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On examples of $\mathbb{R}$-holonomic complexes

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Finding out and analyzing "meaningful" examples of $\mathbb{R}$-holonomic complexes (cf. [SKK2]) seems to be one of the most challenging problems in "microlocal analysis in the future". Examples so far known are related to the θ-zerovalues, and they are constructed with the help of the so-called "Jacobi structure", that is, a set $\{p_j\}_{1\leqq j,k\leqq 2n}$ of microdifferential operators of order < 1 that satisfy

$$[p_j, p_k] = 2\pi\sqrt{-1}\, e_{jk} \qquad (1 \leqq j, k \leqq 2n)$$

with $(e_{jk})_{1\leqq j,k\leqq 2n}$ being a non-degenerate matrix whose entries are all integers; the infinite order system we are interested in is then constructed as $(\exp p_j - 1)u = 0$ $(j = 1, \cdots, 2n)$, roughly speaking. (Cf. [S]). In order to construct a Jacobi structure making use of matrices of differential operators, we usually need to consider some auxiliary systems. (Cf. [SKK1]. Actually the condition on the order of p_j also becomes somewhat more delicate; $\mathrm{ord}(\sum c_j p_j) < 1$ for any $c = (c_1, \cdots, c_{2n}) \in \mathbb{C}^{2n}$ is the one employed in [SKK1].) An example of this sort is explicitly written down in [KKT] (for $n = 2$), and detailed analysis of the example is given there; as is known by a general result on $\mathbb{R}$-holonomic complexes, the $\mathcal{O}$-solution complex of the system in question is $\mathbb{R}$-constructible. Still more important is the fact that we can determine its structure explicitly ([KKT], §3); in particular, its first cohomology group is a locally constant sheaf of rank 1 on some stratum, and it has a non-trivial "monodromic" structure. As the system discussed there can be regarded as the counterpart of the de Rham system in the category of $\mathbb{R}$-holonomic complexes (cf. [K2] §3.5), I dare say the $\mathbb{R}$-holonomic complexes studied in [K1] (for $n = 1$) and in [KKT] (for $n = 2$) should be the starting point of a concrete study of $\mathbb{R}$-holonomic complexes.

References

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