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Laplace Operators Associated with Hida Derivative in White Noise Analysis*

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§1. Introduction

We are greatly interested in analysis in the space of Gaussian white noise functionals, which roughly means the study in one of branches for analysis in infinite dimensional spaces in connection with the problems arising in mathematical physics. This note includes several versions of the so-called de Rham-Hodge-Kodaira decomposition theorem (DR-H-K Thm for short) associated with Hida derivative in white noise analysis or *Hida calculus*.

For a separable complex Hilbert space K, let $\wedge^p K$ be the space of exterior product of order p, completed by an equipped proper metric. $\mathcal{E} \subset E_0 \subset \mathcal{E}^*$ is the Gelfand triple, where E_0 is a given normal Hilbert space with the usual setting in white noise analysis (e.g. [17-20], [3], [4], [15]; see also [8–14] for more general setting of Hida calculus). Consider a nonnegative selfadjoint operator A on E_0 (e.g. [3], [4], [7], [15]), and we denote by the symbol Θ the linear closed operator: $E_0 \longrightarrow K$, determined regarding A (cf. [7], [13], [14]). Then the operator $\mathcal{D}_p \equiv \mathcal{D}_p(\Theta)$ from $\mathcal{P}(\wedge^p K)$ into $\mathcal{P}(\wedge^{p+1} K)$, depending on Θ, is able to be realized by making use of Hida's differential operator (e.g. [17–21]; see also [11], [12], [13]). The de Rham complex is formed by it, with the result that the corresponding Laplace operator can be constructed when we take advantage of the adjoint operator and have resort to functional analytical method. By virtue of closedness of the sequence of complexes we can obtain the DR-H-K Thm in L^2-sense (cf. [14]). Moreover it is easy to see that DR-H-K type theorem holds for the space of smooth test functionals, induced by the Laplacian: i.e.

$$H^{2,\infty}(\wedge_2^p(K)) = \mathrm{Im}[L_p(\Theta, \partial_t) \restriction H^{2,\infty}(\wedge_2^p(K))] \bigoplus \mathrm{Ker}[L_p(\Theta, \partial_t)].$$

On this account we may employ the Arai-Mitoma method [1,1991] to derive a similar type decomposition theorem even for the category $(S)(\wedge^p K)$, just corresponding to the

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space of Brownian test functionals. It is quite interesting to note that this sort of result leads to the study of Dirac operator on the BF Fock space, and also that our analysis could be another key to the supersymmetric quantum field theory. The related topics may be found in [1], [7], [13], [14], [16] and [25].

§2. Notation and Preliminaries

Let E_0 be a separable Hilbert space with the norm $\|\cdot\|_0$ in the usual setting of white noise analysis. We denote by A a nonnegative selfadjoint operator in E_0 such that the inverse operator A^{-1} is a Hilbert-Schmidt type one. We call such an operator a standard operator. $\mathfrak{C}$ is the collection of finite linear combinations of ζ_k, where ζ_k is a complete orthonormal eigenvector of the operator A. For $\xi \in E_0$, we define the norm

$$\|\xi\|_p := \|A^p \xi\|_0$$

for every $p \in \mathbf{R}$. E_p is a completion of the space $\mathfrak{C}$ with respect to the above-mentioned norm. Moreover, $\mathcal{E} \equiv E_\infty$ is a projective limit of E_p, and $\mathcal{E}^* \equiv E_{-\infty}$ is an inductive limit of E_p. Then we have a Gelfand triple

$$\mathcal{E} \subset E_0 \subset \mathcal{E}^* .$$

For a positive definite functional $C(\xi)$ on $\mathcal{E}$, the Bochner-Minlos theorem determines a probability measure μ on $\mathcal{E}^*$ such that

$$C(\xi) = \int_{\mathcal{E}^*} \exp[i\langle x, \xi\rangle]\, \mu(dx).$$

Especially when $C(\xi) = \exp[-\|\xi\|_0^2/2]$, the corresponding measure μ is called a Gaussian white noise (WN for short) measure.

Next we consider an operator $\mathcal{D}_p \equiv \mathcal{D}_p(\Theta)$ on the space of polynomials. $\mathcal{P}$ is the collection of complex-valued polynomials on $\mathcal{E}^*$ (with complex coefficients) of the form:

$$P(x) = \sum_{n=0}^{k} \langle : x^{\otimes n} :, f_n\rangle, \qquad x \in \mathcal{E}^*, \quad f_n \in \mathcal{E}_{\mathbf{C}}^{\widehat{\otimes} n}.$$

Then $\mathcal{P}$ is dense in $(L^2) = L^2(\mathcal{E}^*, \mu)$. Let K be a complex Hilbert space. For $p \in \mathbf{N}_+$, $\wedge^p K$ is the exterior product space of order p. We can define a metric in $\wedge^p K$ as follows: namely, for every $\omega, \gamma \in \wedge^p K$,

$$\langle \omega, \gamma\rangle^{\wedge^p K} = \sum_{\sigma \in \mathcal{S}_p} \mathrm{sgn}(\sigma) \cdot \prod_{k=1}^{p} \langle f_k, g_{\sigma(k)}\rangle_K,$$

where $\omega = f_1 \wedge \cdots \wedge f_p$, $\gamma = g_1 \wedge \cdots \wedge g_p$ $(f_k, g_k \in K)$. By the symbol $\wedge^p K^c$ we denote a completion of $\wedge^p K$ with respect to the aforementioned metric, with $\wedge^0 K^c = \mathbf{C}$. $\mathcal{P}(\wedge^p K^c)$ is the collection of $\wedge^p K$-valued polynomials on $\mathcal{E}^*$ of the form

$$\omega(x) = \sum_{n=1}^{k} \tilde{P}_n(x) \cdot \xi_n, \quad (x \in \mathcal{E}^*)$$

where $\tilde{P}_n \in \mathcal{P}$, $\xi_n \in A_p(\otimes^p D^\infty(T))$. A_p is an alternating operator from $\otimes^p K \longrightarrow \wedge^p K$, and $T = \Theta\Theta^*$, where $\Theta : (E_0)_\mathbf{C} \longrightarrow K$ is a densely defined, closed linear operator. We define

$$D^\infty(T) := \bigcap_{m \in \mathbf{N}} \mathrm{Dom}(T^m).$$

Note that A is then expressed by the product operator $\Theta^*\Theta$. Then it follows that $\mathcal{P}(\wedge^p K^c)$ is dense in the space $\wedge_2^p(K) := (L^2) \otimes \wedge^p K^c$. Now we are in a position to state an operator $\mathcal{D}_p \equiv \mathcal{D}_p(\Theta)$. Actually $\mathcal{D}_p$ is a linear operator from $\mathcal{P}(\wedge^p K^c)$ into $\mathcal{P}(\wedge^{p+1} K^c)$ defined by

$$\begin{aligned} \mathcal{D}_p\omega(x) &\equiv \mathcal{D}_p(\Theta)\omega(x) \equiv \mathcal{D}_p(\Theta, \partial_t)\omega(x) \\ &:= (p+1) \sum_{n=1}^{k} A_{p+1}(\Theta \cdot \partial_t \tilde{P}_n(x) \otimes \xi_n) \end{aligned}$$

for any $\omega \in \mathcal{P}(\wedge^p K^c)$, especially when the polynomial has the form $\omega(x) = \sum_{n=1}^{k} \tilde{P}_n(x) \cdot \xi_n$, and ∂_t is the Hida differential in white noise analysis (cf. [17–21]; see also [11], [12], [13], [16]).

§3. De Rham Complex Subordinate to Hida Differential

The previously mentioned operator $\mathcal{D}_p(\Theta, \partial_t)$ is well-defined for all elements of $\wedge^p K^c$-valued polynomials. Furthermore it is easy to see that $\mathcal{D}_p(\Theta, \partial_t)$ is densely defined, linear operator from $\wedge_2^p(K)$ into $\wedge_2{}^{p+1}(K)$. The range $\mathrm{Ran}(D_p)$ of $\mathcal{D}_p(\Theta, \partial_t)$ is contained in $\mathcal{P}(\wedge^{p+1} K^c)$. It is interesting to note that

$$\mathcal{D}_{p+1}(\Theta, \partial_t) \circ \mathcal{D}_p(\Theta, \partial_t) = 0$$

holds on $\mathcal{P}(\wedge^p K^c)$. Now when we set

$$\mathcal{P}(\wedge^* K^c) := \sum_{p=0}^{\infty} \mathcal{P}(\wedge^p K^c),$$

then a sequence $(\mathcal{P}(\wedge^* K^c), \{\mathcal{D}_p(\Theta, \partial_t)\})$ is the de Rham complex of $\wedge^p K^c$-valued polynomials.

The formal adjoint operator $\mathcal{D}_p^*(\Theta, \partial_t)$ is a linear operator from $\wedge_2^{p+1}(K)$ into $\wedge_2^p(K)$ defined by

$$\langle \mathcal{D}_p(\Theta, \partial_t)\omega,\ \gamma\rangle_{\wedge_2{}^{p+1}(K)} = \langle \omega,\ \mathcal{D}_p^*(\Theta, \partial_t)\gamma\rangle_{\wedge_2^p(K)}$$

for every $\omega \in \wedge_2^p(K)$, $\gamma \in \wedge_2{}^{p+1}(K)$. It follows immediately that

$$\mathcal{D}_p^*(\Theta, \partial_t) \circ \mathcal{D}_{p+1}^*(\Theta, \partial_t) = 0$$

on the domain $\mathrm{Dom}(D_{p+1}^*)$ of $\mathcal{D}_{p+1}^*(\Theta, \partial_t)$. Clearly, $\mathcal{D}_p^*(\Theta, \partial_t)$ is well-defined on $\mathcal{P}(\wedge^{p+1}K^c)$, and the domain of $\mathcal{D}_p^*(\Theta, \partial_t)$ coincides with it. Hence we may deduce that $\mathcal{D}_p^*(\Theta, \partial_t)$ is a densely defined, closed linear operator from $\wedge_2^{p+1}(K)$ into $\wedge_2^p(K)$. Thus we attain that the operator $\mathcal{D}_p(\Theta, \partial_t)$ is closable. We will denote its extension by the same symbol $\mathcal{D}_p(\Theta, \partial_t)$. Set

$$\wedge_2^*(K) := \sum_{p=0}^{\infty} \wedge_2^p(K).$$

Then we get the de Rham complex $(\wedge_2^*(K),\ \{\mathcal{D}_p(\Theta, \partial_t)\})$.

§4. Laplacian Subordinate to Hida Differential

By taking the above-mentioned results in the section 3 into consideration, we can define a Laplacian. Set

$$\mathfrak{D}(J_p) := \mathrm{Dom}\,(\mathcal{D}_p) \bigcap \mathrm{Dom}\,(\mathcal{D}_{p-1}^*)$$

as the domain of the form J_p, which is dense in $\wedge_2^p(K)$. We define

$$J_p[\Theta, \partial_t](\omega, \gamma) := \langle \mathcal{D}_p(\Theta)\omega,\ \mathcal{D}_p(\Theta)\gamma\rangle_{\wedge_2^{p+1}(K)}$$
$$+ \langle \mathcal{D}_{p-1}^*(\Theta)\omega,\ \mathcal{D}_{p-1}^*(\Theta)\gamma\rangle_{\wedge_2^{p-1}(K)}$$

for any $\omega, \gamma \in \mathfrak{D}(J_p)$. This $J_p[\Theta, \partial_t]$ turns to be a sesquilinear form on $\wedge_2^p(K) \times \wedge_2^p(K)$. Note that this formalism indicates the Laplacian $L_p(\Theta, \partial_t)$ to be roughly given by $\{\mathcal{D}_p^* \circ \mathcal{D}_p + \mathcal{D}_{p-1} \circ \mathcal{D}_{p-1}^*\}(\Theta, \partial_t)$. As a matter of fact, it is easy to see that the form J_p is a nonnegative, densely defined, closed form on $\mathfrak{D}(J_p)$. Consequently, there is a unique nonnegative selfadjoint operator $L_p(\Theta, \partial_t)$ in $\wedge_2^p(K)$ such that the equality

$$\langle L_p(\Theta, \partial_t)^{1/2}\omega,\ L_p(\Theta, \partial_t)^{1/2}\gamma\rangle_{\wedge_2^p(K)} = J_p[\Theta, \partial_t](\omega, \gamma)$$

holds for every $\omega, \gamma \in \mathrm{Dom}\,({L_p}^{1/2}) = \mathfrak{D}(J_p)$. This operator $L_p(\Theta, \partial_t)$ is a Laplacian of $\{\mathcal{D}_p(\Theta, \partial_t)\}$. We write this operator as $\Delta_{p,t} \equiv \Delta_{p,t}(\Theta)$ $(= L_p(\Theta, \partial_t))$, because it is obviously a Θ-dependent operator. Hence we can get the following decomposition theorem of the space $\wedge_2^p(K)$ in L^2-sense: that is, THEOREM 1 (*L^2-Decomposition of DR-H-K Type*; [7], [13], [14], [16]). *For any $p \in \mathbb{N}_+$, we have*

$$\wedge_2^p(K) = \mathrm{Im}[\mathcal{D}_{p-1}(\Theta,\partial_t)] \bigoplus \mathrm{Im}[\mathcal{D}_p^*(\Theta,\partial_t)] \bigoplus \mathrm{Ker}\,[\Delta_{p,t}(\Theta)] .$$

We shall state a sketch of proof below. The above decomposition assertion is valid for $p \geq 0$ with $\mathcal{D}_{-1} = \mathbf{0}$. In fact, since $\mathcal{D}_p \circ \mathcal{D}_{p-1} = \mathbf{0}$ holds for any element of $\mathrm{Dom}(\mathcal{D}_{p-1})$, it can be said that $\mathrm{Im}[\mathcal{D}_{p-1}(\Theta,\partial_t)]$ is orthogonal to $\mathrm{Im}[\mathcal{D}_p^*(\Theta,\partial_t)]$ in L^2-sense. First of all we can decompose $\wedge_2^p(K)$ as a direct sum of $\mathfrak{Y}$ and $\mathfrak{M}$, where $\mathfrak{Y}$ is a direct sum of $\mathrm{Im}[\mathcal{D}_{p-1}(\Theta,\partial_t)]$ and $\mathrm{Im}[\mathcal{D}_p^*(\Theta,\partial_t)]$, and $\mathfrak{M}$ is an orthogonal complement of $\mathfrak{Y}$. Next we have only to say that $\mathfrak{M}$ is equal to $\mathrm{Ker}\,[\Delta_{p,t}(\Theta)]$. However, it follows immediately from definitions of the sesquilinear form and kernel of operator. This concludes the assertion.

§5. DR-H-K Thm Associated with Hida Derivative

Recall that $\Delta_{p,t}(\Theta)$ is an operator in the Hilbert space $\wedge_2^p(K)$. Note that the canonical isometry

$$\wedge_2^p(K) \cong L^2(\mathcal{E}^* \to \wedge^p K^c; \mu).$$

We set

$$D^\infty(\Delta_{p,t}) := \bigcap_{m \in \mathbb{N}} \mathrm{Dom}\,(\Delta_{p,t}(\Theta)^m),$$

and define

$$\|\omega\|_k^2 = \sum_{j=0}^{k} \int_{\mathcal{E}^*} \left\|(I + \Delta_{p,t}(\Theta))^j\,\omega\right\|^2_{\wedge^p K^c}\ \mu(dx)$$

for any $\omega \in D^\infty(\Delta_{p,t})$, $p \in \mathbb{N}_+$. Then $H^{2,k}(\wedge_2^p(K))$ denotes the completion of $D^\infty(\Delta_{p,t})$ with respect to the above norm ($k \in \mathbb{N}_0$), and $H^{2,\infty}(\wedge_2^p(K))$ is given as follows:

$$H^{2,\infty}(\wedge_2^p(K)) := \bigcap_{k=0}^{\infty} H^{2,k}(\wedge_2^p(K)).$$

Now we have a complete, countably normed space $(H^{2,\infty}(\wedge_2^p(K)), \|\cdot\|_k)$. We write $\Delta_{p,t}(\Theta)(D^\infty(\Delta_{p,t}))$ as $\mathrm{Im}[\Delta_{p,t}(\Theta) \restriction D^\infty(\Delta_{p,t})]$. Suppose

$$\inf \sigma(\Delta_{p,t}(\Theta)) \backslash \{0\} > 0,$$

where $\sigma(B)$ is the spectrum of a linear operator B on a Hilbert space. Then we have

THEOREM 2 (*De Rham-Hodge-Kodaira Decomposition Theorem*; [7], [13], [16]). *Under the above assumption, the space $H^{2,\infty}(\wedge_2^p(K))$ allows the following decomposition*

$$H^{2,\infty}(\wedge_2^p(K)) = \mathrm{Im}[\Delta_{p,t}(\Theta) \restriction H^{2,\infty}(\wedge_2^p(K))] \bigoplus \mathrm{Ker}\,[\Delta_{p,t}(\Theta)]$$

for any $p \in \mathbb{N}_+$.

The proof is greatly due to the so-called "Method of heat equation". As a matter of fact, a remarkable property of our Laplacian induces existence of a corresponding positive semigroup $\mathcal{T}_s(p,\Theta)$ on $\wedge_2^p(K)$ (for $s \geq 0$). Therefore the spectral representation theorem permits an integral expression of the semigroup with respect to a spectral family $\{E(\lambda; p, \Theta)\}$:

$$\mathcal{T}_s(p,\Theta) = \int_0^\infty \exp(-s\lambda)\, dE(\lambda;\, p, \Theta).$$

By virtue of this expression and a convergence result in the general theory of integration, there exists a limit point φ_0 of $\{\mathcal{T}_s\varphi\}$ in strong topology as $s \to \infty$. Further it can be said that φ_0 belongs to Ker $[\Delta_{p,t}(\Theta)]$. When we define a bounded operator $\mathcal{Q} \equiv \mathcal{Q}(p,\Theta)$ in $\wedge_2^p(K)$ as

$$\mathcal{Q}(p,\Theta)\varphi \;=\; \int_0^\infty (\mathcal{T}_t(p,\Theta)\varphi - \varphi_0)\, dt,$$

then our assumption deduces the fact that the k-norm of Q is estimated majorantly by some constant, which depends only on the index k and the infimum of spectrum. This estimate is, however, valid even for any k, implying that $\mathcal{Q}(p,\Theta)\varphi$ lies in $D^\infty(\Delta_{p,t})$. So that, we can operate the Laplacian to it so as to obtain

$$\Delta_{p,t}(\Theta)\mathcal{Q}(p,\Theta)\varphi(x) = \varphi(x) - \varphi_0(x),$$

where computation of the integral is essentially due to the heat equation method. Thus we attain

$$D^\infty(\Delta_{p,t}) = \mathrm{Im}\,[\Delta_{p,t}(\Theta) \restriction D^\infty(\Delta_{p,t})] \bigoplus \mathrm{Ker}\,[\Delta_{p,t}(\Theta)]\,.$$

Note that $H^{2,\infty}(\wedge_2^p(K)) \cong D^\infty(\Delta_{p,t})$ as a vector space, which concludes the assertion.

§6. DR-H-K Type Decomposition in $(S)(\wedge^p K)$-Category

Recall that Θ is a densely defined, closed linear operator from $(E_0)_{\mathbf{C}}$ into K. We define the second quantization operator $d\Gamma_1(A)$ as

$$d\Gamma_1(A)(\omega(x)) = \sum_{k=1}^{n} \langle :x^{\otimes n}:, A^{\otimes I}[k] f_n\rangle$$

for $\omega \in \mathcal{P}$, $x \in \mathcal{E}^*$, where

$$A^{\otimes I}[k] = I \otimes \cdots \otimes I \otimes \overset{k}{\check{A}} \otimes I \otimes \cdots \otimes I.$$

This operator is selfadjoint on (L^2). On the other hand a nonnegative selfadjoint operator $d\Gamma_2(T)$ on $\wedge^p K^c$ is defined by

$$d\Gamma_2(T) = \sum_{k=1}^{p} T^{\otimes I}[k].$$

It follows that $\mathcal{L}(p,\Theta)$ is essentially selfadjoint if we set

$$\mathcal{L}(p,\Theta) = d\Gamma_1(A) \otimes I + I \otimes d\Gamma_2(T)$$

acting on $\wedge_2^p(K)$. Then we get a very important result, namely, for any positive p, $\Delta_{p,t}(\Theta)$ is equivalent to $\mathcal{L}(p,\Theta)$.

It is well-known that there is a unique nonnegative selfadjoint operator $\Gamma_1(A)$ on (L^2), which is described by

$$S^{-1}\left(\sum_{n=0}^{\infty} A^{\otimes n}\right) S$$

with the S-transform in white noise calculus (cf. [19], [20], [22]). For each $p \geq 0$, $\Gamma_2(T) := \otimes^p T$ proves to be, too, nonnegative and selfadjoint in $\wedge^p K^c$. Let us define

$$\Gamma_p(\Theta) := \Gamma_1(A) \otimes \Gamma_2(T),$$

and

$$|\!|\!|\omega|\!|\!|_k := \left\|(I + \Gamma_p(\Theta))^k \omega\right\|_{\wedge_2^p(K)}$$

for any $\omega \in \mathrm{Dom}\,(\Gamma_p(\Theta)^k)$, $k \geq 1$. $(S)_k(\wedge^p K)$ denotes a completion of $\mathrm{Dom}(\Gamma_p(\Theta)^k)$ relative to the above norm. Then we define

$$(S)(\wedge^p K) := \bigcap_{k=1}^{\infty} (S)_k(\wedge^p K).$$

Suppose $A \geq I + \varepsilon$ ($\varepsilon > 0$), and basically according to the idea of [1] we obtain

THEOREM 3. *Under those assumptions stated above, the following decomposition of de Rham-Hodge-Kodaira type*

$$(S)(\wedge^p K) = \mathrm{Im}[\Delta_{p,t}(\Theta) \upharpoonright (S)(\wedge^p K)] \bigoplus \mathrm{Ker}[\Delta_{p,t}(\Theta)]$$

holds for every $p \in \mathbb{N}_+$.

Although this is a direct result from Theorem 2, it is partly because our Laplacian is successfully realized as a smooth operator having a nice property. That is to say, the range of $\Delta_{p,t}(\Theta)$ restricted on $(S)(\wedge^p K)$ remains even in it.

§7. Concluding Remarks

As we have stated in the section 1 : Introduction, this formalism is possibly regarded as a key to open a new pass towards analysis of Dirac operators in quantum field theory through the framework of Hida calculus. Further analysis of Gaussian white noise functionals related to Dirac operators will be reported by the author in his next article.

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