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Free Boson Representations of the Quantum Affine Algebra $U_q(\widehat{\mathfrak{sl}}_2)$

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1. Introduction

It is established that conformal field theories play an important role in studies on two dimensional models. In particular the Wess-Zumino-Witten(WZW) model provides us with powerful tools to investigate models in the language of affine Kac-Moody algebras. It is also recognized that free field realization is indeed useful to investigate representation of Virasoro and affine Kac-Moody algebras. Wakimoto [1] first introduced free realization of the affine Kac-Moody algebra $\widehat{\mathfrak{sl}}_2$ and there have existed many works on this realization.

Recently Frenkel and Reshetikhin [2] have constructed certain q-deformed chiral vertex operators(qVOs) of the WZW model based on the representation theory of the quantum affine algebra $U_q(\hat{\mathfrak{g}})$. They showed that the correlation functions satisfy a q-difference equation called the q-deformed Knizhnik-Zamolodchikov(qKZ) equation, and the connection matrix of the qKZ equation indeed corresponds to the elliptic solution of the Yang-Baxter equation. As an application XXZ models are analyzed by the technique of q-vertex operators [3,4,5].

In these situations it is desirable to construct a concrete realization of quantum affine algebras and q-deformed chiral vertex operators. Frenkel and Jing first found a q-deformation of the Frenkel-Kac construction which corresponds to boson representation of $U_q(\widehat{\mathfrak{sl}}_2)$ of level one [6]. They show the Drinfeld realization [7] can be treated in terms of currents in which the technique of operator product expansions(OPEs) is powerful. Following this work Jimbo et al. [4] introduced explicit forms of q-deformed chiral vertex operators and calculated the trace of the product of the vertex operators. Recently some papers appear to attempt to extend to the case of arbitrary level. There are two kinds of boson realization of the quantum affine algebra $U_q(\widehat{\mathfrak{sl}}_2)$. One is based on the Wakimoto construction with a bosonization of bosonic ghost system [8] and the other on the realization in terms of bosonized parafermions [9,10]. In this talk we construct another boson representation à la Wakimoto following the line of the Matsuo's realization [9]. In this formulation it is useful to investigate the module of $U_q(\widehat{\mathfrak{sl}}_2)$.

2. Quantum Affine Algebra $U_q(\widehat{\mathfrak{sl}}_2)$

We begin with realization of the algebra $U_q(\mathfrak{sl}_2)$. It is known that the algebra $U_q(\mathfrak{sl}_2)$ can be realized in terms of a q-deformed harmanic oscillator. First we introduce q-deformed oscillators of annihilation and creation, a and $a^\dagger$, which satisfy following relations:

$$\begin{aligned} &aa^\dagger - qa^\dagger a = q^{-N}, \\ &aa^\dagger - q^{-1}a^\dagger a = q^N, \\ &[N, a^\dagger] = a^\dagger, \\ &[N, a] = -a, \end{aligned} \tag{1}$$

where q is a deformation parameter. Hamiltonian of a q-deformed harmonic oscillator is given by $\mathcal{H} = [N] + \frac{1}{2}$, where we use the notation $[N] = \frac{q^N - q^{-N}}{q-q^{-1}}$. One can obtain the q-deformed $\mathfrak{sl}(2)$algebra from these oscillators.

$$\begin{aligned} &J^+ = a^\dagger, \\ &J^- = a[\lambda + 1 - N], \\ &J^3 = N - \frac{\lambda}{2}, \end{aligned} \tag{2}$$

where $\frac{\lambda}{2}$ is constant corresponding to a value of spin. In the case of the irreducible representation $\lambda = j(j\text{: integer})$, bases of weights are given by

$$| \, l >= \frac{(a^\dagger)^l}{\sqrt{[l]!}} \, | \, 0 >, \qquad [N] \, | \, 0 >= 0, \qquad l = 0, 1, 2, \cdots, j. \tag{3}$$

These are eigenvectors of the q-deformed number operator$[N]$ belonging to eigenvalues $[l]$. It is easy to check that these operators satisfy the commutation relations of the quantum $\mathfrak{sl}(2)$ algebra:

$$\begin{aligned} &[J^3, J^\pm] = \pm J^\pm, \\ &[J^+, J^-] = [2J^3] = \frac{q^{2J^3} - q^{-2J^3}}{q - q^{-1}}. \end{aligned} \tag{4}$$

Now we are in a position to construct the q-deformation of the affine Kac-Moody algebra $\widehat{\mathfrak{sl}}_2$ in the same spirit as the $\mathfrak{sl}_2$ case. It is useful to begin with the following bosonized representation of the Wakimoto description of the algebra $\widehat{\mathfrak{sl}}_2$[12].

$$\begin{aligned} &J^+(z) = - : \partial\chi(z) \exp\{-\chi(z) + i\sigma(z)\} :, \\ &J^-(z) =: \left[(k+2)\partial\{\chi(z) - i\sigma(z)\} - \partial\chi(z) + \sqrt{2(k+2)}\partial\varphi(z)\right] \exp\{\chi(z) - i\sigma(z)\} :, \\ &J^3(z) = -i\partial\sigma(z) + \sqrt{\frac{k+2}{2}}\partial\varphi(z). \end{aligned} \tag{5}$$

First we introduce q-deformation of the boson $\varphi(z)$:

$$\varphi(z) = \alpha - \alpha_0 \ln z + \sum_{n\neq 0} \frac{\alpha_n}{[n]} z^{-n},$$
$$[\alpha_0, \alpha] = 2,$$
$$[\alpha_m, \alpha_n] = \delta_{m+n,0} \frac{[2m][m]}{m}. \tag{6}$$

In the case of a system with degrees of finite freedom, one has to use q-deformed oscillators. However, in field theories one can use ordinary oscillators with adequate normalization. As the Wakimoto currents contain three bosons $\{\sigma(z), \chi(z), \varphi(z)\}$, we prepare three kinds of oscillators$\{a_n, \bar{a}_n, b_n; n \in Z; a, \bar{a}, b\}$ satisfying the following commutation relations:

$$\begin{aligned}
[a_m, a_n] &= -\delta_{m+n,0} \frac{[2m][2m]}{m}, & [a_0, a] &= -4, \\
[\bar{a}_m, \bar{a}_n] &= \delta_{m+n,0} \frac{[2m][2m]}{m}, & [\bar{a}_0, \bar{a}] &= 4, \\
[b_m, b_n] &= \delta_{m+n,0} \frac{[2m][(k+2)m]}{m}, & [b_0, b] &= 2(k+2).
\end{aligned} \tag{7}$$

The other commutation relations of oscillators are equal to zero. We define currents of the algebra $U_q(\widehat{\mathfrak{sl}}_2)$ by using the oscillators (7) as follows:

$$\begin{aligned}
K_+(z) =& \exp\Big\{(q - q^{-1}) \sum_{m=1}^{\infty} z^{-m}(a_m + b_m)\Big\} q^{(a_0+b_0)}, \\
K_-(z) =& \exp\Big\{-(q - q^{-1}) \sum_{m=1}^{\infty} z^{m}(q^{-(k+2)m} a_{-m} + q^{-2m} b_{-m})\Big\} q^{-(a_0+b_0)}, \\
X^+(z) =& \frac{1}{q - q^{-1}} : \Big\{Y^+(z) Z_+(q^{-\frac{k+2}{2}} z) - Z_-(q^{\frac{k+2}{2}} z) Y^+(z)\Big\} :, \\
X^-(z) =& -\frac{1}{q - q^{-1}} : \Big\{Y^-(z) Z_+(q^{\frac{k+2}{2}} z) U_+(q^{\frac{k}{2}} z) W_+(q^{\frac{k}{2}} z) \\
& - Z_-(q^{-\frac{k+2}{2}} z) U_-(q^{-\frac{k}{2}} z) W_-(q^{-\frac{k}{2}} z) Y^-(z)\Big\} : .
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
Y^+(z) =& \exp\Big\{-\sum_{m=1}^{\infty} q^{-\frac{k}{2}m} \frac{z^m}{[2m]} (a_{-m} + \bar{a}_{-m})\Big\} e^{-\frac{a+\bar{a}}{2}} z^{-\frac{a_0+\bar{a}_0}{2}} \\
& \times \exp\Big\{\sum_{m=1}^{\infty} q^{-\frac{k}{2}m} q^{(k+2)m} \frac{z^{-m}}{[2m]} (a_m + \bar{a}_m)\Big\}, \\
Y^-(z) =& \exp\Big\{\sum_{m=1}^{\infty} q^{\frac{k}{2}m} \frac{z^m}{[2m]} (a_{-m} + \bar{a}_{-m})\Big\} e^{\frac{a+\bar{a}}{2}} z^{\frac{a_0+\bar{a}_0}{2}} \\
& \times \exp\Big\{-\sum_{m=1}^{\infty} q^{\frac{k}{2}m} q^{(k+2)m} \frac{z^{-m}}{[2m]} (a_m + \bar{a}_m)\Big\},
\end{aligned} \tag{9}$$

$$
\begin{aligned}
W_+(z) &= \exp\left\{(q-q^{-1})\sum_{m=1}^{\infty} z^{-m} b_m\right\} q^{b_0},\\
W_-(z) &= \exp\left\{-(q-q^{-1})\sum_{m=1}^{\infty} q^{-2m} z^m b_{-m}\right\} q^{-b_0},
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
Z_+(z) &= \exp\left\{-(q-q^{-1})\sum_{m=1}^{\infty} z^{-m}\frac{[m]}{[2m]}\bar{a}_m)\right\} q^{-\frac{1}{2}\bar{a}_0},\\
Z_-(z) &= \exp\left\{(q-q^{-1})\sum_{m=1}^{\infty} q^{-(k+2)m} z^m\frac{[m]}{[2m]}\bar{a}_{-m})\right\} q^{\frac{1}{2}\bar{a}_0},
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
U_+(z) &= \exp\left\{(q-q^{-1})\sum_{m=1}^{\infty} q^{km} z^{-m}\frac{[(k+2)m]}{[2m]}(a_m+\bar{a}_m)\right\} q^{\frac{k+2}{2}(a_0+\bar{a}_0)},\\
U_-(z) &= \exp\left\{-(q-q^{-1})\sum_{m=1}^{\infty} q^{-2m} z^m\frac{[(k+2)m]}{[2m]}(a_{-m}+\bar{a}_{-m})\right\} q^{-\frac{k+2}{2}(a_0+\bar{a}_0)}.
\end{aligned}
\tag{12}
$$

As we arrange operators in the way of normal ordering except for the zero modes, :: denotes the normal ordering with respect to the zero modes, $\alpha < \alpha_0, \bar{\alpha} < \bar{\alpha}_0$ and $\beta < \beta_0$. The essential idea of this construction is that we define q-deformations of derivatives, $\partial\chi(z)$, $\partial(\chi(z)-\sqrt{-1}\sigma(z))$ and $\partial\varphi(z)$ by

$$
\begin{aligned}
&\frac{-2}{q-q^{-1}}\left[Z_+(z)-Z_-(z)\right],\\
&\frac{1}{q-q^{-1}}\left[U_+(z)-U_-(z)\right],\\
&\frac{1}{q-q^{-1}}\left[W_+(z)-W_-(z)\right],
\end{aligned}
\tag{13}
$$

respectively *. By taking a limit $q \to 1$, they become

$$
\sum_{m\in Z}\bar{a}_m z^{-m}, \qquad \frac{k+2}{2}\sum_{m\in Z}(a_m+\bar{a}_m)z^{-m}, \qquad \sum_{m\in Z} b_m z^{-m},
$$

which correspond to the fields $z\partial\chi(z), z\partial(\chi(z)-\sqrt{-1}\sigma(z))$ and $z\partial\varphi(z)$, respectively. The

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* This idea is due to the paper[9].

currents of $U_q(\widehat{\mathfrak{sl}}_2)$ satisfy the following relations [6]:

$$
\begin{aligned}
&K_+(z)X^\pm(w) = \left(\frac{q^2 z - q^{\mp\frac{k}{2}} w}{z - q^{2\mp\frac{k}{2}} w}\right)^{\pm 1} X^\pm(w)K_+(z),\\
&K_-(z)X^\pm(w) = \left(\frac{q^2 w - q^{\mp\frac{k}{2}} z}{w - q^{2\mp\frac{k}{2}} z}\right)^{\mp 1} X^\pm(w)K_-(z),\\
&\frac{q^{2+k} z - w}{q^k z - q^2 w} K_-(z)K_+(w) = \frac{q^{2-k} z - w}{q^{-k} z - q^2 w} K_+(w)K_-(z),\\
&X^\pm(z)X^\pm(w) = \frac{q^{\pm 2} z - w}{z - q^{\pm 2} w} X^\pm(w)X^\pm(z),\\
&X^+(z)X^-(w) \sim \frac{1}{q - q^{-1}}\left(\frac{z}{z - q^k w} K_+(q^{\frac{k}{2}} w) - \frac{z}{z - q^{-k} w} K_-(q^{-\frac{k}{2}} w)\right).
\end{aligned}
\tag{14}
$$

Here we define the mode expansions of these currents as

$$
\begin{aligned}
&K_+(z) = \sum_{m\in Z_{\geq 0}} \psi_m z^{-m} = q^{a_0+b_0} \exp\left\{(q - q^{-1}) \sum_{m=1}^{\infty} H_m z^{-m}\right\},\\
&K_-(z) = \sum_{m\in Z_{\geq 0}} \varphi_{-m} z^{m} = q^{-(a_0+b_0)} \exp\left\{-(q - q^{-1}) \sum_{m=1}^{\infty} H_{-m} z^{m}\right\},\\
&H(z) = \sum_{m\in Z} H_m z^{-m}, \qquad X^\pm(z) = \sum_{m\in Z} X^\pm_m z^{-m}.
\end{aligned}
\tag{15}
$$

Putting $K = q^{a_0+b_0}$ we obtain the relations of the Drinfeld realization of $U_q(\widehat{\mathfrak{sl}}_2)$ for level k [7]:

$$
\begin{aligned}
&[H_m, H_n] = \delta_{m+n,0}\frac{1}{m}[2m][km], \qquad m \neq 0,\\
&[H_m, K] = 0,\\
&K X^\pm_m K^{-1} = q^{\pm 2} X^\pm_m,\\
&[H_m, X^\pm_n] = \pm\frac{1}{m}[2m] q^{\mp\frac{k|m|}{2}} X^\pm_{m+n},\\
&X^\pm_{m+1} X^\pm_n - q^{\pm 2} X^\pm_n X^\pm_{m+1} = q^{\pm 2} X^\pm_m X^\pm_{n+1} - X^\pm_{n+1} X^\pm_m,\\
&[X^+_m, X^-_n] = \frac{1}{q - q^{-1}}\left(q^{\frac{k(m-n)}{2}} \psi_{m+n} - q^{\frac{k(n-m)}{2}} \varphi_{m+n}\right).
\end{aligned}
\tag{16}
$$

The Drinfeld realization of $U_q(\widehat{\mathfrak{sl}}_2)$ corresponds to one of the q-deformation of the algebra $\widehat{\mathfrak{sl}}_2$.

Next we introduce the Fock module $F_{l,m_1,m_2}(l \in \frac{1}{2}Z; m_1, m_2 \in Z)$ freely generated by $\{a_n, \bar{a}_n, b_n; n \in Z_{>0}\}$ from a vector

$$
\mid l, m_1, m_2 >= \exp\left\{l\frac{b}{k+2} - m_1\frac{a}{2} + m_2\frac{\bar{a}}{2}\right\} \mid 0 > . \tag{17}
$$

Here a vector $\mid 0>$ has the following properties:

$$b_n \mid 0 >= 0, \qquad a_n \mid 0 >= 0, \qquad \bar{a}_n \mid 0 >= 0, \qquad n \geq 0.$$

The vector $\mid l, m_1, m_2 >$ is a eigenvector of b_0, a_0 and $\bar{a}_0$ belonging to eigenvalues $2l, 2m_1$ and $2m_2$, respectively. From the following relations:

$$[H(z), a_0 + \bar{a}_0] = 0, \qquad [X^{\pm}(z), a_0 + \bar{a}_0] = 0, \tag{18}$$

we can restrict the Fock module F_{l,m_1,m_2} to the sector in which the eigenvalue of $a_0 + \bar{a}_0$ is equal to zero [11]. There is another operator Q^+ which commute with the currents. Q^+ was introduced as a screening operator in the case of bosonization of the bosonic ghast system. In our case Q^+ is defined as

$$Q^+ = \frac{1}{2\pi\sqrt{-1}} \oint S^+(z)dz, \tag{19}$$

where

$$\begin{aligned} S^+(z) = \exp&\left\{ -\sum_{m=1}^{\infty} q^{-\frac{k+2}{2}m} \frac{z^m}{[2m]} \bar{a}_{-m} \right\} e^{\frac{\bar{a}}{2}} z^{\frac{\bar{a}_0}{2}} \\ \times \exp&\left\{ \sum_{m=1}^{\infty} q^{\frac{k+2}{2}m} \frac{z^{-m}}{[2m]} \bar{a}_m \right\}. \end{aligned} \tag{20}$$

We get following OPEs between $S^+(z)$ and currents:

$$\begin{aligned} &S^+(z)X^+(w) \sim 0, \\ &S^+(z)X^-(w) \sim w \frac{\partial_q}{\partial_q z} \left(\frac{R(z)}{w - z} \right), \\ &S^+(z)H(w) \sim 0. \end{aligned} \tag{21}$$

Here $\frac{\partial_q}{\partial_q z} f(z)$ is the difference operator defined as

$$\frac{\partial_q}{\partial_q z} f(z) = \frac{f(qz) - f(q^{-1}z)}{(q - q^{-1})z}, \tag{22}$$

and

$$\begin{aligned} R(z) = \exp&\left\{ -\sum_{m=1}^{\infty} q^{-\frac{k}{2}m} \frac{z^m}{[2m]} a_{-m} \right\} e^{-\frac{a}{2}} z^{-\frac{a_0}{2}} \\ \times \exp&\left\{ \sum_{m=1}^{\infty} q^{-\frac{k}{2}m} q^{(k+2)m} \frac{z^{-m}}{[2m]} a_m \right\}. \end{aligned} \tag{23}$$

Since $S^+(z)$ is single valued on $F_{l,m,m}$ and $R(z)$ is a meromorphic function on $F_{l,m,m}$, Q^+ commute with the currents. We also show the properties, $Q^{+2} = 0$ and the trivial cohomology of the following complex

$$\cdots \overset{Q^+}{\longrightarrow} F_{l,m,m} \overset{Q^+}{\longrightarrow} F_{l,m,m+1} \overset{Q^+}{\longrightarrow} \cdots$$

Then the representation module of $U_q(\widehat{\mathfrak{sl}}_2)$ can be restricted as

$$F_l = \oplus_{m=l+Z}\mathrm{Ker}(Q^+ : F_{l,m,m} \to F_{l,m,m+1}). \tag{24}$$

When $q = 1$, F_l is isomorhpic to the Wakimoto module of $\widehat{\mathfrak{sl}}_2$. It is easy to check that the vector $\mid \frac{j}{2}, 0, 0 >$ satisfies the highest weight conditions:

$$\begin{aligned} &H_n \mid \frac{j}{2}, 0, 0 >= j\delta_{n,0} \mid \frac{j}{2}, 0, 0 >, \qquad n \geq 0, \\ &X_n^+ \mid \frac{j}{2}, 0, 0 >= 0, \qquad n \geq 0, \\ &X_n^- \mid \frac{j}{2}, 0, 0 >= 0, \qquad n > 0. \end{aligned} \tag{25}$$

3. Relations to other realizations

Now we will give relations between realization of $U_q(\widehat{\mathfrak{sl}}_2)$ in section 2 and another one based on the following currents of $\widehat{\mathfrak{sl}}_2$ [13]:

$$\begin{aligned} &J^\pm(z) =: \frac{1}{\sqrt{2}}\left[\sqrt{k+2}\partial\phi_1(z) \pm \sqrt{-1}\sqrt{k}\partial\phi_2(z)\right] \exp\left\{\pm\sqrt{\frac{2}{k}}[\sqrt{-1}\phi_2(z) - \phi_0(z)]\right\} :, \\ &J^3(z) = -\sqrt{\frac{k}{2}}\partial\phi_0(z). \end{aligned} \tag{26}$$

We define the following linear transformation of oscillators $\{a_n, \bar{a}_n, b_n; n \in Z; a, \bar{a}, b\}$:

$$\begin{pmatrix} \alpha_m \\ \bar{\alpha}_m \\ \beta_m \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ -\frac{[(k+2)m]}{[2m]}q^{km} & -\frac{[km]}{[2m]}q^{(k+2)m} & -1 \\ \frac{[(k+2)m]}{[2m]}q^{(k+1)m} & \frac{[(k+2)m]}{[2m]}q^{(k+1)m} & q^m \end{pmatrix} \begin{pmatrix} a_m \\ \bar{a}_m \\ b_m \end{pmatrix}, \qquad m \geq 1,$$

$$\begin{pmatrix} \alpha_{-m} \\ \bar{\alpha}_{-m} \\ \beta_{-m} \end{pmatrix} = \begin{pmatrix} q^{-(k+2)m} & 0 & q^{-2m} \\ -\frac{[(k+2)m]}{[2m]}q^{-2m} & -\frac{[km]}{[2m]} & -q^{-2m} \\ \frac{[(k+2)m]}{[2m]}q^{-m} & \frac{[(k+2)m]}{[2m]}q^{-m} & q^{-m} \end{pmatrix} \begin{pmatrix} a_{-m} \\ \bar{a}_{-m} \\ b_{-m} \end{pmatrix}, \qquad m \geq 1,$$

$$\begin{pmatrix} \alpha_0 \\ \bar{\alpha}_0 \\ \beta_0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ -\frac{k+2}{2} & -\frac{k}{2} & -1 \\ \frac{k+2}{2} & \frac{k+2}{2} & 1 \end{pmatrix} \begin{pmatrix} a_0 \\ \bar{a}_0 \\ b_0 \end{pmatrix}.$$

$$\begin{pmatrix} \alpha \\ \bar{\alpha} \\ \beta \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ -\frac{k+2}{2} & -\frac{k}{2} & -1 \\ \frac{k+2}{2} & \frac{k+2}{2} & 1 \end{pmatrix} \begin{pmatrix} a \\ \bar{a} \\ b \end{pmatrix}. \tag{27}$$

From the commutation relations (7) one can obtain

$$\begin{aligned} &[\alpha_m, \alpha_n] = \delta_{m+n,0}\frac{[2m][km]}{m}, \qquad [\alpha_0, \alpha] = 2k, \\ &[\bar{\alpha}_m, \bar{\alpha}_n] = -\delta_{m+n,0}\frac{[2m][km]}{m}, \qquad [\bar{\alpha}_0, \bar{\alpha}] = -2k, \qquad m, n \neq 0 \\ &[\beta_m, \beta_n] = \delta_{m+n,0}\frac{[2m][(k+2)m]}{m}, \qquad [\beta_0, \beta] = 2(k+2). \end{aligned} \tag{28}$$

Substituting the inverse transformations of (27) into the currents (8), we can obtain

$$\begin{aligned} K_+(x) &= \exp\left\{(q-q^{-1})\sum_{m=1}^{\infty} z^{-m}\alpha_m\right\}q^{\alpha_0}, \\ K_-(x) &= \exp\left\{-(q-q^{-1})\sum_{m=1}^{\infty} z^{m}\alpha_{-m}\right\}q^{-\alpha_0}, \\ X^+(z) &= \frac{1}{q-q^{-1}} : \Big\{Y^+(z)Z_+(q^{-\frac{k+2}{2}}z)W_+(q^{-\frac{k}{2}}z) \\ &\quad - W_-(q^{\frac{k}{2}}z)Z_-(q^{\frac{k+2}{2}}z)Y^+(z)\Big\} :, \\ X^-(z) &= -\frac{1}{q-q^{-1}} : \Big\{Y^-(z)Z_+(q^{\frac{k+2}{2}}z)W_+(q^{\frac{k}{2}}z)^{-1} \\ &\quad - W_-(q^{-\frac{k}{2}}z)^{-1}Z_-(q^{-\frac{k+2}{2}}z)Y^-(z)\Big\} :, \end{aligned} \tag{29}$$

where

$$\begin{aligned} Y^+(z) &= \exp\left\{\sum_{m=1}^{\infty} q^{-\frac{k}{2}m}\frac{z^m}{[km]}(\alpha_{-m}+\bar{\alpha}_{-m})\right\}e^{\frac{\alpha+\bar{\alpha}}{k}}z^{\frac{\alpha_0+\bar{\alpha}_0}{k}} \\ &\times \exp\left\{-\sum_{m=1}^{\infty} q^{-\frac{k}{2}m}\frac{z^{-m}}{[km]}(\alpha_m+\bar{\alpha}_m)\right\}, \\ Y^-(z) &= \exp\left\{-\sum_{m=1}^{\infty} q^{\frac{k}{2}m}\frac{z^m}{[km]}(\alpha_{-m}+\bar{\alpha}_{-m})\right\}e^{-\frac{\alpha+\bar{\alpha}}{k}}z^{-\frac{\alpha_0+\bar{\alpha}_0}{k}} \\ &\times \exp\left\{\sum_{m=1}^{\infty} q^{\frac{k}{2}m}\frac{z^{-m}}{[km]}(\alpha_m+\bar{\alpha}_m)\right\}, \end{aligned} \tag{30}$$

$$\begin{aligned} W_+(z) &= \exp\left\{-(q-q^{-1})\sum_{m=1}^{\infty} z^{-m}\frac{[m]}{[2m]}\beta_m\right\}q^{-\frac{\beta_0}{2}}, \\ W_-(z) &= \exp\left\{(q-q^{-1})\sum_{m=1}^{\infty} z^{m}\frac{[m]}{[2m]}\beta_{-m}\right\}q^{\frac{\beta_0}{2}}, \end{aligned} \tag{31}$$

$$Z_+(z) = \exp\left\{-(q-q^{-1})\sum_{m=1}^{\infty} z^{-m}\frac{[m]}{[2m]}\bar{\alpha}_m)\right\}q^{-\frac{1}{2}\bar{\alpha}_0},$$
$$Z_-(z) = \exp\left\{(q-q^{-1})\sum_{m=1}^{\infty} z^{m}\frac{[m]}{[2m]}\bar{\alpha}_{-m})\right\}q^{\frac{1}{2}\bar{\alpha}_0}. \tag{32}$$

These currents correspond to those in the paper [9] except for a small change because of a normalization of zero mode.

Shiraishi's representation is also based on the Wakimoto currents. The main difference consists in how to treat a treatment for q-deformation of derivative. He extends differentiation to a q-difference operator defined as

$$_n\partial_z f(z) \equiv \frac{f(q^n z) - f(q^{-n} z)}{(q-q^{-1})z}. \tag{33}$$

There is a correspondence between $H(z)$, $X^{\pm}(z) = X^{\pm}_{+}(z) + X^{\pm}_{-}(z)$ in (8)(15) and $zJ^3(z)$, $zJ^{\pm}(z) = J^{\pm}_{I}(z) + J^{\pm}_{II}$ in the appendix of the paper [14].

Finally the aim of this talk is restricted on the forms of currents and their relations. In the line of our construction we can obtain screening currents, qVOs and n-point correlation functions with one screening charge on sphere. In the case of the two-point correlation function, we have confirmed the correspondence with the results in the paper [9,14]. Detailed accounts of our results will be given in a forthcoming paper. It is necessary to investigate cohomological structure of the Fock module [15,16] in order to derive irreducible representations and correlation functions on torus. These are under investigation.

References

[1] M. Wakimoto, Commun. Math. Phys. **104** (1986) 605.

[2] I.B. Frenkel and N.Yu. Reshetikhin, Commun. Math. Phys. **146** (1992) 1.

[3] B. Davies, O. Foda, M. Jimbo, T. Miwa, A. Nakayashiki, Diagonalization of the XXZ Hamiltonian by Vertex Operators. preprint RIMS-873 (1992) to appear in Comm. Math. Phys..

[4] M. Jimbo, K. Miki, T. Miwa and A. Nakayashiki, Phys. Lett. **A168** (1992) 256.

[5] M. Idzumi, K. Iohara, M. Jimbo, T. Miwa, T. Nakashima, T. Tokihiro, Quantum Affine Symmetry in Vertex Models, RIMS preprint (1992) to appear in Int. J. Mod. Phys. A.

[6] I.B. Frenkel and N.H. Jing, Proc. Nat'l Acad. Sci. USA **85** (1988) 9373.

[7] V.G. Drinfeld, Sov. Math. Dokl. **36** (1987) 212.

[8] J. Shiraishi. Free boson Representation of $U_q(\widehat{\mathfrak{sl}}_2)$, preprint UT-617 (1992).

[9] A. Matsuo, Free field Represetation of Quantum Affine Algebra $U_q(\widehat{\mathfrak{sl}}_2)$; Free Field Realization of q-deformed Primary fields for $U_q(\widehat{\mathfrak{sl}}_2)$, preprint (1992).

[10] A. Abada, A.H. Bougourzi, M.A. El Gradechi, Deformation of the Wakimoto Construction, preprint (1992).

[11] D. Friedan, E. Martinec, S. Shenker, Nucl. Phys. **B271** (1986) 93.

[12] A.Gerasimov, A.Marshakov, A.Morozov, M.Olshanetsky and S.Shatashvili, Int. J. Mod. Phys. **A5** (1992) 2495.

[13] D. Nemeschansky, Phys. Lett. **B224** (1989) 121.

[14] A. Kato, Y. Quano, J. Shiraishi, Free Boson Representation of q-Vertex Operators and Their Correlation Functions, preprint UT-618 (1992).

[15] D. Bernard and G. Felder, Commun. Math. Phys. **127** (1990) 145.

[16] B. Feigin and E. Frenkel, Commun. Math. Phys. **128** (1990) 161.