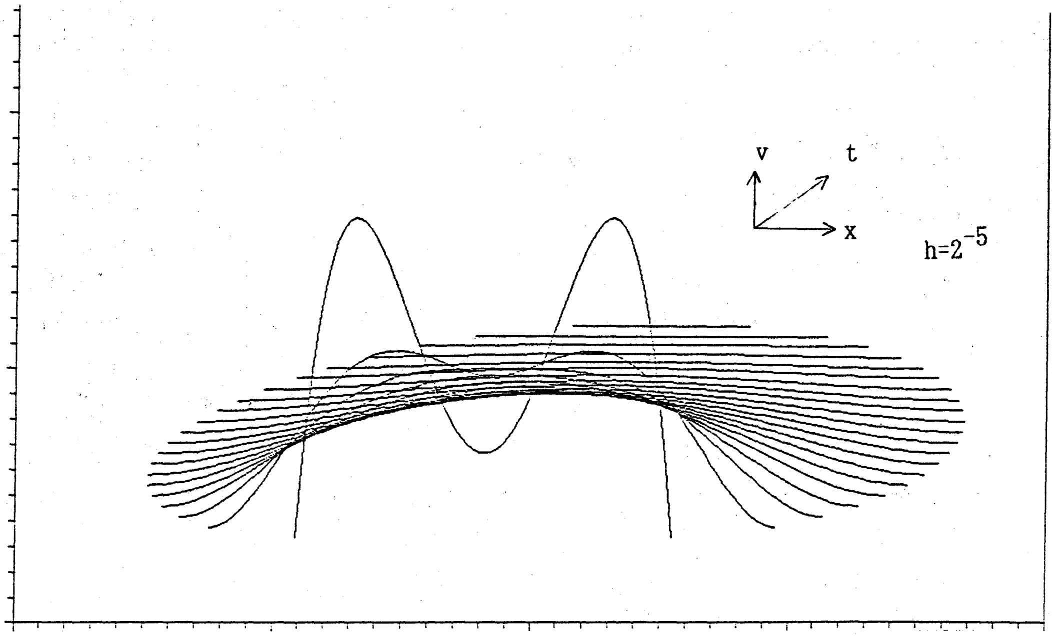
$$b_1 = \frac{2m(m+1)}{m-1}, \quad b_2 = (m-1).$$

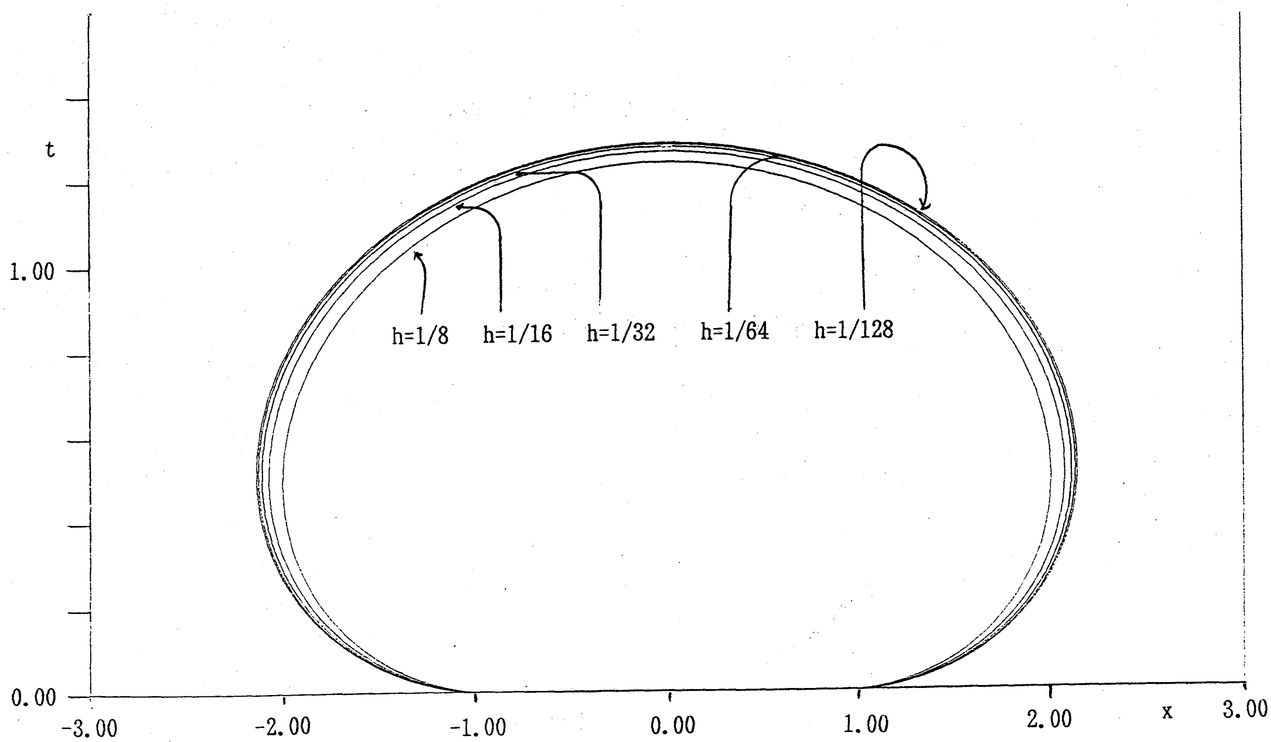
h	Numerical Extinction Time
2^{-3}	2.013478
2^{-4}	2.044670
2^{-5}	2.059979
2^{-6}	2.067771
2^{-7}	2.071648
2^{-8}	2.073709
2^{-9}	2.074431
Exact Extinction Time	2.07537...

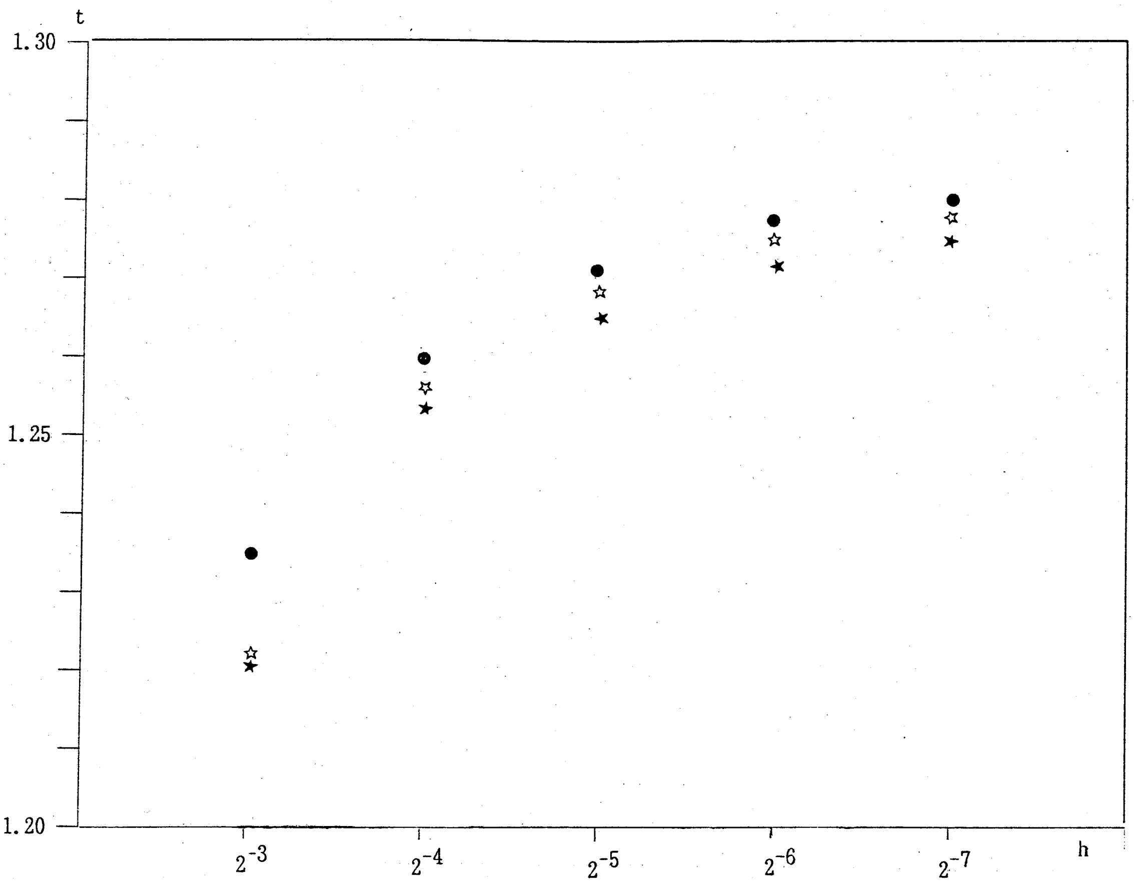


Numerical solutions and interface curves with m=1.5, p=0.5, c=1 and

$$v(0,x) = \begin{cases} 0.75(x^2-1)^2 - 4(x^2-1)(x^2-0.125) & \text{for } x^2 \leqq 1, \\ 0 \quad \text{for } x^2 > 1 . \end{cases}$$







●:Numerical extinction time.

☆:Numerical time T_h^* with M=50 for each h such that $\dot{\ell}_h$, $-\dot{r}_h \leqq$ M on [0, T_h^*].

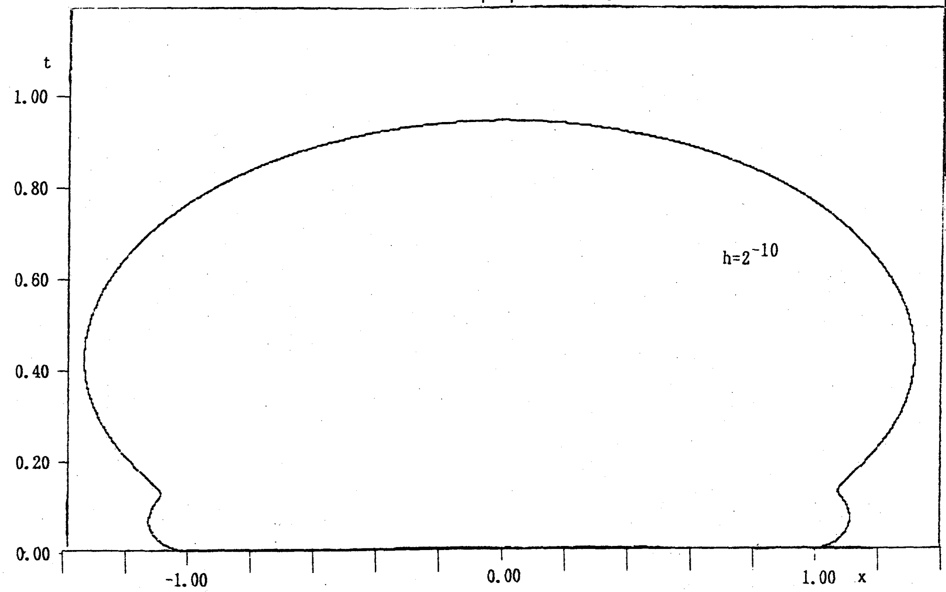
★:Numerical time T_h^* with M=25 for each h such that $\dot{\ell}_h$, $-\dot{r}_h \leqq$ M on [0, T_h^*].

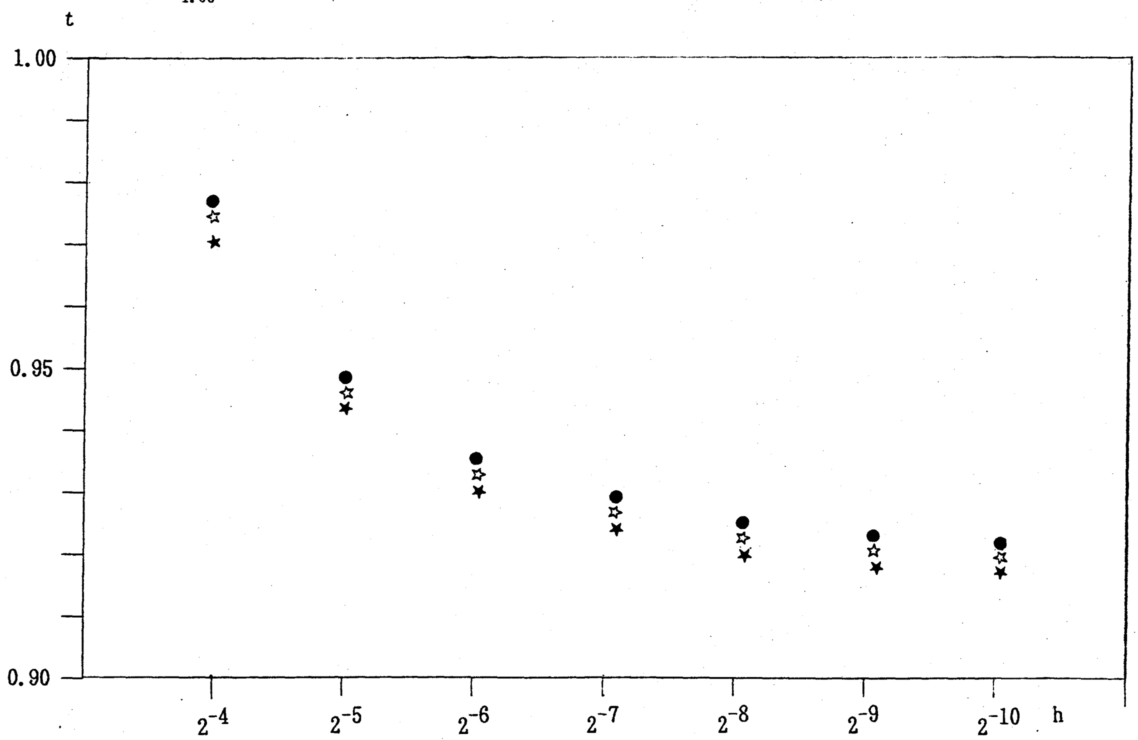
h	Numerical Extinction Time
2^{-3}	1.234850
2^{-4}	1.258987
2^{-5}	1.270498
2^{-6}	1.276316
2^{-7}	1.279211
2^{-8}	1.280641
2^{-9}	1.281343

Numerical interface curves and extinction time with m=1.5, p=0.5 and c=1. The initial value is given by

$$u(0,x) = \begin{cases} 1 - 1.8|x| & \text{for } |x| \leqq 0.5\ , \\ 0.1 & \text{for } 0.5 \leqq |x| \leqq 0.99\ , \\ 10(1 - |x|) & \text{for } 0.99 \leqq |x| \leqq 1\ , \\ 0 & \text{for } |x| \geqq 1\ . \end{cases}$$

h	Numerical Extinction Time
2^{-4}	0.976625
2^{-5}	0.947809
2^{-6}	0.934925
2^{-7}	0.928547
2^{-8}	0.924544
2^{-9}	0.922440
2^{-10}	0.921341





●:Numerical extinction time.

☆:Numerical time T_h^* with M=50 for each h such that $\dot{\ell}_h$, $-\dot{r}_h \leqq M$ on $[0, T_h^*]$.

★:Numerical time T_h^* with M=25 for each h such that $\dot{\ell}_h$, $-\dot{r}_h \leqq M$ on $[0, T_h^*]$.

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