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<th>Title</th>
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</thead>
<tbody>
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A group generated by the Lévy Laplacian

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1. INTRODUCTION

The Lévy Laplacian $\Delta_L$ is one of infinite dimensional Laplacians introduced by P. Lévy in his book [Lé 22]. In his book, he mentioned that $\Delta_L$ comes from the singular part $f''_s$ of the second derivative $f''$, i.e.,

$$\Delta_L f(x) = \int_0^1 f''_s(x;u)du.$$

This Laplacian has been studied by many authors. In 1975, T.Hida introduced $\Delta_L$ into the theory of generalized white noise functionals in [Hi 75]. H.-H. Kuo [Ku 83,89,92a,92b] defined the Fourier-Mehler transform on the space $(S)^*$ of generalized white noise functionals and gave a relation between its transform and $\Delta_L$. An interesting characterization of $\Delta_L$ in terms of rotation groups was obtained by N. Obata [Ob 90]. Recently, T. Hida [Hi 92b] applied $\Delta_L$ to S. Tomonaga’s many time theory in quantum physics.

The purpose of this paper is to construct a group generated by $\Delta_L$.

In §2, we will explain a construction of the space of generalized white noise functionals and define the Lévy Laplacian $\Delta^T_L$ for a finite interval $T$ in $\mathbb{R}$ in that space. Moreover, we introduce an operator $\Delta$ and prove that $\Delta$ coincides with $2\Delta^T_L$ on a domain $D^T_L$ in $(S)^*$. In §3, we will construct a $(C_0)$-group $\{G_t\}_{t \in \mathbb{R}}$ generated by $\Delta^T_L$. In the last section, we will give a relation between the adjoint operator of Kuo’s Fourier-Mehler transform and a group $\{G_{it}\}_{t \in \mathbb{R}}$.

2. THE LÉVY LAPLACIAN IN THE WHITE NOISE CALCULUS

In this section, we introduce a space of Hida distributions following [Hi 80], [KT 80-82] and [PS 91] (See also, [HKPS 93], [HOS 92] and [Ob 92]) and the Lévy Laplacian defined on a domain in this space.

1) Let $L^2(\mathbb{R})$ be the Hilbert space of real square-integrable functions on $\mathbb{R}$ with norm $|\cdot|_0$. Consider a Gel’fand triple

$$S = S(\mathbb{R}) \subset L^2(\mathbb{R}) \subset S^* = S^*(\mathbb{R}),$$
where $\mathcal{S}(\mathbb{R})$ is the Schwartz space consisting of rapidly decreasing functions on $\mathbb{R}$ and $\mathcal{S}^*(\mathbb{R})$ is the dual space of $\mathcal{S}(\mathbb{R})$.

Let $A$ be the following operator

$$A = -(d/dx)^2 + x^2 + 1.$$ 

For each $p \in \mathbb{Z}$, we define $|f|_p = |A^p f|_0$ and let $\mathcal{S}_p$ be the completion of $\mathcal{S}$ with respect to the norm $| \cdot |_p$. Then the dual space of $\mathcal{S}_p'$ of $\mathcal{S}_p$ is the same as $\mathcal{S}_{-p}$.

2) Let $\mu$ be a probability measure on $\mathcal{S}^*$ with the characteristic functional given by

$$C(\xi) \equiv \int_{\mathcal{S}^*} \exp\{i < x, \xi > \} \, d\mu(x) = \exp\{-\frac{1}{2} |\xi|^2_0\}, \quad \xi \in \mathcal{S}.$$ 

Let $(L^2) = L^2(\mathcal{S}^*, \mu)$ be the space of complex-valued square-integrable functionals defined on $\mathcal{S}^*$ and define the $S$-transform by

$$S\varphi(\xi) = C(\xi) \int_{\mathcal{S}^*} \exp\{< x, \xi > \} \varphi(x) \, d\mu(x), \quad \varphi \in (L^2).$$

The Hilbert space admits the well-known Wiener-Itô decomposition:

$$(L^2) = \bigoplus_{n=0}^{\infty} H_n,$$

where $H_n$ is the space of multiple Wiener integrals of order $n \in \mathbb{N}$ and $H_0 = C$. From this decomposition theorem, each $\varphi \in (L^2)$ is uniquely represented as

$$\varphi = \sum_{n=0}^{\infty} I_n(f_n), \quad f_n \in L^2_\mathbb{C}(\mathbb{R})^\otimes n,$$

where $I_n \in H_n$ and $L^2_\mathbb{C}(\mathbb{R})^\otimes n$ denotes the $n$-th symmetric tensor product of the complexification of $L^2(\mathbb{R})$.

For each $p \in \mathbb{Z}, p \geq 0$, we define the norm $\|\varphi\|_p$ of $\varphi = \sum_{n=0}^{\infty} I_n(f_n)$, by

$$\|\varphi\|_p = \left( \sum_{n=0}^{\infty} n! |f_n|_{p,n} \right)^{1/2},$$

where $| \cdot |_{p,n}$ is the norm of $\mathcal{S}_p^\otimes n$ (the $n$-th symmetric tensor product of the complexification of $\mathcal{S}_p$). The norm $\| \cdot \|_0$ is nothing but the $(L^2)$-norm. We put

$$(\mathcal{S})_p = \{ \varphi \in (L^2); \|\varphi\|_p < \infty \}$$
for \( p \in \mathbb{Z}, p \geq 0 \). Let \((S)^*_p\) be the dual space of \((S)_p\). Then \((S)_p\) and \((S)^*_p\) are Hilbert spaces with the norm \( \| \cdot \|_p \) and the dual norm of \( \| \cdot \|_p \), respectively.

Denote the projective limit space of the \((S)_p, p \in \mathbb{Z}, p \geq 0\), and the inductive limit space of the \((S)^*_p, p \in \mathbb{Z}, p \geq 0\), by \((S)\) and \((S)^*\), respectively. Then \((S)\) is a nuclear space and \((S)^*\) is nothing but the dual space of \((S)\). The space \((S)^*\) is called the space of Hida distributions (or generalized white noise functionals).

Since \( \exp<\cdot, \xi> \in (S)\), the \(S\)-transform is extended to an operator \(U\) defined on \((S)^*\):

\[
U \Phi(\xi) = C(\xi) \ll \Phi, \exp<\cdot, \xi> \gg, \xi \in S,
\]

where \( \ll \cdot, \cdot \gg \) is the canonical pairing of \((S)\) and \((S)^*\). We call \(U \Phi\) the \(U\)-functional of \(\Phi\).

3) We next introduce the definition of the Lévy Laplacian following Kuo [Ku 92] (see also [HKPS 93]). Let \(U\) be a Fréchet differentiable function defined on \(S\), i.e. we assume that there exists a map \(U'\) from \(S\) to \(S^*\) such that

\[
U(\xi + \eta) = U(\xi) + U'(\xi)(\eta) + o(\eta), \eta \in S,
\]

where \(o(\eta)\) means that there exists \(p \in \mathbb{Z}, p \geq 0\), depending on \(\xi\) such that \(o(\eta)/|\eta|_p \to 0\) as \(|\eta|_p \to 0\). Then the first variation

\[
\delta U(\xi; \eta) = dU(\xi + \lambda \eta)/d\lambda|_{\lambda=0}
\]

is expressed in the form

\[
\delta U(\xi; \eta) = \int_R U'(\xi; u) \eta(u) du
\]

for every \(\eta \in S\) by using the generalized function \(U'(\xi; \cdot)\). We define the Hida derivative \(\partial_t \Phi\) of \(\Phi\) to be the generalized white noise functional whose \(U\)-functional is given by \(U'(\xi; t)\).

Definition. (I) A Hida distribution \(\Phi\) is called an \(L\)-functional if for each \(\xi \in S\), there exist \((U \Phi)'(\xi; \cdot) \in L_{loc}^1(\mathbb{R}), (U \Phi)'_s(\xi; \cdot) \in L_{loc}^1(\mathbb{R})\) and \((U \Phi)'_r(\xi; \cdot, \cdot) \in L_{loc}^1(\mathbb{R}^2)\) such that the first and second variations are uniquely expressed in the forms:

\[
(U \Phi)'(\xi)(\eta) = \int_R (U \Phi)'(\xi; u) \eta(u) du,
\]

and

\[
(U \Phi)''(\xi)(\eta, \zeta) = \int_R (U \Phi)''_s(\xi; u) \eta(u) \zeta(u) du + \int_{\mathbb{R}^2} (U \Phi)''_r(\xi; u, v) \eta(u) \zeta(v) dudv,
\]

for each \(\eta, \zeta \in S\), respectively and for any finite interval \(T, \int_T (U \Phi)'_s(\cdot; u) du\) is a \(U\)-functional.
(II) Let $D_L$ denote the set of all $L$-functionals. For $\Phi \in D_L$ and any finite interval $T$ in $\mathbb{R}$, the Lévy Laplacian $\Delta^T_L$ is defined by

$$\Delta^T_L \Phi = U^{-1} \left[ \frac{1}{|T|} \int_T (U\Phi)'(\cdot; u) \, du \right].$$

Remark. Explicit conditions for the uniqueness of the above decomposition (2.1) is given in [HKPS 93, chapter 6].

Let $T$ be a finite interval in $\mathbb{R}$. Take a smooth function $e$ defined on $\mathbb{R}$ satisfying $0 \leq e(u) \leq 1$ for all $u \in \mathbb{R}$, $e(u) = 1$ for $|u| \leq 1/2$ and $e(u) = 0$ for $|u| \geq 1$. We define an operator $\Delta$ for a Hida distribution $\Phi$ by

$$U[\Delta \Phi](\xi) = \lim_{n \to \infty} \int_{S^k} U\Phi''(\xi)(\theta_n^T e_n(\rho_n \ast x), \theta_n^T e_n(\rho_n \ast x)) \, d\mu(x),$$

if the limit exists in $U[(S)^*]$. From now on, we denote $e_n(\rho_n \ast x)$ by $j_n(x)$. Let $D_L^T$ denote the set of all $L$-functionals $\Phi$ satisfying $U\Phi(\eta) = 0$ for $\eta$ with $\text{supp}(\eta) \subset T^c$. In [Sa 94], we obtained the following result. (For the proof, see [Sa 94].)

**Theorem 1.** Let $T$ be a finite interval in $\mathbb{R}$ and $\Phi$ an $L$-functional in $D_L^T$. Then, we have $\Delta \Phi = 2 \Delta^T_L \Phi$.

### 3. The Lévy Laplacian as the Infinitesimal Generator

A generalized functional $\Phi$ is called a normal functional if its $U$-functional $U\Phi$ is given by a finite linear combination of

$$\int_{A^k} f(u_1, \ldots, u_k) \xi(u_1)^{p_1} \cdots \xi(u_k)^{p_k} \, du_1 \cdots du_k, \quad (3.1)$$

where $f \in L^1(A^k), p_1, \ldots, p_k \in \mathbb{N} \cup \{0\}, k \in \mathbb{N}$, and $A$ : a finite interval in $\mathbb{R}$. This functional $\Phi$ is in $D_L$.

Let $\mathcal{N}_T$ denote the set of all normal functionals in $D_L^T$. For $p > 1$ and $\Phi \in D_L^T$, we define a $-p$-norm $\| \cdot \|_{-p}$ by

$$\| \Phi \|_{-p}^2 = \sum_{k=0}^\infty \| (\Delta^T_L)^k \Phi \|_p^2 (\in [0, \infty])$$

and denote the completion of $\mathcal{N}_T$ with respect to the norm $\| \cdot \|_{-p}$ by $D_L^{(-p)}$. Then $D_L^{(-p)}$ is the Hilbert space with the norm $\| \cdot \|_{-p}$ and $\Delta^T_L$ is a bounded linear operator.
on $D_{L}^{(-p)}$. Hence a $(C_{0})$-group $\{G_{t}, t \in \mathbb{R}\}$ is given by

$$G_{t} = \lim_{n \to \infty} \sum_{k=0}^{n} \frac{t^{k}}{k!} (\triangle_{L}^{T})^{k},$$

in the sense of the operator norm. It is easily checked that $\|G_{t}\| \leq e^{|t|}$, for any $t \in \mathbb{R}$.

Define an operator $g_{t}$ on $\mathcal{N}_{T}$ for $t \geq 0$ by

$$U[g_{t}\Phi](\xi) = \lim_{n \to \infty} \int_{S^{*}} U\Phi(\xi + \sqrt{nT} j_{n}(x)) d\mu(x), \Phi \in \mathcal{N}_{T}.$$ 

For a normal functional $\Phi$ which $U\Phi$ is given as in (3.1) with the domain $A^{k} \subset T^{k}$, it is easily checked that

$$U[g_{t}\Phi](\xi) = \sum_{\nu_{1}=0}^{[p_{1}/2]} \cdots \sum_{\nu_{k}=0}^{[p_{k}/2]} \frac{p_{1}! \cdots p_{k}!}{(2\nu_{1})!(p_{1}-2\nu_{1})! \cdots (2\nu_{k})!(p_{k}-2\nu_{k})!}$$

$$(\frac{2t}{|T|})^{\nu_{1}+\cdots+\nu_{k}} \int_{A^{k}} f(u_{1}, \ldots, u_{k}) \xi(u_{1})^{p_{1}-2\nu_{1}} \cdots \xi(u_{k})^{p_{k}-2\nu_{k}} du_{1} \cdots du_{k}.$$ 

Therefore, $g_{t}$ is a linear operator from $\mathcal{N}_{T}$ to itself. By Theorem 1, it can be checked that $G_{t} = g_{t}$ on $\mathcal{N}_{T}$. Since $\mathcal{N}_{T}$ is dense in $D_{L}^{(-p)}$, we have the following

**Theorem 2.** For any $t \geq 0$, $g_{t}$ is extended to the operator $G_{t}$.

**4. The Fourier-Mehler Transform and the Lévy Laplacian**

An characterization of Hida distributions was obtained by J. Potthoff and L. Streit [PS 91]. From [PS 91], we see that for any $U$-functional $F$, and $\xi, \eta$ in $\mathcal{S}$, the function $F(\lambda \xi + \eta)$, $\lambda \in \mathbb{R}$, extends to an entire function $F(z \xi + \eta)$, $z \in \mathbb{C}$. Then we can define an operator $g_{it}$, $t \in \mathbb{R}$, by

$$U[g_{it}\Phi](\xi) = \lim_{n \to \infty} \int_{S^{*}} U\Phi(\xi + \sqrt{it} j_{n}(x)) d\mu(x),$$ 

if the limit exists. Since $\mu$ is symmetric, the integral is defined independent of choices of the branch of $\sqrt{it}$. As in (3.2), we can naturally define $G_{it}$, $t \in \mathbb{R}$, by

$$G_{it} = \lim_{n \to \infty} \sum_{k=0}^{n} \frac{(it)^{k}}{k!} (\Delta_{L}^{T})^{k},$$ 

on $D_{L}^{(-p)}$. 

An infinite dimensional Fourier-Mehler transform \( F_{\theta}, \theta \in \mathbb{R} \), on \((S)^{*}\) was defined by H.-H. Kuo [Ku 91] as follows. The transform \( F_{\theta} \Phi, \theta \in \mathbb{R} \) of \( \Phi \in (S)^{*} \) is defined by the unique generalized white noise functional with the \( U \)-functional

\[
U[F_{\theta} \Phi](\xi) = U\Phi(e^{i\theta} \xi) \exp \left[ \frac{i}{2} e^{i\theta} \sin \theta |\xi|^{2}_{0} \right], \quad \xi \in S.
\]

Moreover, the adjoint operator \( F_{\theta}^{*} \) of \( F_{\theta} \) is given by

\[
F_{\theta}^{*} \Phi = \sum_{n=0}^{\infty} I_{n}(h_{n}(\Phi; \theta)) \text{ for } \Phi = \sum_{n=0}^{\infty} I_{n}(f_{n}) \in (S),
\]

where

\[
h_{n}(\Phi; \theta) = \sum_{m=0}^{\infty} \frac{(n + 2m)!}{n! m!} \left( \frac{i}{2} \sin \theta \right)^{m} e^{i(m+n)\theta} \tau^{\otimes m} * f_{n+2m};
\]

\[
\tau^{\otimes m} = \int_{\mathbb{R}^{m}} \delta_{t_{1}} \otimes \delta_{t_{1}} \otimes \cdots \otimes \delta_{t_{m}} \otimes \delta_{t_{m}} dt_{1} \cdots dt_{m}.
\]

This operator \( F_{\theta}^{*} \) is a continuous linear operator on \((S)\). (For details, see [Ku 91] and also [HKO 90]) On \((S)\), the Gross Laplacian \( \Delta_{G} \) (See [Gr 65, 67]) and the number operator \( N \) is given by

\[
\Delta_{G} \Phi = \int_{\mathbb{R}} \partial_{t}^{2} \Phi dt
\]

and

\[
N \Phi = \int_{\mathbb{R}} \partial_{t}^{*} \partial_{t} \Phi dt,
\]

respectively (see [Ku 86]). The operator \( e^{i\theta N} \) is called the Fourier-Wiener transform (see [HKO 90]). Now, we introduce an operator \( e^{i\theta \Delta_{G}} \) from \((S)\) into itself given by

\[
e^{i\theta \Delta_{G}} \Phi = \sum_{n=0}^{\infty} I_{n}(\ell_{n}(\Phi; \theta)); \quad (4.1)
\]

\[
\ell_{n}(\Phi; \theta) = \sum_{m=0}^{\infty} \frac{(n + 2m)!}{n! m!} \left( \frac{i}{2} \theta \right)^{m} \tau^{\otimes m} * f_{n+2m},
\]

for \( \Phi = \sum_{n=0}^{\infty} I_{n}(f_{n}) \in (S) \). Then we have the followings.

**Lemma 1.**

\[
F_{\theta}^{*} = e^{i\theta N} \circ e^{i\theta \Delta_{G}}.
\]

**Proof:** Take \( \Phi = \sum_{n=0}^{\infty} I_{n}(f_{n}) \in (S) \). From (4.1), we see that

\[
e^{i\theta \Delta_{G}} \Phi = \sum_{n=0}^{\infty} I_{n}(\ell_{n}(\Phi; e^{i\theta})).
\]

Hence,

\[
e^{i\theta N} \left( e^{i\theta \Delta_{G}} \Phi \right) = \sum_{n=0}^{\infty} I_{n}(e^{i\theta} \ell_{n}(\Phi; e^{i\theta})).
\]
Since $e^{in\theta}h_n(\Phi;e^{i\theta}\sin\theta) = h_n(\Phi;\theta)$, we obtain (4.2).

**Lemma 2.** For any $\Phi \in (S)$, we have

$$U[e^{i\theta \Delta G} \Phi](\xi) = \int_{S^*} U\Phi(\xi + \sqrt{i\theta} y) \, d\mu(y). \quad (4.3)$$

**Remark.** For any $\Phi \in (S)$, $\xi \in S$ and $z_1, z_2 \in \mathbb{C}$, the functional $U\Phi(z_1 \xi + z_2 \eta)$, $\eta \in S$, can be extended to a functional $\overline{U}\Phi(z_1 \xi + z_2 \eta)$, same symbol $U\Phi(z_1 \xi + z_2 \eta)$.

**Proof:** For $\Phi = \sum_{n=0}^{\infty} I_n(f_n) \in (S)$, the right-hand side of (4.3) has the following expansion:

$$\sum_{n=0}^{\infty} \int_{\mathbb{R}^n} f_n(u) \int_{S^*} \{\xi(u_1) + \sqrt{i\theta} x(u_1)\} \cdots \{\xi(u_n) + \sqrt{i\theta} x(u_n)\} \, d\mu(x) \, du$$

$$= \sum_{n=0}^{\infty} \sum_{\nu=0}^{[n/2]} \frac{n!}{(2\nu)!!(n-2\nu)!} (i\theta)^\nu <\xi^\otimes(n-2\nu), \tau^\nu * f_n> = \sum_{m=0}^{\infty} <\xi^\otimes m, \ell_m(\Phi;\theta)>.$$  

From (4.1), we see that the last series is equal to $U[e^{i\theta \Delta G} \Phi](\xi)$. \qed

Define an operator $J_n$ by

$$U[J_n \Phi](\xi) = U\Phi \circ j_n(\xi), \quad \Phi \in D_L^{(-p)}, \quad \xi \in S.$$  

For all $n \in \mathbb{N}$ and $\Phi \in D_L^{(-p)}$, we can easily check $J_n \Phi \in (S)$. Then we have the following.

**Theorem 3.** Let $\Phi \in D_L^{(-p)}$ be a generalized white noise functional with the $U$-functional given by $\psi(F_1, \ldots, F_n)$, where $\psi$ is an entire function on $C$ and $F_1, \ldots, F_n \in U[N_T]$. We assume the condition

$$\sum_{k_1, \ldots, k_n=0}^{\infty} \frac{1}{k_1! \cdots k_n!} |\partial_{u_1}^{k_1} \cdots \partial_{u_n}^{k_n} \psi(0, \ldots, 0)|.$$  

$$\sup_{N} \int_{S^*} \left| (F_1 \circ j_N)^{k_1} \cdots (F_n \circ j_N)^{k_n} (ie^{i\alpha_N(t)} \xi + \sqrt{i\epsilon \alpha_N(t)} \sin \alpha_N(t) x \right| \, d\mu(x) < \infty$$

holds for all $t > 0$ and $\xi \in S$, where $\alpha_N(t) = t(\theta_N^T)^2$. Then

$$\lim_{N \to \infty} U[F^*_\alpha_N(t) J_N \Phi](\xi) = U[G_{it} \Phi](\xi), \quad \xi \in S.$$  

**Proof:** From Lemma 2, we have

$$U[e^{i\alpha_N(t) \sin \alpha_N(t) \Delta G} J_N \Phi](\xi) = \int_{S^*} U[J_N \Phi](\xi + \sqrt{i\epsilon \alpha_N(t) \sin \alpha_N(t) y} \, d\mu(y).$$
This functional is expressed in the form given by
\[ \sum_{\ell=0}^{\infty} \langle \xi^{\otimes \ell}, f_{N,\ell} \rangle, \]
where \( f_{N,\ell} \in S_{C}^{\otimes \ell} \). Hence, from Lemma 1, we get
\[
U[F_{\alpha_{N}(t)}^{\star}J_{N}\Phi](\xi) = \sum_{\ell=0}^{\infty} e^{i\alpha_{N}(t)\ell} \langle \xi^{\otimes \ell}, f_{N,\ell} \rangle.
\]
From the condition of this theorem and the Lebesgue convergence theorem, we can calculate as follows:
\[
\lim_{N \to \infty} U[F_{\alpha_{N}(t)}^{\star}J_{N}\Phi](\xi) = \lim_{N \to \infty} U[e^{\frac{i}{2}e^{i\alpha_{N}(t)}\sin \alpha_{N}(t)\Delta_{G}}J_{N}\Phi](e^{i\alpha_{N}(t)}\xi)
\]
\[= \lim_{N \to \infty} \int_{S^{*}} (F_{\alpha_{N}(t)}^{\star}J_{N}\Phi)(\xi + \sqrt{\frac{i}{e^{i\alpha_{N}(t)}}\sin \alpha_{N}(t)}x) d\mu(x)
\]
\[= \sum_{k_{1},\ldots,k_{n}=0}^{\infty} \frac{1}{k_{1}! \cdots k_{n}!} \partial_{u_{1}}^{k_{1}} \cdots \partial_{u_{n}}^{k_{n}} \psi(0,\ldots,0).
\]
By the direct calculations, it is easily checked that
\[
\lim_{N \to \infty} \int_{S^{*}} (F_{1} \circ j_{N})^{k_{1}} \cdots (F_{n} \circ j_{N})^{k_{n}} (\xi + \sqrt{\frac{i}{e^{i\alpha_{N}(t)}}\sin \alpha_{N}(t)}x) d\mu(x)
\]
\[= U[g_{it}U^{-1}(F_{1}^{k_{1}} \cdots F_{n}^{k_{n}})](\xi) = U[g_{it}U^{-1}F_{1}](\xi)^{k_{1}} \cdots U[g_{it}U^{-1}F_{n}](\xi)^{k_{n}}.
\]
Consequently, we obtain (4.4).

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