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Spherically symmetric solutions to the compressible Euler equation with an external force

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1. Introduction

Compressible Euler equation in R^n is a $(n+1)\times(n+1)$ system of conservation laws which describes the motion of isentropic gas.

$$(1.1)\quad \begin{aligned} \rho_t \;+\; \sum_{j=1}^{n}\frac{\partial}{\partial x_j}(\rho u_j) \;&=\; 0\,, \\ (\rho u_i)_t \;+\; \sum_{j=1}^{n}\frac{\partial}{\partial x_j}(\rho u_i u_j+\delta_{ij}p) \;&=\; \rho f_i \;\;(i=1,2,\cdots,n),\;\; p \;=\; a^2\rho^\gamma \end{aligned}$$

where ρ is a density, $\vec{u} ={}^t (u_1,u_2,\cdots,u_n)$ is a velocity, p is a pressure with δ_{ij} the Kronecker delta and $\vec{f} ={}^t (f_1,f_2,\cdots,f_n)$ is an external force. γ is a given constant. This is a famous example of system of conservation laws and there are many works related to this equation. For one dimensional case, Nishida [14] established the existence of global weak solutions, for the first time, for the case $\gamma = 1$ by using the Glimm's method. DiPerna [3] extended the result to the case of $\gamma = 1 + 2/(2m+1)$ ($m \geq 2$ integers) using the theory of compensated compactness. Ding et al [1], [2] removed this restriction and established the existence of global weak solutions for $1<\gamma\leq 5/3$. On the other hand, little is known for the case $n\geq 2$. No global solutions have known to exist, but only classical solutions. In [9], Makino, Mizohata and Ukai presented the global solutions first for this case without an external force. Here, we shall present global solutions first for this case with an external force.

Let us consider the initial and boundary value problem for (1.1) in $t\geq 0$, $x\in\Omega\subset R^n$ with the following conditions.

$$(1.2)\quad \vec{u}(0,x) \;=\; \vec{u}_0(x)\,,\; \rho(0,x) \;=\; \rho_0(x)\,,$$

$$(1.3)\quad \vec{u}\cdot\vec{n} \;=\; 0 \;\text{ if } x\in\partial\Omega\,.$$

In this paper we shall investigate the typical example of (1.1). Suppose that a solid star with radius 1 and mass M is surrounded by an isothermal gas ($\gamma = 1$). Then (1.1) becomes

$$(1.4)\quad \begin{aligned} \rho_t \;+\; \sum_{j=1}^{3}\frac{\partial}{\partial x_j}(\rho u_j) \;&=\; 0\,, \\ (\rho u_i)_t \;+\; \sum_{j=1}^{3}\frac{\partial}{\partial x_j}(\rho u_i u_j+\delta_{ij}p) \;&=\; \rho f_i \;\;(i=1,2,3),\; p \;=\; a^2\rho^\gamma, \end{aligned}$$

$$\vec{u}\cdot\vec{n} = 0 \ \text{if}\ x \in \partial\Omega\ ,\ \Omega = \{x|\ |x| > 1\}. \tag{1.5}$$

where $\vec{f}$ is a gravitational force given by

$$\begin{aligned}\vec{f} &= (f_1(t,x), f_2(t,x), f_3(t,x)) \\ &= \left(-\frac{x_1}{|x|}\cdot\frac{M}{|x|^2}, -\frac{x_2}{|x|}\cdot\frac{M}{|x|^2}, -\frac{x_3}{|x|}\cdot\frac{M}{|x|^2}\right) .\end{aligned} \tag{1.6}$$

We restrict ourselves to the case where this motion is spherically symmetric and $\vec{u}$ is normal to the surface of the star. We then obtain, denoting $r = |x|$ and $u_i(t,x) = \frac{x_i}{|x|}u(t,|x|)$,

$$\begin{aligned}\rho_t + (\rho u)_r + \frac{2}{r}\rho u &= 0, \\ \rho(u_t + uu_r) + p_r &= -\frac{\rho M}{r^2}, \\ p &= a^2\rho,\end{aligned} \tag{1.7}$$

on $t \geq 0$ and $1 \leq r < \infty$. Note that the Neumann condition (1.5) becomes

$$u(t,1) = 0 . \tag{1.8}$$

Let us adopt new function $\tilde{\rho}$. Put $\tilde{\rho} = r^2\rho$. Then (1.7) becomes

$$\begin{aligned}\tilde{\rho}_t + (\tilde{\rho}u)_r &= 0, \\ (\tilde{\rho}u)_t + (\tilde{\rho}u^2 + a^2\tilde{\rho})_r &= \frac{2a^2}{r}\tilde{\rho} - \frac{M\tilde{\rho}}{r^2}.\end{aligned} \tag{1.9}$$

Next we introduce Lagrangian mass coordinate

$$\tau = t\ ,\quad \xi = \int_1^r \tilde{\rho}(t,s)ds . \tag{1.10}$$

We then obtain, from (1.9),

$$\begin{aligned}\tilde{\rho}_\tau + \tilde{\rho}^2 u_\xi &= 0\ , \\ u_\tau + a^2\tilde{\rho}_\xi &= \frac{2a^2}{r} - \frac{M}{r^2}.\end{aligned} \tag{1.11}$$

Put $v = 1/\tilde{\rho}$. Then (1.13) becomes, after changing τ to t,

$$\begin{aligned}v_t - u_\xi &= 0\ , \\ u_t + \left(\frac{a^2}{v}\right)_\xi &= \frac{K}{r} - \frac{M}{r^2}\ ,\end{aligned} \tag{1.12}$$

where $r = 1 + \int_0^\xi v(t,\zeta)d\zeta$ and $K = 2a^2$. Let us consider the initial boundary value problem for (1.12) in $t \geq 0, \xi \geq 0$ with the following initial and boundary conditions.

$$u(0,\xi) = u_0(\xi)\ ,\quad v(0,\xi) = v_0(\xi)\ ,\quad for\ \xi > 0, \tag{1.13}$$

$$u(t,0) = 0\ ,\quad for\ t > 0. \tag{1.14}$$

We call that $u(t,\xi)$ and $v(t,\xi)$ are weak solutions of initial boundary value problem (1.12), (1.13) and (1.14) if $u,\ v\ \in\ L^\infty((0,T)\times(0,\infty))$ and if they satisfy the integral identities

$$
\begin{aligned}
&\int_0^T\int_0^\infty v\phi_t - u\phi_\xi\, d\xi dt + \int_0^\infty v_0(\xi)\phi(0,\xi)d\xi = 0,\\
(1.15)\qquad &\int_0^T\int_0^\infty u\psi_t + \left(\frac{a^2}{v}\right)\psi_\xi + \int_0^\infty u_0(\xi)\psi(0,\xi)d\xi\\
&= -\int_0^T\int_0^\infty \left(\frac{K}{1+\int_0^\xi v(t,\zeta)d\zeta} - \frac{M}{(1+\int_0^\xi v(t,\zeta)d\zeta)^2}\right)\cdot\psi\, d\xi dt
\end{aligned}
$$

for any test function $\phi\in C_0^\infty([0,T)\times[0,\infty))$ and $\psi\in C_0^\infty([0,T)\times(0,\infty))$ and for any $T>0$.

Here is our first result.

Theorem 1.1. *Suppose that $v_0(\xi)$ and $u_0(\xi)$ are of bounded variation, and that $v_0(\xi)$ satisfy $\delta_0 < v_0(\xi) < M_0$ for some positive constants δ_0 and M_0. Then (1.12), (1.13) and (1.14) admit global weak solutions. (i.e. T does not depend on the initial data.) which satisfy*

$$\| u \|_\infty < \infty\ ,\quad 0 < \delta_1 < v(t,\xi) < M_1 \ \text{a.e. for some}\ \delta_1\ \text{and}\ M_1.$$

We want to emphasize the fact that Theorem 1.1 only shows the existence of global solutions for the Lagrangian equation. It is not clear that this Theorem 1.1 implies the existence of global solutions of (1.4). If solutions are smooth, we can prove that $\vec{u}$ and ρ deduced from $u(t,\xi)$ and $v(t,\xi)$ satisfy (1.4) by using the chain rule. But if solutions are weak solutions, we must be more careful. Instead of using the chain rule, we use the fact that Lagrangian transformation is Lipschitz continuous to prove the equivalence.

Theorem 1.2. *Suppose that $u(t,\xi)$ and $v(t,\xi)$ are weak solutions of (1.12) satisfying*

$$
\begin{aligned}
(1.16)\qquad &u,\ v\ \in\ L^\infty((0,T)\times(0,\infty))\ ,\\
&0<\delta_1\le v(t,\xi),\ |v(t,\xi)|\le M_1,\ |u(t,\xi)|\le M_2 \ \text{a.e. in}\ (0,T)\times(0,\infty).
\end{aligned}
$$

with δ_1, M_1 and M_2 are given positive constants. Then $\vec{u}(t,x)$ and $\rho(t,x)$ deduced from $u(t,\xi)$ and $v(t,\xi)$ by using (1.10) are weak solutions of (1.4) with spherical symmetry.

Conversely, if $\vec{u}(t,x)$ and $\rho(t,x)$ are weak solutions of (1.4) with spherical symmetry satisfying

$$
\begin{aligned}
(1.17)\qquad &u_i,\ \rho\ \in\ L^\infty((0,T)\times\Omega)\ (i=1,2,\cdots,n),\\
&0<\frac{\delta_2}{|x|^2}\le\rho(t,x)\le\frac{M_1'}{|x|^2},\ |u(t,x)|\le M_2' \ \text{a.e. in}\ (0,T)\times\Omega\ ,
\end{aligned}
$$

with δ_2, M_1' and M_2' are given positive constants. Then $u(t,\xi)$ and $v(t,\xi)$ deduced from $\vec{u}(t,x)$ and $\rho(t,x)$ are also weak solutions of (1.12).

Combining Theorem 1.1 and Theorem 1.2, we obtain our main result.

Theorem 1.3. *Suppose that $u_0(x)$ and $\rho_0(x)$ are spherically symmetric and satisfy*

$$(1.18)\qquad 0<\frac{\delta_0}{|x|^2}\le\rho_0(x)\le\frac{M_0}{|x|^2},\ |u(t,x)|\le M_0' \ \text{a.e. in}\ \Omega\ ,$$

with $\delta>0$ and M_0, $M_0'\le\infty$. Then there exist global weak solutions of (1.4).

This is the first result of the existence of global solutions of (1.1) with an external force.

2. Outline of proof of Theorem 1.1 and Theorem 1.2

The proof of Theorem 1.1 consists of three steps. First, we shall construct approximate solutions by using modified Glimm's scheme. In the second step we shall get uniform estimates of the variation of the approximate solutions. Finally, by using the uniform estimates which we have obtained in the second step, we obtain the global weak solutions.

Let us construct approximate solutions $u^l(t,\xi)$ and $v^l(t,\xi)$ by using a modified Glimm's scheme. For $l, h > 0$, define

$$\begin{aligned} Y &= \{(n\,,\,m);\;\; n = 1,\,2,\,3,\,\cdots,\; m = 1,\,3,\,5,\,\cdots\}, \\ A &= \prod_{(m,n)\in Y} \left[\,\{nh\} \times ((m-1)l,\,(m+1)l)\,\right], \end{aligned} \tag{2.1}$$

where l/h will be determined later. Choose a point $\{\, a_{nm} \,\} \in A$ randomly, and write $a_{nm} = (nh, c_{nm})$. For $n = 0$, we put $c_{0m} = ml$. Mesh lengths l and h are chosen so that $l/h > a/(inf\, v^l)$ for any given $T > 0$. Suppose that u^l and v^l are defined for $0 \le t < nh$. We are going to define u^l and v^l for $nh \le t < (n+1)h$. For $ml \le \xi < (m+2)l$, m : odd, we define

$$\begin{aligned} u^l(t,\xi) &= u^l_0(t,\xi) + U^l(t,\xi)\cdot(t-nh), \\ v^l(t,\xi) &= v^l_0(t,\xi), \end{aligned} \tag{2.2}$$

where u^l_0 and v^l_0 are the solutions of

$$\begin{aligned} v_t - u_\xi &= 0\,, \\ u_t + \left(\frac{a^2}{v}\right)_\xi &= 0, \end{aligned} \tag{2.3}$$

with initial data (t=nh)

$$\begin{aligned} u^l_0(nh,\xi) &= \begin{cases} u^l(nh-0,c_{nm}), & \xi < (m+1)l, \\ u^l(nh-0,c_{n\,m+2}), & \xi > (m+1)l, \end{cases} \\ v^l_0(nh,\xi) &= \begin{cases} v^l(nh-0,c_{nm}), & \xi < (m+1)l, \\ v^l(nh-0,c_{n\,m+2}), & \xi > (m+1)l, \end{cases} \end{aligned} \tag{2.4}$$

and

$$\begin{aligned} U^l(t,\xi) &= \frac{K}{1 + \sum_{j=1}^{\frac{m+1}{2}} v^l(nh-0,c_{n\,2j-1})\cdot 2l} \\ &\quad - \frac{M}{\left(1 + \sum_{j=1}^{\frac{m+1}{2}} v^l(nh-0,c_{n\,2j-1})\cdot 2l\right)^2}\,. \end{aligned} \tag{2.5}$$

For $0 \le \xi < l$, we define u^l and v^l as (2.2) where u^l_0 and v^l_0 are the solutions of (2.3) with initial (t=nh) boundary data

$$u^l_0(nh,\xi) = u^l(nh-0,c_{n1}),\; v^l_0(nh,\xi) = v^l(nh-0,c_{n1}),\; \xi > 0, \tag{2.6}$$

$$u(t,0) = 0, \;\; t > nh, \tag{2.7}$$

and

$$U^l(t,\xi) = K - M. \tag{2.8}$$

System (2.3) is hyperbolic, provided $v > 0$, with the characteristic roots and Riemann invariants given by

$$(2.9)\qquad \begin{aligned} \lambda &= -\frac{a}{v}, \quad r = u + a\log v,\\ \mu &= \frac{a}{v}, \quad s = u - a\log v. \end{aligned}$$

In order to obtain the uniform estimates, we shall estimate the negative variation of Riemann invariants of u^l and v^l. This is the main idea of our proof. Concerning shock waves and Riemann invariants, the following four lemmas are well known.

Lemma 2.1. *The 1-shock wave curve S_1 and 2-shock wave curve S_2, starting from (r_0, s_0) can be expressed in the form*

$$(2.10)\qquad \begin{aligned} S_1 &: s - s_0 = f(r - r_0) \text{ for } r \le r_0,\\ S_2 &: r - r_0 = f(s - s_0) \text{ for } s \le s_0, \end{aligned}$$

where

$$0 \le f'(x) < 1,\ f''(x) \le 0,\ \lim_{x \to -\infty} f'(x) = 1.$$

The 1-rarefaction wave curve R_1 and 2-rarefaction wave curve R_2, starting from (r_0, s_0) can also be expressed in the form

$$(2.11)\qquad \begin{aligned} R_1 &: s - s_0 = 0 \text{ for } r \ge r_0,\\ R_2 &: r - r_0 = 0 \text{ for } s \ge s_0. \end{aligned}$$

Let us consider (2.3) with following initial data

$$(2.12)\qquad u_0(\xi) = \begin{cases} u_l, \\ u_r, \end{cases} \quad v_0(\xi) = \begin{cases} v_l, & x < 0,\\ v_r, & x > 0. \end{cases}$$

Lemma 2.2. *Let u and v are the solutions of (2.3) and (2.12). Then,*

$$(2.13)\qquad \begin{cases} r(t,\xi) \equiv r(u(t,\xi), v(t,\xi)) \ge r_0 \equiv min\,(r(u_r, v_r), r(u_l, v_l)),\\ s(t,\xi) \equiv s(u(t,\xi), v(t,\xi)) \le s_0 \equiv max\,(s(u_r, v_r), s(u_l, v_l)). \end{cases}$$

Next consider (2.3) in $t \ge 0$, $\xi \ge 0$ with following initial and boundary conditions

$$(2.14)\qquad u(0,\xi) = u_0^+, \quad v(0,\xi) = v_0^+, \quad for\ \xi > 0,$$

$$(2.15)\qquad u(t,0) = 0, \quad for\ t > 0.$$

Lemma 2.3. *Let u and v are the solutions of (2.3), (2.14) and (2.15). Then,*

$$(2.16)\qquad \begin{cases} r(t,\xi) \equiv r(u(t,\xi), s(t,\xi)) \ge r(u_0^+, v_0^+),\\ s(t,\xi) \equiv s(u(t,\xi), s(t,\xi)) \le max\,(-r(u_0^+, v_0^+), s(u_0^+, v_0^+)). \end{cases}$$

Let us consider Riemann problem (2.3) and (2.12). Denote by Δr (resp Δs) the absolute value of the variation of the Riemann invariant r (resp s) in the first (resp second) shock wave. We denote $P(u_l, v_l, u_r, v_r) = \Delta r + \Delta s$. Then the following lemma is known. For the proof, see [9].

Lemma 2.4.

$$P(u_1, v_1, u_3, v_3) \le P(u_1, v_1, u_2, v_2) + P(u_2, v_2, u_3, v_3), \tag{2.17}$$

where u_1, u_2 and u_3 are arbitrary constants and v_1, v_2 and v_3 are arbitrary positive constants.

The above four lemmas were proved in [9]. Using these lemmas and the fact that $U^l(t, x) \le K + M$, we can prove the following lemma.

Lemma 2.5. *Put $r_0 = min\ r(u_0(\xi), v_0(\xi))$ and $s_0 = max\ s(u_0(\xi), v_0(\xi))$. Then, for $0 < t < T$,*

$$\begin{cases} r^l(t,\xi) \equiv r\left(u^l(t,\xi), s^l(t,\xi)\right) \ge min(r_0, r_0 + (K-M)T), \\ s^l(t,\xi) \equiv s\left(u^l(t,\xi), s^l(t,\xi)\right) \\ \quad \le max\,(-r_0, -r_0 - (K-M)T, s_0) + (K+M)T \end{cases} \tag{2.18}$$

Proof. By using Lemma 2.2 and Lemma 2.3, we get

$$\begin{cases} r^l(t,\xi) \ \ge\ min\ (r_0, r_0 + (K-M)h), \\ s^l(t,\xi) \ \le\ max\ (-r_0, s_0) \ +\ (K+M)h, \end{cases} \tag{2.19}$$

for $0 \le t < h$. Thus we obtain, for $h \le t < 2h$

$$\begin{aligned} r^l(t,x) \ &\ge\ min\ (r_0, r_0 + 2(K-M)h), \\ s^l(t,x) \ &\le\ max\ (-min(r_0, r_0 + (k-M)h), max(-r_0, s_0) + (k+M)h) + (K+M)h \\ &\le\ max\ (-min(r_0, r_0 + (k-M)h) + (K+M)h, max(-r_0, s_0) + (k+M)h) + (K+M)h \\ &\le\ max\ (-r_0, -r_0 - (K-M)h, s_0) + 2(K+M)h\ . \end{aligned}$$

Continuing similar calculations successively until $T = Nh$, we can obtain (2.17). □

Lemma 2.5 describes the invariant region of approximate solutions. Next, we shall estimate the variation near the boundary. Denote by $i_0^{n\pm}$ the straight line segments joining the points $(0, (n \pm \frac{1}{2})h)$ and a_{1n}. Let $F(i_0^{n\pm})$ be the absolute value of the variation of the Riemann invariants for all shocks on $i_0^{n\pm}$. Then we also have the following Lemma.

Lemma 2.6.

$$F(i_0^{n+}) \le F(i_0^{n-}) + 2|M - K|h. \tag{2.20}$$

Proof. For simplicity, we restrict ourselves to the typical case. Suppose that S_2 and S_1 cross i_0^{n-}. (See Fig.1)

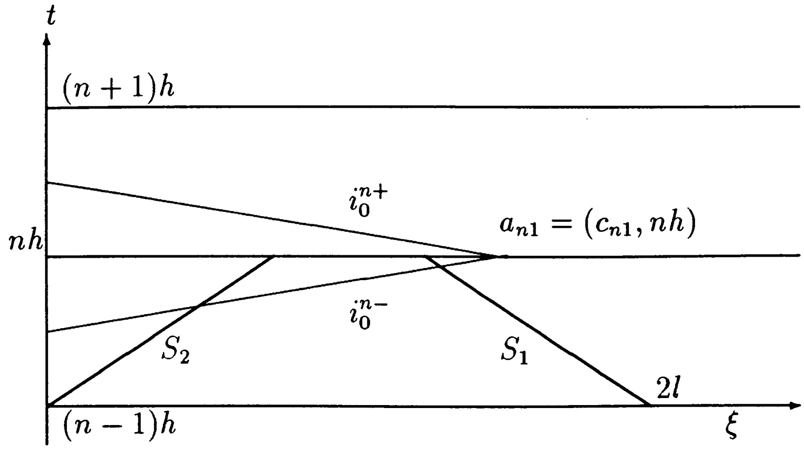

Figure.1

Let us estimate the variation of Riemann invariants of shocks which cross i_0^{n+}. Put $r_+^{n-1} = r^l(a_{1\,n-1})$, $s_+^{n-1} = s^l(a_{1\,n-1})$, $r_-^{n-1} = -s_-^{n-1}$ $= r^l((n-1)h+0,0)$, and $\delta_{n-1} = U^l(a_{1\,n-1})$.
Put $r_+^{n-1'} = r^l((n-1)h+0,2l)$ and $s_+^{n-1'} = s^l((n-1)h+0,2l)$.
Put $A = (r_-^{n-1}, s_-^{n-1})$, $B = (r_+^{n-1}, s_+^{n-1})$ and $B' = (r_+^{n-1'}, s_+^{n-1'})$.
Put $C = (r_+^{n-1'} + \delta_{n-1}h, s_+^{n-1'} + \delta_{n-1}h$. There are two cases we must consider. The first case is $\delta_{n-1} \geq 0$. Denote by I (resp. II) the half space $\{(r,s)|r+s < 0\}$(resp.$\{(r,s)|r+s \geq 0\}$). In this case when $C \in I$, S_2 crosses i_0^{n+} and we must estimate the negative variation of Riemann invariants caused by this shock. Denote by $V(P,Q)$ the absolute value of the total variation of r and s by the line segment PQ.

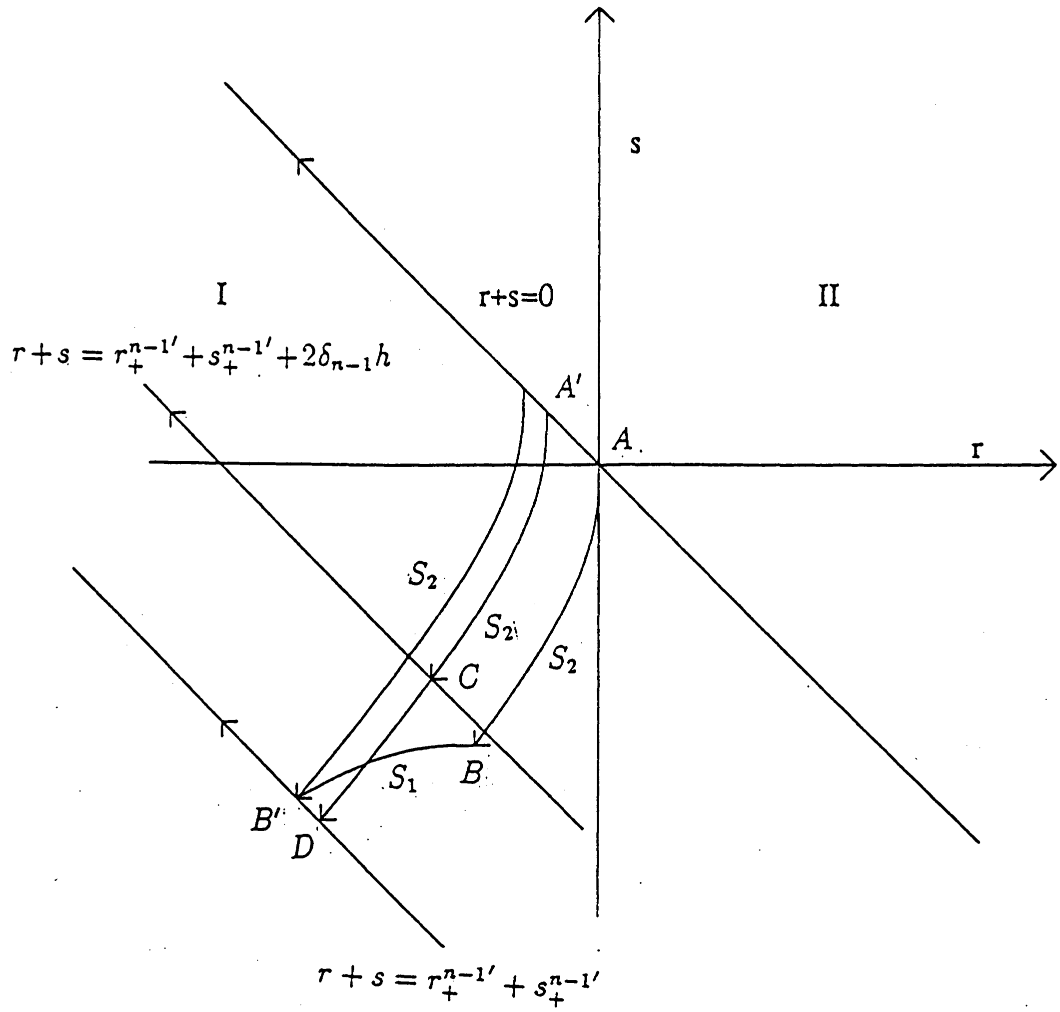

Figure.2

From figure.2,

$$F(i_0^{n+}) = V(A'C) \leq V(A'D) = V(AD) = V(AB') = F(i_0^{n-}).$$

When $C \in II$, R_2 cross i_0^{n+}. Since Riemann invariants increases when rarefaction wave crosses, $F(i_0^{n+}) = 0$. Thus we get

$$F(i_0^{n-}) \geq F(i_0^{n+}) = 0\,.$$

which satisfies (2.20).

The second case is $\delta_{n-1} < 0$. In this case $C \in I$. Thus S_2 crosses i_0^{n+}. Note that if $\delta_{n-1} < 0$, $K - M \leq \delta_{n-1} < 0$. From figure.3,

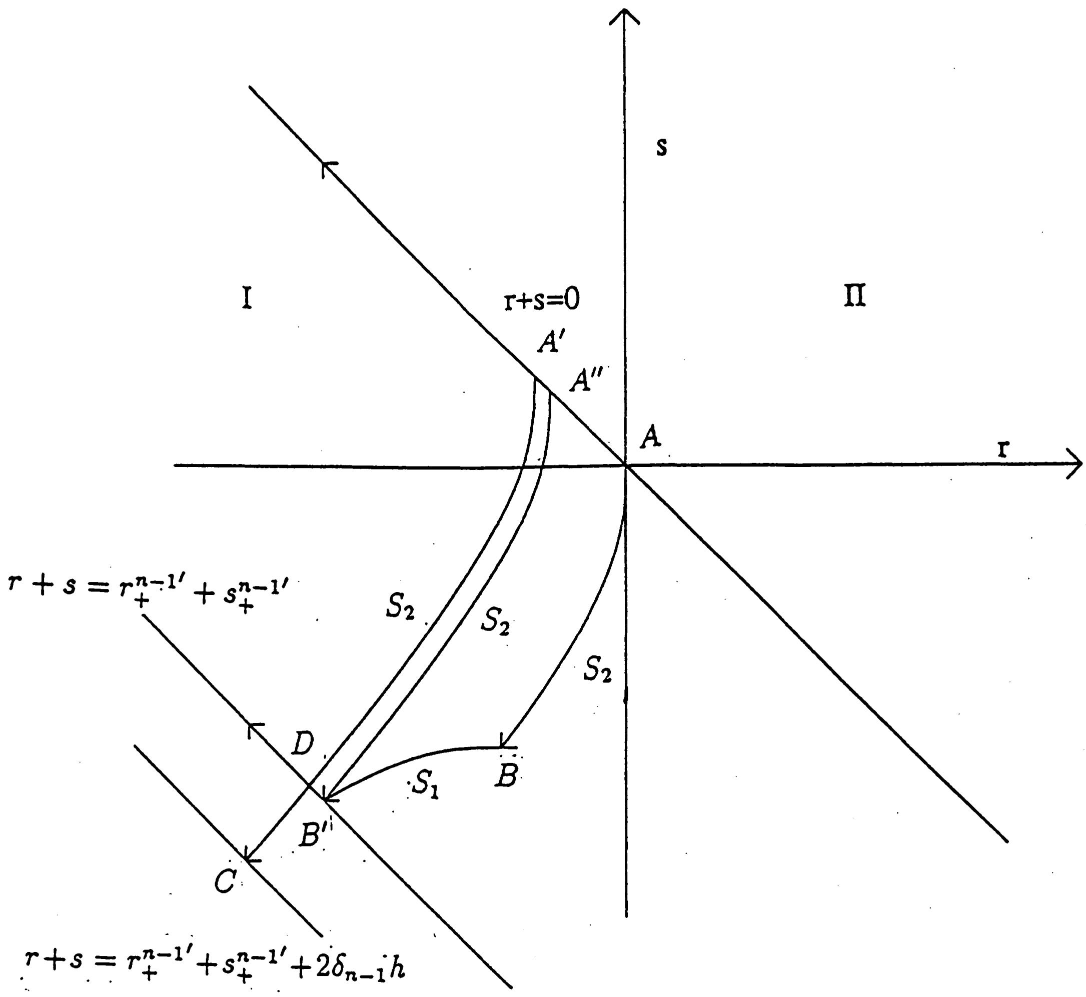

Figure.3

$$F(i_0^{n+}) = V(A'C) = V(A'D) + V(DC) = V(A''B') + 2|\delta_{n-1}|h$$
$$= V(AB') + 2|\delta_{n-1}|h \leq F(i_0^{n-}) + 2|K - M|h.$$

Thus we obtain (2.20). □

By using Lemma 2.4, Lemma 2.5 and Lemma 2.6, we get uniform estimates of the variation of approximate solutions u^l and v^l and especially an uniform estimate of $inf v^l$. Thus by applying standard arguments of Glimm's theory, we can prove Theorem 1.1. For the detail, see [13].

Next, let us describe the outline of the proof of the former part of Theorem 1.2. We can prove the equivalence of weak solutions of (1.1) and (1.9) by standard arguments. But that of (1.9) and (1.12) is very delicate. Suppose that u and v are weak solutions of (1.12). Put $\tilde{\rho} = 1/v$ Denote by Λ the mapping $(t, \xi) \to (t, r)$, namely,

(2.21) $$\Lambda(t, \xi) = (t, r)$$

Our first observation is that Λ is bi-Lipschits homeomorphism. Calculating the distribution derivatives of Λ, we get

$$\frac{\partial r}{\partial \xi} = \frac{1}{\tilde{\rho}}, \quad \frac{\partial r}{\partial t} = u . \tag{2.22}$$

(2.21) implies $r \in W^{1,\infty}$, and the following lemma shows that Λ is Lipschits continuous.

Lemma 2.7. *Let Ω be an open convex subset of R^n. Then*

$$W^{1,\infty}(\Omega) \hookrightarrow Lip(\Omega) .$$

More careful calculation shows that Λ is homeomorphism and Λ^{-1} is also Lipschits continuous. Therefore we can use the following lemma.

Lemma 2.8. *Let X, Y be measurable subsets of R^n and P be a mapping from X onto Y. If P is Lipschitz continuous, JP (Jacobian) is defined a.e.. Moreover, if P is bi-Lipschitz homeomorphism and satisfies $|JP| \geq \delta$ a.e. for some $\delta > 0$, then we have, for any $u(x) \in L^1(R^n)$,*

$$\int_X u(x)dx = \int_Y \frac{u \circ P^{-1}}{JP} dy . \tag{2.23}$$

The second observation is that u and v also satisfy (1.15) for any Lipshitz test function instead of smooth test function. We can prove it by using the mollifier. Since Λ is bi-Lipschits homeomorphism, Λ is a bijection on the set of Lipshitz test functions.

These observations allow us to prove the former part of Theorem 1.2. We can prove the other part of Theorem 1.2 similarly. For the detail, see [12].

3. Remark

By using the similar arguments, we can easily construct global weak solutions for more general external force. But there are some open problems.

Open Problem I : For the case Ω includes the origin.

Indeed, we are not able to estimate the singularity at the origin until now.

Open Problem II : For the case $\gamma > 1$.

By using compensated compactness arguments, Makino and Takeno have proved the existence of local weak solutions for (1.9) in the case $\gamma > 1$. But they are not able to get uniform estimates of approximate solutions due to the inhomogeneous terms. Recently, Makino has informed me that Glimm and Chen had succeeded to construct global weak solutions for this case. But unfortunately, I do not known their results and ideas explicitly.

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